COST-EFFECTIVE OPTIMIZATION OF RUBBLE-MOUND BREAKWATER CROSS SECTIONS

by

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February 1986
Final Report

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US Army Corps of Engineers
Washington, DC 20314-1000
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# COST-EFFECTIVE OPTIMIZATION OF RUBBLE-MOUND BREAKWATER CROSS SECTIONS

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**Prepared By:**
Civil Works Research
Work Unit 31234

**Report Date:**
February 1986

**Number of Pages:**
130

**Security Class:**
Unclassified

**Distribution Statement:**
Approved for public release; distribution unlimited.

**Key Words:**
Breakwater, Optimization
Criteria, Planning
Design, Rubble-mound

**Abstract:**
This report discusses design criteria, design procedures, and practical considerations involved in planning, design, and construction of rubble-mound breakwaters. Currently available methods for estimating rates of damage to breakwater armor and for predicting wave transmission characteristics are also described. Using information typically available today to most breakwater designers, a step-wise procedure is presented which can identify an optimum breakwater cross section, both in terms of structural integrity and functional performance.
The investigations summarized in this report were authorized by the Office, Chief of Engineers (OCE), US Army Corps of Engineers, and performed as a part of Civil Works Research Work Unit 31234, "Developing Functional and Structural Design Criteria." Funds were provided through the Coastal Structures Evaluation and Design Research and Development Program administered by the Coastal Design Branch of the Coastal Engineering Research Center (CERC) at the US Army Engineer Waterways Experiment Station (WES).

The author gratefully acknowledges the assistance of the following individuals: Messrs. Robert B. Lund and Perry Holman and Ms. Debra L. Rouse, co-op students in the Coastal Design Branch, for their help in software development; Dr. Michael E. Andrew, of the Prototype Measurement and Analysis Branch, for his assistance in the probability and statistics considerations; the staff of the Wave Research Branch, under the direction of Mr. D. D. Davidson, for their suggestions and review comments; and Ms. Shirley A. J. Hanshaw, of the Publications and Graphic Arts Division, WES, for her editing.

The work was conducted under general direction of Dr. Frederick E. Camfield, Chief, Coastal Design Branch; Mr. C. E. Chatham, Jr., Chief, Wave Dynamics Division; Mr. Charles C. Calhoun, Jr., Assistant Chief, CERC; and Dr. James R. Houston, Chief, CERC.

COL Allen F. Grum, USA, was Director of WES during the preparation and publication of this report, and Dr. Robert W. Whalin was Technical Director.
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PART I: INTRODUCTION

Objectives

1. The primary objective of this report is to introduce a systematic method by which planners and designers of rubble-mound breakwaters, specifically those in the US Army Corps of Engineers (Corps) District offices, can formulate an optimum cross-section configuration and verify its effectiveness, both in terms of structural integrity and functional performance. Rubble-mound breakwaters, the most common coastal structures worldwide, are built to provide protection from direct wave attack to boat harbors (Figure 1) and to port facilities. Recent advances in coastal oceanography have greatly improved the understanding of wave generation, propagation, and transformation into shallow water. These advances, along with greater availability of measured and hindcast wave data, have allowed procedures for design of rubble-mound structures to become much more complex than in previous years. The

Figure 1. A rubble-mound breakwater protecting a boat harbor
guidance available in the Shore Protection Manual (SPM) (1984) provides the basic tools for planning and designing breakwaters. This paper is intended to supplement that guidance by providing a practical perspective to the wide variety of environmental data now available to coastal engineers for rubble-mound breakwater design.

**Scope**

2. A brief review is presented of past and present criteria development procedures, design techniques, and related practical considerations, followed by a more detailed discussion of breakwater damage prediction and estimation of wave transmission characteristics. A systematic procedure is proposed to formulate alternative cross-section designs, evaluate their structural and functional effectiveness, and determine detailed dimensions which realize maximum net incremental benefits.

**Definition and Purposes of Rubble-Mound Breakwaters**

3. Breakwaters and, to some degree, jetties and groins are designed as barriers to sea waves, providing calmer water in their lees. Wave barriers can be constructed in many different ways, including vertical-sided concrete caissons, sheet-pile walls, wooden crib structures, and floating bodies. The oldest and most common type of wave barrier is the rubble-mound breakwater because of its typical economy and constructibility in harsh coastal conditions. The long history of rubble-mound breakwaters has proven them quite reliable in a wide range of environments (Bruun 1985). A rubble-mound breakwater consists of sloped layers of stone or concrete shapes that are sized to withstand wave attack, excess settlement or loss of fill material, and to prevent scour, as shown in the typical cross section in Figure 2. Their inherent flexibility

![Figure 2. A typical rubble-mound breakwater cross section](image-url)
tends to prevent catastrophic failure, even in the event of underdesign. The design parameters for rubble-mound breakwaters are rather inexact compared to those of most rigid civil engineering structures; thus, conservative overdesign is quite common.

4. Rubble-mound breakwaters can have a number of secondary purposes that are related to their primary purpose as a wave barrier. A breakwater protecting a harbor entrance and mooring area from wave attack might serve to divert currents and longshore transport of sediments. Also, it could be designed to provide access by people and equipment to the outer or deeper portions of the harbor. A breakwater protecting port facilities where cargo is being discharged and loaded might have these additional purposes and could even serve as a foundation for the port facilities themselves. This paper concentrates on considerations surrounding the wave barrier function. Furthermore, the perspective of the Corps as a public works agency is maintained since, in this case, the owners of the structure and the beneficiaries of its protection are the same (i.e. the taxpayers). The discussion to follow could also easily apply to a rubble-mound breakwater financed by private enterprise for commercial purposes, since tangible public benefits can, in many instances, be translated as profits. Many features of the planning and design procedures discussed later in this report can be extrapolated to planning and design of facilities other than rubble-mound breakwaters. The emphasis and most computational aspects will apply specifically to rubble-mound breakwaters intended as wave barriers.

The Need for Optimization

5. The construction cost for rubble-mound breakwaters is usually on the order of millions of dollars for smaller harbor or shore protection projects and on the order of tens of millions of dollars for larger harbor or port projects. The consequences of a dramatic structural failure include costs for repair of the breakwater which may approach the order of magnitude of the original construction costs due in part to expensive mobilization. Also, such consequences may include costs from property damage and inconvenience to port and harbor operations which occurred during the storm that damaged the breakwater. These latter costs would typically be of a lower order of magnitude than the breakwater construction costs. All of these costs of rubble-mound
breakwater failure are minimized by the tendency for this type of structure not to fail catastrophically. Catastrophic failure of flood control structures (dams and levees) causes tremendous adverse consequences for the property and people in their flood plains, often including loss of life. The costs of these consequences can easily exceed the order of magnitude of the construction costs for the flood protection. This comparison illustrates that, in comparison to some other civil engineering works, a certain small risk of failure for rubble-mound breakwaters can be tolerated.

6. Federal public works agencies in the United States have the statutory constraint for project authorization that the tangible benefits realized by the proposed plan must exceed all the life-cycle costs. This constraint has been further defined to apply to the incremental benefits and costs of each major feature of a proposed project. A rubble-mound breakwater built as a part of a federally funded project must "carry its own weight" in terms of its incremental net benefits. Recent administrative policies have provided additional restrictive criteria for federal financing of public works projects by requiring cost sharing with regional or local governments. These policies force planners to carefully consider the financeability of a project as well as its overall economic feasibility. Local sponsors of federally funded navigation projects commonly have severe limits on what costs they can share. A proposed breakwater project may be theoretically justified by a wide margin, but if it is not affordable it will not be built. Conversely, a sponsor may have the luxury of ample funding sources for cost sharing, but if a breakwater plan does not achieve enough incremental benefits, federal participation will not be possible. It is therefore critical that rubble-mound breakwaters be designed to provide the optimum trade-off between life-cycle costs and incremental benefits. This paper will deal with methods of formulating such an optimum plan without extending planning schedules and budgets beyond reason. A commitment, both in time and money, is necessary, however, to address enough key questions for systematic optimization to be possible.

Organization of the Report

7. This introduction will be followed by a review of design principles for structural stability, including some of the many practical considerations involved in rubble-mound breakwater design. Current references offering more
detailed discussions of various specific design considerations are given wherever possible, and readers are urged to consult these works. Review of design procedures is necessary in this paper to place an appropriate perspective on simplifying assumptions made in this and other discussions of optimization procedures. An introduction to a number of methods now in use to predict damages to rubble-mound structures will be presented as tools to estimate future maintenance and repair costs for a breakwater design. Similarly, a discussion of methods to predict the wave transmission characteristics of breakwaters will follow to show how the structure's functional performance may be evaluated. The main paper will be concluded with a procedure to guide planners and designers of rubble-mound breakwaters from the choice of design criteria to determination of final dimensions. Appendixes will document the software available to accomplish some steps of this procedure.
PART II: BASIC DESIGN PRINCIPLES

Design Criteria

8. There is a well-known tendency for subjective judgments to creep into supposedly systematic project planning endeavors in the earliest phases. A proven method to order your thinking in the conceptual phase of a project is to first thoroughly define the problems and opportunities at the site in terms of desirable goals to be achieved. This has long been the first step in the civil works planning process as practiced by the Corps. Two types of design criteria or "planning objectives," as stated in Corps planning guidance (Board of Engineers for Rivers and Harbors 1985 and Water Resources Council 1983), can be identified at this point relative to the function of a breakwater as a wave barrier. The first, and most familiar, is a criterion which defines the structure's ability to withstand the effects of extreme storms without itself suffering significant damages. This type of criterion can be referred to as the "structural integrity" or "survival" criterion. The second type, referred to as the "functional performance" criterion, deals with the effectiveness of the structure at its intended function which is to provide protection from waves.

9. The structural integrity criterion determines the breakwater's life-cycle costs to the extent that a certain level of investment is necessary to prevent damages from an extreme event. There will always be a finite probability that any storm, no matter how extreme, will be exceeded in intensity, so this criterion also determines the expected repair costs during the project's life. The most extreme sea state in which a particular breakwater design will suffer no damages cannot, in practice, be precisely defined, as will be discussed later. The statement of a structural integrity criterion should be phrased with this in mind. It should be stated in terms of the desired effect, that is, prevention of breakwater damages (and associated repair costs). An example would be "damages to more than 5 percent of the breakwater armor will occur with less than 2 percent probability per year." There are, of course, numerous complications in achieving such a goal, including definition of the types of possible damages and determination of the combined probability per year of the physical parameters (wave height, wave period, wave direction, water level, storm duration, and others) which could cause them.
Nevertheless, this is a workable statement in terms of an objective which is adaptable to more than one means of determining structural dimensions.

10. The functional performance criterion determines the incremental economic benefits of a breakwater design since it defines the structure's level of effectiveness as a wave barrier. It also affects the cost since a certain additional increment of investment may be necessary to achieve a given level of effectiveness. This level of effectiveness can usually be stated in terms of a maximum transmitted wave condition during a given extreme event. The probability of exceedance for this event can in turn be related to property damage and other economic losses. Probability of exceedance is usually stated in terms of any single year, but it can also be stated in terms of all or some portion of the life of the project. A workable statement of a functional performance criterion might be that "10 percent of transmitted waves in any storm will exceed 1 m with less than 5 percent probability per year." This statement assumes that "10 percent of transmitted waves" can be related to some level of unacceptable property damage or operational disruption inside the breakwater. An even more general statement might be that "navigational delays and property damages from transmitted waves shall occur with less than 5 percent probability per year."

11. Criteria of both types need to be defined for each section of the breakwater where either the environment (water depth, wave exposure, or other factors) or the required level of protection significantly differs. These sections can essentially be treated independently until a point when economy of breakwater materials, related constructibility constraints, and the transition requirements become apparent. Usually the breakwater head, any elbows, and one particular section of trunk will take precedence over other sections. Head and trunk designs do not as yet lend themselves to reliable analytical methods and typically require much subjective judgment and extensive physical modeling. Most remarks in the rest of this paper will refer to the critical trunk section, with the understanding that other less critical trunk sections may have different design criteria.

The Hudson Formula

12. Investigations into the stability of rubble-mound coastal structures were performed in the decade before the second World War by a Spanish
engineer named Cavanilles Iribarren. Iribarren (1938) presented the first widely used empirical formula for estimating a stable armor unit weight, given incident wave height, seaward slope of the structure, density of the seawater, and certain characteristics of the armor material. He assumed that stones on the outer slope were subject to gravity and wave forces, the latter of which included buoyant, impact, and friction components. The Iribarren formula was intended to predict the minimum weight stone which would remain in place when subject to waves of a given height. This height, defined in scale model tests as the level of "incipient damage," indicated that over the entire slope no more than 1-5 percent of the stones was displaced (d'Angremond 1975). The Iribarren formula is coming back into use in its original and in modified forms and will be discussed again later in this report.

13. During and after World War II, the approach of Iribarren was continued by Robert Hudson, a Corps investigator at the Waterways Experiment Station (WES) in Vicksburg, Mississippi. Hudson performed a great number of scale model tests on a variety of rubble-mound breakwater configurations. He also published a paper (Hudson 1958) which presented an armor unit weight prediction formula with many of the same features and assumptions as those of Iribarren. This formula is still in almost universal use by coastal engineers because of its relative simplicity and the many experimental and prototype tests of its reliability. The Hudson formula is

\[ W = \frac{\rho_r g H^3}{\Delta^3 K_d \cot \theta} \]  

(1)

where

- \( W \) = weight of armor unit at the level of incipient damage*
- \( \rho_r \) = mass density of the armor material
- \( g \) = acceleration of gravity
- \( H \) = incident wave height
- \( \Delta = (\rho_r - \rho_w)/\rho_w \)
- \( \rho_w \) = mass density of the water
- \( K_d \) = an empirical stability coefficient
- \( \theta \) = the angle from horizontal of the seaward slope of the structure

* For convenience, symbols and abbreviations are listed in the Notation (Appendix E).
14. Table 7-8 in the SPM (1984) presents the values for $K_d$ recommended by the Corps for use in the Hudson formula. Values are presented for a variety of quarried materials and artificial concrete shapes. Each value is associated with a number of factors, including:

a. Shape characteristics of the armor units (i.e., smooth, rough, round, or elongated rock).

b. Position of units on the trunk or head of the breakwater.

c. Wave form (i.e., whether or not the wave is breaking directly on the structure).

d. Slope or range of slopes (in some cases).

e. Method of placement (random versus special individual placement).

f. Number of layers of armor units to be placed on the slope.

g. Relative gradation and smoothness (for quarried rock).

15. An important point to note about the $K_d$ values in the SPM (1984) is that 58 percent of them were derived from monochromatic wave model test results, while the rest are interpolated values. Another factor of importance is that some of the armor unit types for which $K_d$ values are presented have actually been used in only a small number of prototype breakwaters. All of the units lack systematically documented prototype verification of their relative stability, though efforts are currently under way to consolidate historical performance of Corps constructed breakwaters. Uniform rough angular quarystone, riprap (graded rough angular quarystone), and dolosse have been most extensively tested in scale models and currently have the best documentation of prototype experience (Jackson 1968a and Carver 1983).

16. The coefficient $K_d$, as applied in the Hudson formula with its basis in the assumptions of Iribarren, does not directly account for as many as 20 or more design conditions (Ligteringen and Heijdra 1984) that are now known (in at least a qualitative sense) to affect breakwater stability. Some investigators (Brorsen, Burcharth, and Larsen 1974 and Burcharth 1979) have questioned whether the Hudson formula is reliable for predicting stability of dolosse and other slender concrete armor units. In the future, these units may require variable $K_d$ factors related to slope and other conditions not now inherent in the values presented in Table 7-8 of the SPM (1984). Some of the other conditions of concern include:

a. Influence of wave period or the steepness of individual waves (Ahrens and McCartney 1975 and Losada and Gimenez-Curto 1979).
b. Influence of wave groupiness in natural irregular seas (Burcharth 1979)

c. Effect of the foreshore or the breakwater toe on wave transformation (Bruun 1979 and Kjelstrup 1979)


e. Interaction of waves with monolithic crest elements or densely packed underlayers, such as resonance of reflected waves with incident waves (Jensen 1983).

f. Friction of outer armor material with underlayers (Hedges 1984)

g. Mechanical strength (resistance to tension, compression, impact, fatigue, etc.) of individual armor units (Poole et al. 1984 and Groeneveld, Mol, and Zwelsloot 1983).

h. Potential settlement, foundation failure, and related geotechnical problems (Thorpe 1984).

i. Seismic stability.

17. The Hudson formula can be applied to interpret scale tests of proposed designs to measure the "actual" $K_d$ of an armor unit in a particular breakwater configuration, in which case many of the above factors would be addressed. A series of successive tests on the same configuration with varying monochromatic wave period can determine the critical period when waves of that height would break directly on the face of the armor slope. Likewise, this "sensitivity analysis" approach (vary one parameter while holding others constant) can provide estimates for the reliability of the point of incipient damage and damage rates for more severe wave height and period combinations. Tests with irregular waves are also possible and should be considered, even though the procedures involved and interpretation of results in terms of Hudson formula parameters are less standardized. Physical modeling is an essential step in the cost-effective design of rubble-mound breakwaters and should not be neglected for any except the smallest, most inconsequential structures (Paape and Ligteringinen 1980). Some specific techniques for verifying armor stability and damage rates by scale model testing will be discussed later in this report.

**Alternative Stability Relations**

18. The Iribarren formula, as mentioned earlier, has recently been receiving renewed attention worldwide because of some spectacular failures of
The Iribarren formula in its original form appears as follows (d'Angremond 1975):

\[ W = \frac{N \rho g \mu^3 H^3}{\Delta^3 (\mu \cos \theta + \sin \theta)^3} \]  

where

- \( N \) = empirical coefficient related to the armor material characteristics (comparable to \( K_d \) in Equation 1)
- \( \mu \) = coefficient of static friction between individual armor units (equivalent to the tangent of the angle \( \theta \) at which armor would slide from gravity alone; values found by Graveson, Jensen, and Sorensen (1980) are presented in Table 1)

### Table 1

<table>
<thead>
<tr>
<th>Type of Armor</th>
<th>Coefficient ( \mu )</th>
<th>Angle of Repose ( \theta )</th>
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<tbody>
<tr>
<td>Round seastones</td>
<td>1.0</td>
<td>45</td>
</tr>
<tr>
<td>Quarrrystones</td>
<td>1.1</td>
<td>48</td>
</tr>
<tr>
<td>Concrete cubes</td>
<td>1.2</td>
<td>50</td>
</tr>
<tr>
<td>Concrete tetrapods</td>
<td>-1.4</td>
<td>-55</td>
</tr>
<tr>
<td>Concrete dolosse</td>
<td>-2.7</td>
<td>-70</td>
</tr>
</tbody>
</table>

19. The original Iribarren formula has only one additional parameter \( \mu \) with \( N \) being essentially equivalent to \( K_d \). This additional explicit parameter appears to have little advantage to offer, except that static friction has recently been investigated as a potentially critical factor in the overall stability of complex artificial shapes such as dolosse (Price 1979). It allows the Iribarren formula to account for the marginal stability of materials placed at their natural angle of repose. This factor might be used also in the future as a measure of seismic stability of rubble-mound structures.

20. Engineers at the Danish Hydraulics Institute (DHI) have proposed a modification to the Iribarren formula for application with scale model tests using irregular waves (Graveson, Jensen, and Sorensen 1980). This DHI-Iribarren formula is

\[ W = \frac{\rho_l \mu^3 H^2 L}{K_o \Delta^3 (\mu \cos \theta - \sin \theta)^3} \]
where

\[ H_s = \text{significant wave height of the incident irregular waves} \]

\( L_p = \text{wave length at the site corresponding to the period of peak energy density for the incident irregular waves} \)

\[ K_o = \text{alternate stability coefficient} = L_p/NH_s \]

21. The principal modification of the original Iribarren formula is the substitution of an alternate stability coefficient on the basis that the original stability coefficient \( N \) in the numerator of Equation 2 is a function of the wave steepness \( H_s/L_p \). A number of other investigators have proposed similar stability relations (Rybtchevsky 1964, Jensen 1984, and Ahrens 1984). A similar modification to the Hudson formula could be made by substituting \( K_d = K_d'H_s/L_p \). Ahrens (1984) found that stability by "reef type breakwaters," or low-crested breakwaters without traditional multi-layered cross sections (basically homogeneous rubble-mounds), was reflected with greater confidence using a modified Hudson formula with \( H_{s-p}^2 \) in the numerator than with the original Hudson formula (Figures 3 and 4).

22. Engineers at Delft Hydraulics Laboratory in The Netherlands recently performed an extensive series of scale model tests of the stability of rock slopes under random wave attack (Van der Meer and Pilarczyk 1984). These tests resulted in the formulation of a set of stability formulae for quarry-stone armor of breakwaters and revetments. Their tests also gave information on how to predict damage rates as a function of the number of incident waves. Armor layer gradation was found to have a lesser effect than that found by other investigators (Ahrens and McCartney 1975). Slope angle was found to have an effect on stability similar to that predicted by the Hudson formula. Wave period effect was investigated as a function of the "Iribarren number" or surf parameter as follows:

\[ \xi = \frac{\tan \theta}{\left( \frac{H_s}{L_o} \right)^{1/2}} \]  

(4)

where \( L_o = gT_z^2/2\pi \), based on the average wave period \( T_z \).

23. The influence of wave period was found to correspond roughly with the traditional distinction between breaking and nonbreaking waves. The effect of variations in the incident wave spectral shape, as measured in various
Figure 3. Model data plotted by Hudson stability number

Figure 4. Model data plotted by spectral stability number
ways to reflect both irregularity and groupiness, was found to be minimal. This result differs from the conclusions of other tests relative to the influence of wave groupiness on stability (Burcharth 1979). A major influence by core permeability was found. The stability formulae proposed for rubble-mound (quarrystone) structures with permeable cores ($D_{50}$ armor/$D_{50}$ core = 3.2, as tested) for breaking waves ($\xi < 2.5 - 3.5$) was

$$\frac{H_S}{\Delta D_{n50}} = 5.8 \left[ \frac{S_2}{N^{1/2}} \right]^{0.22} \xi^{-0.54}$$

(5)

or, equivalently,

$$\frac{H_S}{\Delta D_{n50}} \left( \frac{gT^2}{D_{n50}} \right)^{1/2} \tan \theta = 49 \left( \frac{S_2}{N^{1/2}} \right)^{1/3}$$

(6)

The formula proposed for nonbreaking waves ($\xi > 2.5 - 3.5$) with $\cot \theta \leq 3$ was

$$\frac{H_S}{\Delta D_{n50}} = 1.65 (\cot \theta)^{1/2} \left( \frac{S_2}{N^{1/2}} \right)^{1/6} \xi^{0.1}$$

(7)

whereas for nonbreaking waves ($\xi > 2.5 - 3.5$) with $\cot \theta > 3$ the formula was

$$\frac{H_S}{\Delta D_{n50}} = 2.86 \left( \frac{S_2}{N^{1/2}} \right)^{1/6} \xi^{0.1}$$

(8)

where

- $H_S$ = the significant wave height of the incident spectrum
- $D_{n50}$ = the nominal diameter, based on the mass of the 50th percentile $W_{50}$ from the armor material mass distribution curve
  
  $$= (W_{50}/\rho_r)^{1/3}$$

- $S_2$ = a dimensionless damage level, defined as the number of equivalent $D_{n50}$ cubes eroded over a width of $D_{n50}$
  
  - = 2-3 for incipient damage (as with the Hudson formula)
  - = 8 to 17 for armor layer "failure" (significant exposure or underlayers)
- $N$ = number of incident waves

The range of $\xi$ values from 2.5 to 3.5 for the transition from breaking to nonbreaking wave conditions apparently represents the difficulty in describing an irregular sea state as either breaking or nonbreaking, since both
breaking and nonbreaking waves can occur in the same sea state. Others who have investigated breakwater stability as a function of the surf parameter (Equation 4) include Gunbak (1976) and Losada and Gimenez-Curto (1980).

24. A relatively complete list of rubble-mound breakwater stability formulae, proposed by various investigators over the years, was published by the Permanent International Association of Navigation Congresses (PIANC) (1976). The variety of model tests and prototype experience inherent in these formulae and those developed since 1976 is but a small fraction of the many thousands of monochromatic wave tests conducted to determine Hudson formula parameters used to design hundreds of breakwaters all over the world. Use of these other stability relations should, therefore, be applied only in conjunction with traditional procedures using the Hudson formula for comparison. A conservative choice can then be made between the stable armor weights and damage rates predicted by the Hudson formula and these alternate methods.

Practical Considerations for Stability

25. The analytical methods available for predicting rubble-mound breakwater stability have been shown not to include many important considerations that could cause a structure to fail. Breakwater design has always involved a great deal of subjective judgment and probably always will. Some of the most pertinent practical considerations that must be made in determining rubble-mound breakwater material characteristics and dimensions are reviewed below. Comprehensive review of both practical and analytical considerations is available in the SPM (1984), Angerschou et al. (1983), Institute of Civil Engineers (1984), Jensen (1984), and Bruun (1985).

Incident wave conditions

26. The incident wave conditions are traditionally defined as the wave height at the seaward face of the structure with a further distinction as to whether or not the waves are breaking. This breaking versus nonbreaking criterion has been argued extensively over the years. The convention remains in practice, however, due to obvious differences in design conditions for rubble-mound structures built in shallow water, where wave heights are depth limited, and in deeper water (depth > ~15 m), where waves have not transformed to the point of breaking in front of the structure. The natural irregularity of sea states can be fairly well represented by a single height and period related to
some specified exceedance value, but it must be acknowledged that both height and period will vary in any storm. Consequently, some incident waves will be breaking on the structure, and others will not. The succession of high and low waves (wave groupiness) and of breaking and nonbreaking waves can be a critical factor. The potential effects of wave groupiness or multiple converging wave trains (multi-peaked spectra) are difficult to assess without a substantial amount of field data and scale model testing with irregular waves.

27. The alternative stability coefficients for the Hudson and Iribarren formulae discussed above which include wave steepness $H/L$ provide one means of making a more explicit description of incident wave conditions. Other descriptive parameters that have been investigated include the surf similarity parameter in Equation 4 (Bruun and Gunbak 1978, Burcharth 1979, Losada and Gimenez-Curto 1980, Van der Meer and Pilarczyk 1984, and Bruun 1985) and the Stokes or Ursell parameter $HL^2/h^3$ (Carver 1983). Estimated values of these wave form parameters can be used as a more systematic means of classifying individual waves as breaking in the critical plunging mode, as spilling, or as nonbreaking. Irregular sea states require further definition in terms of either time domain characteristics or spectral (frequency domain) parameters. A number of useful parameters for characterizing irregular waves are discussed by Rye (1977).

28. Wave transformation effects caused by the proposed construction works themselves cannot be neglected. Breakwaters with shallow slopes or with extensive toe development can also change the wave conditions at the waterline on the seaward face by "tripping" the waves. Scale-model tests are necessary to quantify these effects on the armor layer (Jackson 1968b). Relatively steep and impermeable structures may partially reflect incident waves such that resonance of incident and reflected waves causes scour near the toe. Determination of the sensitivity of a structure to these effects from oblique waves requires scale-model testing in a three-dimensional wave basin. These potential problems make physical modeling critical for reliable estimation of the stability of a proposed rubble-mound breakwater.

29. The duration of a storm at sea is a real world parameter that should be considered in any design effort or laboratory stability analysis. Figure 5 illustrates the time-history of significant wave height, peak spectral wave period, and predominant direction of wave propagation for a storm in the Gulf of Alaska simulated from synoptic weather data at 6-hr intervals.
STORM OF NOVEMBER, 1956 IN THE NORTHERN GULF OF ALASKA NEAR KODIAK, ALASKA AS SIMULATED USING THE HINDCAST DATA BASE OF THE U.S. ARMY ENGINEER WATERWAYS EXPERIMENT STATION

Figure 5. Typical time-history of wave parameters (To convert feet to meters, multiply by a factor of 0.3048)
This rise to and fall from peak conditions over many hours, sometimes days, is typical for severe storms in most areas of the world. The peak condition is typically applied in extremal analyses, but the duration of conditions above a threshold related to the stability of a proposed structure is also important. Simulation of many hours (or many thousands of waves) is performed as standard practice for breakwater stability tests by a number of prominent laboratories (Owen and Allsop 1984 and Van der Meer and Pilarczyk 1984). The effect of duration on breakwater stability is discussed by Graveson, Jensen, and Sorensen (1980), Jensen (1984) and Bruun (1985).

**Foundation considerations**

30. The weight of a rubble-mound breakwater and the hydraulic effects it causes near its foundation are potential factors which can lead to a structural failure. Investigation of gravity related stability problems, such as slip failure of the foundation or excessive (possibly differential) settlement, requires the attention of a geotechnical specialist. Hydraulic problems such as scour at the toe must be addressed in the earliest stages of design. The suitability of a natural foundation and the possibilities for preventive measures can ultimately determine the feasibility of constructing an entire breakwater. Excavation of poor foundation materials and replacement with fill or artificial improvement of the strength of natural materials can amount to a substantial fraction of the project cost. The need to place filter materials or other scour protection along a breakwater can also substantially constrain the geometry of the armor and underlayers. Seismic stability analyses in areas subject to earthquakes should be performed. All of these geotechnical considerations require extensive field data consisting of numerous borings supplemented by acoustic surveys and penetrometer tests.

**Primary armor**

31. During the past 40 years many lengthy journal articles, textbook chapters, and conference papers have been written on the subject of armor design for rubble-mound breakwaters. A discussion of the entire multitude of practical considerations applicable to armor design would be beyond the scope of this report. A comprehensive review is available by Baird and Hall (1984) in which many of the most important factors in armor design are discussed. Rubble-mound breakwaters have a tendency to be designed from the top down because the exigencies of design and construction of those portions exposed to direct wave attack tend to constrain all other features. The stability
formulae, presented as Equations 1 through 8, apply only to the resistance to displacement of individual armor units. The use of concrete armor units also requires the investigation of mechanical strength related to the interaction between the units in the armor layer and the associated impacts, fatigue, and creep (static) effects that occur. Quarrystone can be subject also to fracturing without displacement, but experience shows rock and the bulkier concrete units (such as plain or modified cubes) develop less of this sort of damage than do more slender concrete units (such as dolosse). A number of proof tests and other quality control procedures have been proposed to account for mechanical strength limitations in concrete armor units (Burcharth 1981 and Price 1979) which should be considered for application in any project involving these units. Large concrete armor units should be designed with the advice of a specialist in concrete engineering, particularly where fiber reinforcement is contemplated. The availability of existing forms should be investigated before fabrication of expensive specialized concrete forms is undertaken (Owen 1985). Design formulae indicate a minimum size armor unit, but the availability of existing forms and other practical factors may make slightly larger units more economical.

32. Design considerations related to the geometry of the armor layer are of particular interest in discussions of optimization since armor units are typically the most expensive materials used in a rubble-mound breakwater. The extent to which primary armor extends below the still-water level on the seaward face is typically set subjectively at 1.5 to 2.0 design wave heights (Figure 1). A berm of secondary armor or underlayer material at the toe of the primary armor is considered good practice, enhancing both the accuracy of underwater placement of the primary armor units and their resistance to sliding failure near the toe. The primary armor is usually extended below the waterline on the leeward side by 0.5 to 1.5 wave heights, depending on the degree of overtopping anticipated. If a monolithic wave screen is planned for construction on the crest (as illustrated in Figure 6) and virtually no overtopping is to be allowed, armor on the lee side need only be sized to remain stable in the ambient wave climate on that side of the breakwater. Wave screens and monolithic crest structures are sensitive and highly specialized features (Jensen 1983) and will not be dealt with in this paper.

33. The allowances above, along with the crest width, crest height, and number of armor units comprising the thickness of the primary armor layer,
Figure 5. Typical breakwater cross section with a monolithic crest element determine the total volume of primary armor per unit length of structure. The conditions determining these dimensions will change along the length of a breakwater. Transitions should be gradual with conservative allowances for the limited confidence in the predicted variations in design conditions. All of these considerations must account for both extreme high water conditions and the possibility of a low tide condition which could greatly complicate the stability of features near the toe.

34. The dimensions of the armor layer generally are formulated as functions of the primary armor weight. The armor thickness and crest width are related to the weight of the armor unit by the following relation:

\[ r = nK^\Delta \left( \frac{W}{\rho_{rg}} \right)^{1/3} \]  

where

- \( r \) = total average layer thickness or crest width
- \( n \) = number of armor units comprising the thickness or width (usually 2 for the thickness and 3 for the crest width)
- \( K^\Delta \) = "layer coefficient" (see Table 7-13, SPM 1984), an empirical measure of the thickness compared to that of the same number of equivalent cubes

35. The weight of the individual armor units, as determined by the Hudson formula (Equation 1), is a function of the slope, the armor material's density, the \( K_d \) factor, and the wave height cubed. A small increase in design wave height makes a substantial difference in the armor weight, i.e., a 10 percent increase in \( H \) corresponds to a 33 percent increase in \( W \). The armor thickness will increase only 10 percent. The in-place unit price of armor material (both quarystone and concrete) will vary directly with the total weight of the units relating also to the practicalities of quarry
development, concrete unit forming, and difficulties in handling. A reduced slope (increased cot θ in Equation 1) will reduce the armor weight requirement but will change the runup characteristics of the seaward face in a non-linear manner. Overtopping and the associated transmitted wave characteristics are then affected, which in turn affects the required crest elevation for acceptable wave attenuation. The overall volume of a roughly trapezoidal-shaped breakwater (in cross section) increases as the square of the increase in the crest elevation. A significant effort is therefore necessary to determine the most economical combination of slope, armor type, armor weight, and crest elevation for every pair of functional and structural design criteria, even when first cost is the only consideration.

Other breakwater features

36. The constraints involved in primary armor layer design can sometimes overshadow other considerations for design of the secondary armor layers, underlayers, core, foundation filters, and scour protection (Figure 2). The terminology of the SPM (1984) refers to a secondary armor layer as material placed on the face of the breakwater below the primary armor layer. An underlayer is placed between the armor on the exposed face and the core in the interior of the structure. Underlayers serve basically three functions: to keep the core in place through filtering action, to further dissipate wave energy that has penetrated through the primary armor, and to act as a foundation for the primary armor. These functions also apply to underlayers between the primary armor and the natural foundation (sea floor). Multiple underlayers may be required to satisfactorily accomplish all these functions. Material with small enough voids to hold finer core material in place may be too fine to stay in place itself under the larger voids in the primary armor layer. The primary armor also needs a relatively rough surface under it to discourage sliding. A coarser underlayer also provides some protection from waves during placement of the primary armor (Hedges 1984).

37. Filtering criteria developed for water quality or seepage control purposes, such as \( D_{15} \) (filter) \( \leq 5D_{85} \) (foundation) (Sowers and Sowers 1970), can be restrictive, thus the size gradation should be a consideration in evaluation of borrow areas for core material. Efficient use of quarry materials is encouraged in the SPM (1984). Given the practical problems of accurate placement of complex underlayers in the field (especially underwater), this goal may not always prove as economical as relaxing gradations of the various
layers such that their number, complexity, and associated construction quality control requirements are minimized. Unfortunately, breakwater specialists do not agree on a precise filter criterion for rubble-mound breakwater underlayers (Jensen, Graveson, and Kirkegaard 1983), and physical modeling of scour of core material is complicated by scale effects (Hedges 1984).

38. A densely packed core can reflect a significant amount of wave energy back through the underlayers and reduce the stability of the armor or increase scour near the toe of the breakwater. A core and underlayer system that reduces wave energy through turbulence and frictional loss is preferred to a more reflective system. A core that is too permeable can transmit waves as much as 80 percent of the incident wave height (Kogami 1978), and it may pass littoral materials. Some useful experiments with wave transmission through porous rubble-mound breakwaters were performed by Madsen and White (1976) and continued by Seelig (1980a). Their methods are helpful in predicting wave transmission characteristics and will be discussed again later in this report. The effects of variations in permeability are discussed further in Bruun (1985).

Head and elbow construction

39. The inevitable lateral flow across round heads and elbows and the reduced interlocking and compaction in these areas complicate just about every facet of breakwater design. Practical methods to deal with these complications consist primarily of conservative adjustments to analyses as applied to sections of the breakwater trunk. This type of adjustment has limited confidence as evidenced by the frequent need to repair heads and elbows of conventionally designed rubble-mound breakwaters. Model testing in a three-dimensional wave basin is at present the only reliable means of improving this confidence. This is particularly important with slender concrete units (such as dolosse), which may have little or no increased stability over rock or bulky units in lateral flows (Burcharth and Thompson 1982). It is this fact that has caused some investigators to question the reliability of the Hudson formula and the associated \( K_d \) factors published in the SPM (1984) for use in head or elbow design (Angerschou et al. 1983). The detail design of heads and elbows will very likely remain a highly subjective and empirical process for some time.

Toe construction

40. A number of practical problems related to the toe of rubble-mound
breakwaters have already been mentioned. This area of transition from a hopefully stable static environment (the breakwater) to the natural, often dynamic, sea floor is critical to the overall stability of the structure. Toe features not only protect the bottom from scouring (which can lead to undermining) but also support the weight of the armor material above. The need to provide primary armor 1.5 to 2.0 wave heights below the still-water level can conflict with the need to filter foundation sediments at the toe. This is particularly true in high tidal ranges where low water conditions can expose the toe to more extreme wave effects. The support of armor materials is most reliably accomplished with a substantial berm of secondary armor or underlayer material at the toe of the armor slope. This berm should have at least several units or a minimum 3-m top width. Wide differences in the size of the bottom sediments and the breakwater material near the bottom may require excavation of a trench along the toe to accommodate a toe berm with an adequate filtering underlayer, as illustrated in Figure 7. Geotextiles can be used also in some instances to reduce the height of toe features and the associated exposure to more severe wave energy. The concurrent physical modeling of armor stability and toe scour is complicated by scale effects, but model tests can reveal trends which could suggest a compromise of either the filtering criteria or the extent of primary armor. One radical concept in toe design is the "wave reducing berm" (Delft Hydraulics Laboratory 1983) which provides artificially shallow depths for dissipation of wave energy. Suggestions for design of more conventional toe features are discussed by Eckert (1983) and Jensen (1984).

Construction equipment and techniques

41. The constructibility of a rubble-mound breakwater design is an extremely important and practical consideration that can control its overall feasibility. Smaller breakwaters can often be constructed with conventional land-based construction equipment and techniques by building from the shore outward. Detached breakwaters can be constructed in this fashion only if a temporary causeway to the permanent portion is constructed and later removed. Larger or more exposed breakwaters often include features which make construction exclusively with land-based equipment difficult. For example, placement of large armor units in relatively deep water near the toe of a shallow slope (perhaps at the head) may be too far to reach for a mobile crane on the breakwater crest. Another example is the occasional need to build up
underwater features using floating equipment, particularly toe berms, prior to placement of core material, underlayers, and primary armor. The sequence of operations, specific placement techniques, and the associated equipment available to perform this work usually constrain the range of alternate breakwater configurations to some degree. No breakwater configuration should be conceived without thorough attention to its method of construction. More detailed discussions of these considerations are available in Bruun (1979), Kjelstrup (1979), Maquet (1984), and Bruun (1985).

Physical Modeling for Stability

Guiding principles

42. The specific techniques applied in physical modeling of rubble-mound breakwaters by various hydraulic laboratories differ in detail, but the guiding principles of similitude offer the same basic constraints in all cases. Scale models of breakwaters for hydraulic stability are designed according to the Froude scaling relation which requires that the Froude number of the model be equal to that of the full-scale prototype in its intended
natural setting. This relation is expressed as

$$\left( \frac{v^2}{gL} \right)_m = \left( \frac{v^2}{gL} \right)_p$$

(10)

where

- \( V \) = flow velocity
- \( g \) = acceleration of gravity
- \( L \) = a linear dimension associated with the flow

These scaling criteria provide that the linear dimensions of the model are all geometrically similar to those of the prototype. Typical rubble-mound breakwater model scales range from 1:5 to 1:70. The Froude number theoretically represents the ratio of inertial to gravitational forces, an appropriate measure in situations where gravity is the predominant force. It is widely accepted that this is usually the case for rubble-mound breakwaters (Hudson et al. 1979).

Another scaling law sometimes applies, however, which requires that the Reynolds numbers of the model and prototype be equal, or

$$\left( \frac{LV}{\nu} \right)_m = \left( \frac{LV}{\nu} \right)_p$$

(11)

where \( \nu \) is the kinematic viscosity of the fluid. The Reynolds number theoretically represents the ratio of inertial forces to viscous forces. Viscous forces in the primary armor layer, underlayers, and core are now thought to have greater importance than they did in the pioneering days of rubble-mound breakwater design. The Reynolds criterion conflicts in many instances with the Froude criterion in sizing structural materials for models (particularly in smaller, more economical models), and compromising measures are usually necessary. Other scale effects can come into play when model waves are so short that surface tension has a significant effect (seldom a real problem in practice) or when the mechanical strength of armor units is critical. Dealing with these conflicting criteria makes physical modeling of rubble-mound breakwaters a highly specialized practice. Proper execution of a rubble-mound breakwater scale model study requires both specialized equipment and extensive experience available only at a handful of hydraulic laboratories around the world.
Operational procedures

44. The representation of the sea state in scale models continues to improve in modeling facilities because of enhancements of wave generating equipment and improved understanding of the physics of water waves. The earliest wave generators were capable only of a sinusoidal motion generating monochromatic waves. The last decade has seen these facilities replaced in many laboratories with wave generators capable of producing irregular waves which simulate specified prototype energy spectra or irregular time series. The techniques for application of monochromatic waves are somewhat standardized, but presently there are widely differing opinions on the most appropriate application of irregular waves in scale models of rubble-mound stability. The transformation of the waves from deep water to shallow water must be arranged to be equivalent to that in the prototype for both monochromatic and irregular waves. The shallow-water waves of interest for stability are usually taken to be those naturally transformed waves that would exist at the site without the structure in place. This convention usually involves a calibration of the model facility before the model structure is placed in a flume or basin.

45. Complications with reflected waves arise after the structure is in place. Techniques are available for analysis of model wave data which resolve incident and reflected waves (Goda and Suzuki 1976). Some facilities are capable of compensating for reflected waves by modified motion of the wave generator. It is necessary in facilities without this capability to reduce wave reflection as much as possible by various other means. Use of irregular model waves can also result in spurious long-period waves (Jensen and Kirkegaard 1985) which must be compensated for by the generator or in the interpretation of measured results.

46. Model breakwater materials must reflect a number of prototype conditions, including geometry, density, surface roughness, and orientation in the structure. A number of recent tests have also involved attempts (the results of which remain in question) to estimate mechanical stresses within armor units (Timco 1981 and Delft Hydraulics Laboratory 1985). Geometry, density, and surface roughness are controlled by careful choice of model materials and preparation of the units. Minor density scale effects due to use of fresh water in a model of a saltwater site can usually be compensated for by small adjustments to the weight of the model breakwater units. Orientation
in the structure is accomplished with a variety of manual and automatic techniques designed to simulate the realities of full-scale field placement. The placement tolerances of model rubble-mound breakwaters often are smaller than their prototype counterparts, however. The hydraulic characteristics of underlayers and the core can be especially difficult to model by the Reynolds criteria since the shape of the units, their surface friction (particularly between layers), and the shape of the interstices are critical. Erosion of fine foundation material at the toe of breakwaters is also a problem, generally yielding only qualitative conclusions. An account of these and other scale effects is necessary for reliable interpretation of model results (Jensen and Klinting 1983).

47. Scale modeling operational procedures associated with the design of rubble-mound breakwaters can be classified in three general groups: (a) cross-section design tests run in two-dimensional flumes (Figure 8),

(b) tests of heads, elbows, transitions, offshore hydrographic effects, and oblique waves in three-dimensional wave basins, and (c) tests of breakwaters at various stages of construction (in either flumes or wave basins). The first of these is of primary interest to discussions of analytical optimization, since it is this type of scale model testing which has generated most of the analytical relations used by designers. These tests of proposed cross-section designs are intended to verify the predictions of analytical procedures and to refine detailed features of the cross section. They are often
more than a fail/no fail "proof test" of a design and should be arranged to provide the maximum information of use for similar future designs.

48. A procedure used for many years to verify the stability of proposed cross-section designs involves subjecting a model breakwater to a short series of monochromatic waves at the design (stability) wave height at various wave periods above and below the design period. The water level is also varied within the range of possible levels predicted for the prototype site. This sensitivity analysis approach is intended to reveal the breakwater's response to the severe condition when plunging breakers are directly impacting the seaward face, as seen in Figure 8. Displacement of some fraction of the armor layer is measured by before and after soundings of the model structure. This procedure is relatively economical and provides an indication of the design's resistance to armor unit displacement by a group of waves with a "worst case" combination of period and water level. Some statistical confidence is lost since the design criteria for wave period and water level are not held constant in the modeling procedure. Subsequent changes to the cross section in response to unacceptable damages in the model contribute to further departure from initial design criteria and any associated risk analysis. Design criteria must then be reformulated and associated analyses repeated with the new criteria.

49. Tests of cross-section designs with irregular waves typically involve a longer series of waves, since a significant number of waves (100 or more) are necessary to adequately resolve a specified energy spectrum. The added test condition parameters related to reconstructing a specific spectral shape in a wave flume discourage the sensitivity analysis method described above. Hydraulic laboratories differ in their approach to tests for the effect of wave groups with irregular waves, however. Some favor manipulation of spectral shape parameters to enhance wave groupiness, while others prefer spectra that are as natural as possible. Recorded spectra are reproduced in some instances to assure a completely natural incident wave condition in stability tests. Durations of individual tests also vary from relatively short tests of around 100 waves (30 to 45 min) to tests of thousands of waves and many hours simulating the growth and decline of a storm, as illustrated in Figure 5. Further discussion of model tests with irregular waves is available in Jensen (1984) and Bruun (1985).

50. Evaluation of damages after a test is a critical step which
requires special care and can involve sophisticated techniques and equipment. Color coding armor units in their initial placement is a simple way of illustrating the degree of overall displacement of the armor layer. Soundings on a small grid before and after a test will measure the overall volume of material which was moved, though net profile changes can hide more drastic gross movements which may have occurred during the test. Photographic or video procedures have been used to follow actual movements, including rocking in place, of individual units with good success (Delft Hydraulics Laboratory 1985). Detection of rocking is especially important in testing dolosse or other slender concrete armor units since it is known that they experience significant breakage in place from impacts between individual units. Testing for damage rates of these units is therefore a highly subjective process because the excessive mechanical strength of model units prevents evaluation of the stability of a design after some of the armor units have broken. Reduced strength model units (Timco 1981) may eventually provide a better means to measure stability of slender units, but model units fully similar to their prototype units in mechanical strength are not currently available. Stresses in prototype concrete armor units are far from fully understood, but research in this area is under way at most leading hydraulic laboratories.

51. A number of other characteristics are sometimes measured in conjunction with stability tests of breakwater designs, including reserve stability and wave transmission. Reserve stability refers to the extent of damage that occurs when the breakwater is subjected to waves in excess of the design condition, an important consideration in risk analyses. Wave transmission characteristics require additional tests to be fully defined, particularly when the functional performance design criterion (in terms of wave transmission) is substantially different from the structural integrity design criteria. Runup is a useful parameter to measure in conjunction with wave transmission tests, since the ratio of runup to freeboard seems to be the most sensitive parameter in analytically predicting wave transmission by overtopping. Runup is difficult to gage precisely on rough permeable slopes, and traditional visual methods are still common. Measurement of volumetric overtopping rates is also occasionally of interest, but a special setup with provisions for containing overtopped water is necessary. Techniques for measuring and evaluating the detailed relationship of runup, volumetric overtopping, and transmitted waves to incident waves in terms of wave-by-wave effects and
time series analysis parameters need a great deal of further development.

52. Tests in wave basins to determine the overall susceptibility of a rubble-mound breakwater to direct and oblique wave attack (with attention to the head and elbows), to transitions between cross sections, and to the hydrographic features offshore of the structure, are necessary for most projects. Wave basin facilities are larger and more complex than wave flumes (Figure 9) and therefore are more expensive to use. Tests of this nature not only reveal unique information about breakwater stability and other characteristics but also provide important confirmation of conclusions from flume tests. Basin models are typically at smaller scales than most flume models; thus, Reynolds scale effects are exaggerated. Basin model testing confirms the location of the most critical cross section of which more precise stability tests should be performed in a flume at larger scale. Wave transmission by diffraction through the entrance channel or other breakwater gaps is one of the most important measurements in a basin test. Long-period oscillations resulting from the enclosure of a harbor area by a breakwater are also important to detect. Model tests including tidal fluctuations can reveal circulation patterns inside a proposed breakwater (Headquarters, Department of the Army 1984).

53. The last category of breakwater model test is most important for large breakwaters requiring complex construction procedures and many months of construction time. Provisions for interim protection of partially completed breakwaters must be tested to justify what can be a significant additional cost to the project. Wave basin testing is more often appropriate for this work, but flume testing can be quite helpful also.

54. The modeling procedures discussed above are the true basis of virtually all the analytical tools available to rubble-mound breakwater designers. Quantitative measurements of prototype breakwater performance are just now becoming available and have yet to be applied toward reliable analytical design procedures. Each application of analytical procedures is an interpolation or extrapolation of limited prior experience. More often than not, refinements which reduce cost and improve performance result from model tests of a proposed design. Vital confirmation of analytical assumptions, both explicit and implicit, is provided by even the simplest model test. The expense and time are worth it in every case.
Figure 9. Three-dimensional wave basin scale model test
55. A key step in identification of an optimum among alternative rubble-mound breakwater plans is to estimate the expected damages and life cycle costs of related maintenance and repairs. The concept of designing a rubble-mound breakwater for zero damage is unrealistic because a finite risk always exists for the stability criteria to be exceeded in the life of the structure. The stochastic nature of the physical phenomena affecting coastal engineering structures requires that a probabilistic approach be applied, if these maintenance cost estimates are to be more than guesses. The incident wave climate can be characterized by estimating probability distribution functions by a number of relatively well accepted methods (Battjes 1984). The crucial problem for rubble-mound breakwater designs is in relating a given level of damage and associated repair costs to specific incident wave conditions. The rate at which this damage accumulates must also be predicted in order to tentatively schedule maintenance and related cash flows. The following section will review some techniques proposed for making these predictions. Their relative merits will be discussed and areas of ongoing or needed future research identified.

Damage Assessment

56. Damages to rubble-mound breakwaters have been quantified in many ways by researchers and field engineers. The current issue surrounding breakage of concrete armor units has led to a number of recent publications pointed at systematic assessment of damages of all kinds. One useful characterization of prototype damages in terms of displaced primary armor units was proposed by Groeneveld, Mol, and Den Boer (1984) and is presented in Table 2.

Table 2
Classification of Breakwater Damage

<table>
<thead>
<tr>
<th>Type of Failure</th>
<th>Displacement, %</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor</td>
<td>0-3</td>
<td>A few individual units of top layer displaced, but no gaps in top layer larger than 4 units; bottom layer intact</td>
</tr>
</tbody>
</table>

(Continued)
Table 2 (Concluded)

<table>
<thead>
<tr>
<th>Type of Failure</th>
<th>Displacement, %</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate</td>
<td>3-5</td>
<td>No gaps in top layer larger than 6 units; only slight displacements of bottom units</td>
</tr>
<tr>
<td>Major</td>
<td>5-30</td>
<td>Top layer removed over a large area; bottom layer over not more than 2 units</td>
</tr>
<tr>
<td>Total</td>
<td>Over 30</td>
<td>Primary armor and underlayers removed over a large area with exposure of core material</td>
</tr>
</tbody>
</table>

57. This classification of prototype damages is realistic as far as field reconnaissance of a damaged breakwater is concerned, but it departs somewhat from the convention of detecting incipient damage in model tests. It does not take into account any concrete armor units which have broken in place. This inadequacy is compensated for, in part, by the displacement of intact primary armor units being accompanied, in most instances, by concurrent displacement of broken pieces. It is the exposure and, ultimately, the erosion of underlayers and core that spell the actual failure of a rubble-mound breakwater in the functional sense, with the exception of the case when a monolithic crest element has been rendered ineffective. Field investigators should also search for evidence of other modes of failure besides hydraulic displacement, including sliding due to toe failure, excessive foundation settlement, and seismic displacements. Classification of damages as a function of both cause and effect is discussed in detail in Bruun (1985).

58. Laboratory investigations, as pioneered by Iribarren (1938) and Hudson (1958), typically attempt to identify the point of incipient damage. Kogami (1978) defined this criterion as "...the condition in which the number of armor units clearly recognized to have been moved or rocked on the cover layer surface by wave actions was less than 1% of the total of the units on the forward cover layer...." The account of rocking implies that a precise method of measuring the extent of rocking is available. Another interpretation relates to the point at which displacement has reached a depth in the armor layer equal to the equivalent cube dimension of the armor units (Losada and Gimenez-Curto 1979). Techniques developed by WES in the 1950's for measuring model breakwater displacements with before and after soundings have been estimated to have a resolution (repeatability) of ±2 percent (Carver 1983). Identification of incipient damage with this commonly used method
is therefore only meaningful in the range of 0-3 percent primary armor displacement. Nielsen and Burcharth (1983) have indicated that measurements of very low levels of displacement or rocking (0-3 percent) are less reliable than those of higher levels. This trend relates to the resolution of measurement techniques as well as the repeatability of the experimental results themselves.

59. Given a relatively consistent and precise method of measuring displacement, Ahrens (1984) has proposed a useful dimensionless parameter for systematic quantification of breakwater damage:

\[ D' = \frac{A_D}{W} \left( \frac{W}{\rho_r} \right)^{2/3} \]  

where \( A_D \) is the average eroded cross-sectional area for a specific length of model breakwater (Figure 10). Van der Meer and Pilarczyk (1984) applied the following dimensionless damage parameter \( S_2 \) in their model tests of quarrystone, which was mentioned previously (Equations 5 through 8) in the discussion of their conclusions regarding stability:

\[ S_2 = \frac{A_D}{(D_{n50})^2} \]  

It is also important to identify erosion of the underlayers or core that may coincidentally occur with erosion of the armor layer.

Analytical Damage Prediction

60. Scale model studies reported by Jackson (1968a) and Carver and Dubose (in preparation) have addressed, to a limited degree, the level of damage to breakwater armor layers experienced when the design wave height is exceeded. This information was applied to formulate Table 7-9 in the SPM (1984) which predicts the percent damage \( \%D \) for various armor types as a function of the design wave exceedance ratio \( H/H_d \) where \( H \) is a monochromatic incident wave height which is greater than the design wave height \( H_d \). The reserve stability trends, or tendency for damage levels to increase with design wave exceedance ratio, can also be characterized by a function of the following form:
TYPICAL DAMAGE PROFILE
TEST 37, $H_s = 13.76$ cm, $T_p = 3.00$ SEC

Figure 10. Definition sketch for dimensionless damage
\[
\%D\left(\frac{H}{H_d}\right) = \%D(H_d)e^{S_r(H/H_d-1)}
\]  

where

\%D(H/H_d) = damage experienced by a particular armor type from an incident wave height \( H \), given a design wave height \( H_d \)

\%D(H_d) = level of incipient damage detectable in the model tests which identifies the damage trend for a particular armor type (i.e. when \( H/H_d = 1 \))

\( S_r \) = an empirical coefficient fit to the scale model test results for a particular armor unit type

61. A higher \( S_r \) coefficient means that an armor unit type experiences higher damage levels for the same increase in \( H \). Table 3 gives the values

Table 3
Coefficients for Analytical Prediction of Breakwater Damage

<table>
<thead>
<tr>
<th>Armor Unit Type</th>
<th>Wave Condition</th>
<th>%D(H_d)</th>
<th>( S_r )</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarrystone (rough)</td>
<td>Nonbreaking</td>
<td>3.0</td>
<td>6.95</td>
<td>Jackson (1968a)</td>
</tr>
<tr>
<td>Quarrystone</td>
<td>Breaking</td>
<td>2.0</td>
<td>3.65</td>
<td>Carver and Dubose (1985)</td>
</tr>
<tr>
<td>Quadripods</td>
<td>Nonbreaking</td>
<td>3.0</td>
<td>6.00</td>
<td>Jackson (1968a)</td>
</tr>
<tr>
<td>Tribars</td>
<td>Nonbreaking</td>
<td>3.0</td>
<td>4.87</td>
<td>Jackson (1968a)</td>
</tr>
<tr>
<td>Dolosse</td>
<td>Nonbreaking</td>
<td>2.0</td>
<td>1.68</td>
<td>Carver and Dubose (1985)</td>
</tr>
<tr>
<td>Dolosse</td>
<td>Breaking</td>
<td>2.0</td>
<td>3.55</td>
<td>Carver and Dubose (1985)</td>
</tr>
</tbody>
</table>

for %D(H_d) and \( S_r \) found for the armor unit types which have been tested at WES. These coefficients may be used with caution in Equation 14 to predict breakwater damage. The variation in %D(H_d) between armor unit types reflects improvements in the accuracy of damage measurements as well as damage trends which may be related to armor unit characteristics. One or more statistical outliers representing more severe damage than predicted by Equation 14 (presumed to have been caused by weaknesses other than hydraulic stability) were excluded from the analysis of data for each armor type. The damage predicted by Equation 14 at selected levels of design wave exceedance and the associated upper 95 percent statistical confidence limit are presented in

39
Table 4. Equation 14 predicts the statistical mean trend of the experimental data. Since this is the most probable damage level for a given H/H_d ratio based on the empirical evidence available, it is appropriate for application in estimates of expected damage. Designers should be sure also to consider the damage predicted by the upper 95 percent confidence limit of the pertinent model test (as shown in Table 4) and report these predictions in their documentation of the design analysis.

Table 4
Damage Level Predictions at Selected Design Wave Exceedances
in Percent Displacement of the Armor Layer*

<table>
<thead>
<tr>
<th>H/H_d</th>
<th>Quarrystone (Nonbreaking)</th>
<th>Quarrystone (Breaking)</th>
<th>Quadripods (Nonbreaking)</th>
<th>Tribars (Nonbreaking)</th>
<th>Dolosse (Nonbreaking)</th>
<th>Dolosse (Breaking)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>3.0/24.7</td>
<td>2.0/10.2</td>
<td>3.0/18.6</td>
<td>3.0/10.4</td>
<td>2.0/4.3</td>
<td>2.0/7.4</td>
</tr>
<tr>
<td>1.05</td>
<td>4.2/25.7</td>
<td>2.4/10.6</td>
<td>4.0/19.5</td>
<td>3.8/11.1</td>
<td>2.2/4.5</td>
<td>2.4/7.8</td>
</tr>
<tr>
<td>1.10</td>
<td>6.0/27.7</td>
<td>2.9/11.0</td>
<td>5.5/20.7</td>
<td>4.9/12.1</td>
<td>2.4/4.6</td>
<td>2.9/8.2</td>
</tr>
<tr>
<td>1.15</td>
<td>8.5/29.6</td>
<td>3.5/11.6</td>
<td>7.4/22.6</td>
<td>6.2/13.4</td>
<td>2.6/4.9</td>
<td>3.4/8.7</td>
</tr>
<tr>
<td>1.20</td>
<td>12.1/33.1</td>
<td>4.1/12.2</td>
<td>10.0/25.1</td>
<td>7.9/15.1</td>
<td>2.8/5.1</td>
<td>4.1/9.4</td>
</tr>
<tr>
<td>1.25</td>
<td>17.1/38.2</td>
<td>5.0/13.0</td>
<td>13.4/28.6</td>
<td>10.1/17.3</td>
<td>3.0/5.3</td>
<td>4.9/10.1</td>
</tr>
<tr>
<td>1.30</td>
<td>24.2/45.5</td>
<td>6.0/14.0</td>
<td>18.1/33.3</td>
<td>12.9/20.1</td>
<td>3.3/5.6</td>
<td>5.8/11.1</td>
</tr>
<tr>
<td>1.35</td>
<td>34.2/55.8</td>
<td>7.2/15.2</td>
<td>24.5/39.8</td>
<td>16.5/23.7</td>
<td>3.6/5.9</td>
<td>6.9/12.2</td>
</tr>
<tr>
<td>1.40</td>
<td>48.4/70.4</td>
<td>8.6/16.7</td>
<td>33.0/48.5</td>
<td>21.0/28.4</td>
<td>3.9/6.3</td>
<td>8.3/13.6</td>
</tr>
</tbody>
</table>

* Displacement of more than 30-40 percent of the armor layer will often involve erosion of underlayers, which in practice requires a repair effort of greater scope than replacement in kind.

62. Tables 3 and 4 include predictions for only four types of armor units, two of which do not include breaking wave conditions. This is unfortunate, but it leaves the designer with no option but to apply subjective judgment to choose damage coefficients which are close to those of the most similarly shaped armor unit in the same wave conditions. Slender concrete armor units, including nearly all concrete types more complex than plain cubes, are subject to breakage in place from impacts between individual units in the armor slope. This breakage would presumably be accompanied by displacement of the broken pieces during an extreme storm. An increase in S_r of 50-100 percent would provide some allowance for this likelihood, but there are no data currently available with which to more precisely predict breakage or its
above empirical results are from monochromatic model tests of limited duration which do not account for the natural irregularity of ocean waves nor the effect of variable duration of exposure. The many untested or otherwise unresolved questions about breakwater damage modes should not, however, prevent designers from applying the information that is available. The need to confirm analytical predictions of breakwater stability and performance by scale model testing prior to construction cannot be overemphasized.

63. A characterization of damage as a function of incident wave height, with the features of Equation 14, allows the "expected" or long-term average damage to be estimated. The statistical definition of expectation for continuously distributed variables is

\[ E[x] = \int x f(x) \, dx \]  

(15)

where \( f(x) \) is the probability density function (pdf) of \( x \) (DeGroot 1975). A function of \( x \), \( g(x) \) may be substituted for \( x \) in Equation 15 without changing the definition, thus

\[ E[g(x)] = \int g(x)f(x) \, dx \]  

(16)

The long-term distribution of wave heights formulated for most current design exercises to represent the incident wave climate is derived as a cumulative probability distribution (cpd) \( F(H) \) where

\[ f(H) = \frac{dF(H)}{dH} \]  

(17)

The expected annual damage can then be estimated from a damage function \( \%D(H_d) \), such as Equation 14, and a cpd for wave heights \( F(H) \) by using the following equation:

\[ E \frac{\%D}{yr} = \lambda \int \%D \left( \frac{H}{H_d} \right) \left[ \frac{dF(H)}{dH} \right] dH \]  

(18)

where \( \lambda \) = the Poisson parameter or average number per year of extreme events represented by \( H \) values. This formulation assumes that the number of storms per year is a random variable and can be represented by a mean value. It assumes further that this number is independent of the \( H \) values.
which represent the intensity of the individual storms.

64. The availability of synoptic hindcast data base of wave data for most of the US coastline (Corson et al. 1981) accommodates the technique for formulation of $F(H)$ where only the significant wave height $H_s$ values (representing the intensity of a severe storm) above a threshold value are addressed (Battjes 1984). Recent applications of hindcast wave data at WES (Andrew, Smith, and McKee 1985) have yielded good results with a cpd function for significant wave heights above a threshold using the following extremal (Fisher-Tippet) Type I distribution:

$$F(H_s) = e^{-e^{-(e-H_s)/\phi}}$$ (19)

$$\frac{dF(H_s)}{dH_s} = \frac{F(H_s)}{\phi} e^{-(e-H_s)/\phi}$$ (20)

where $F(H_s)$ is the cumulative probability that a significant wave height $H_s$ in a sample is equal to or less than some specified $H_s$, or $P[H_s' \leq H_s]$. $\epsilon$ and $\phi$ are parameters fit to the data by regression. The traditional return period $RT$ can be estimated as (Borgman and Resio 1982)

$$RT = \frac{1}{\lambda[1 - F(H_s)]}$$ (21)

65. Another commonly applied cpd, traditionally used for annual extremes, is the following Weibull distribution:

$$F(H_s) = 1 - e^{-(e-H_s)/\phi}$$ (22)

where $C$ is an additional empirical parameter which must be fit to the data. This distribution is equivalent to a Rayleigh distribution when $C = 2$ and reduces to an exponential distribution when $C = 1$ and $\epsilon = 0$ (Petraukas and Aagaard 1970).

66. Either of these cpd functions could be applied to estimate the expected damages, given a damage function such as Equation 14. These cpd functions are typically applied to present the probability of exceedance for a specified $H_s$ (i.e. $P(H_s' > H_s)$). Assignment of a representative unit price for repair of displaced armor units allows the expected cost of
damages $E(\$/yr)$ to be estimated for a breakwater design, which is the same as the "equivalent annual amount" that might be derived by discounted cash flow analysis. An interactive FORTRAN computer program called "BWDAMAGE" has been developed at WES. This program estimates $E(\$/yr)$ given values of $D(H_d)$ and $S_r$ for Equation 14, $\varepsilon$ and $\phi$ for an Extremal Type I cpd of significant wave heights (Equation 19), representative armor repair unit prices and the volume of the armor layer. This program is documented further in Appendix D of this report. Its intended use is for comparison of alternative plans, and for this purpose the limited statistical confidence of the applied formulae is acceptable. Substitution of a measured damage function from model tests of a particular design would greatly improve the reliability of the program's estimates.

67. A number of refinements to the above scheme of analytical prediction of rubble-mound breakwater damage are conceivable. The effects of wave period and storm duration on stability have been recently investigated by a number of specialists. The effects of wave period and storm duration were incorporated directly into the stability formulae proposed for quarystone rubble-mound breakwaters by Van der Meer and Pilarczyk (1984) (Equations 5 through 8). The joint effect of wave height and period on armor damage, in the form of the surf parameter (Equation 4), and risk analysis in terms of a probability distribution of wave steepness is discussed in Bruun (1985). The DHI-Iribarren stability formula (Equation 3) directly incorporates the effect of wave period as the corresponding wave length (Graveson, Jensen, and Sorensen 1980). This latter work also addresses the effect of storm duration by focusing on the rate at which damage occurs for variations of the other stability related factors ($W$, $H$, $T$, $\cos \theta$, etc.). The following relation of damage was derived from the data of Graveson, Jensen, and Sorensen (1980):

$$D_r = 0.0622 \left( \frac{K_o}{1,000} \right)^{3.17}$$

where

$D_r$ = damage rate $D/t$, in percent armor displacement per hour

$K_o$ = the DHI-Iribarren stability coefficient

$= \rho g \frac{H^2}{p} \frac{L}{W} \frac{\cos \theta - \sin \theta}{3}$ from Equation 3

$K_o$ is a function of wave height and period, so all three parameters ($H$, $T$, $t$) are also included in the DHI approach, since
\[ D(H, T, t) = 0.0622t \left( \frac{K_o}{1000} \right)^{3.17} \]  

where \( t \) can be taken as the average duration of exceedance of \( H_{s1p} \) in Equation 3. This duration is difficult to assess in practice. Investigations of the long-term joint probability distribution \( F(H, T, t) \) are needed for a more precise definition of this parameter. If the pdf \( f(H, T, t) \) could in turn be estimated, a particular rubble-mound breakwater design could be evaluated for its expected annual damages by

\[ E \frac{D}{yr} = \lambda \iiint D(H, T, t) f(H, T, t) dH dT dt \]  

68. The practical problem in applying Equation 25 is estimating the joint pdf \( f(H, T, t) \). An interim approach to account for duration would be to assume an average \( t \) for all storms exceeding the design condition, based on evaluation of hindcast statistics or other long data records. Likewise, characteristic peak periods and water depths \( d \) can be associated with extreme storms in most cases without rigorous definition of the joint pdf or cpd. This is already common practice, since a design wave period and water surface elevation have always been necessary for accomplishment of wave transformation analyses and estimates of runup, overtopping, and wave transmission. Methods for estimating the joint long-term probability distribution of \( H \) and \( T \) are discussed by Sigbjornsson, Haver, and Morch (1976) and Ochi (1980). A practical approach to estimating expected damage by use of the DHI-Iribarren formula, given appropriate wave data, is proposed in Jensen (1984). Assumptions concerning the mean direction of wave propagation \( \phi \) and the associated directional spreading \( \sigma \) are also inherent in current practice for defining the wave climate at a site. The effect of wave direction on rubble-mound stability and damage rate is discussed by Losada and Gimenez-Curto (1982) and Christensen et al. (1984). Estimation of expected damages in terms of \( D(H, T, t, d, \phi, \sigma) \) and \( f(H, T, t, d, \phi, \sigma) \) will be possible only after much additional theoretical, laboratory, and field investigation.
69. Waves are transmitted by rubble-mound breakwaters in three ways: around, over, and through. The first way plainly refers to diffraction of incident waves around the heads of breakwaters at the entrance channel or through other gaps in the structure. Wave transmission by diffraction, the most substantial of the three modes, can be limited by careful orientation of the breakwaters. Diffractions waves combine with waves transmitted over and through a breakwater within the area influenced by diffraction. All three modes must be addressed in this area. Methods to define the limits of penetration of diffractions waves, including estimates involving directional irregular incident waves, are presented in the SPM (1984) (see also Goda, Takayama, and Suzuki 1978).

70. Many projects, such as boat harbors and ports where the breakwater is relatively extensive and the principal physical feature providing wave protection, can deal with optimization of the breakwater in plan as a separate measure. This optimization can precede the optimization of the breakwater cross section and include layout of all the other major features associated with the proposed coastal development. Procedures for systematic optimization of breakwater lengths and orientation with respect to wave penetration by diffraction are discussed by Groeneveld et al. (1983) and in EM 1110-2-1615 (Headquarters, Department of the Army 1984).

71. Rubble-mound breakwaters are designed usually with the intention that waves do not overtop the structure except in the most extreme incident wave conditions. Traditionally this has been a matter of estimating runup on the seaward face for an extreme wave height and period combination and setting the crest just above the maximum runup. This method can provide a crude approximation of crest elevation for concept formulation, but it should not be carried further into the design process. More precise techniques for estimating wave transmission by overtopping were devised by Cross and Sollitt (1971) and later refined by Seelig (1980b).
72. The height of a wave transmitted by overtopping has been found to be a function of incident wave height, period, freeboard (vertical distance from the crest to the mean water level), slope, crest width, and surface characteristics affecting runup. Water depth and bottom slope at the toe of the structure also affect wave transmission by overtopping to the extent that they affect the characteristics of the incident wave. The reflection characteristics and permeability of the structure also have an effect. Figure 11 illustrates incident wave energy being partially reflected, partially dissipated in turbulence at the seaward face, and partially dissipated by viscous effects. Energy not reflected or dissipated in these ways either passes through or over the breakwater, or both. The explicit method developed by Seelig (1980b) for predicting wave heights transmitted by overtopping is as follows:

\[ H_t = K_{to}(H_i) \]  

where

- \( H_t \) = transmitted wave height
- \( K_{to} \) = transmission coefficient (by overtopping)
  \[ = C \left( 1 - \frac{F}{R} \right) \]  
  \( C \) = an empirical coefficient
  \[ = 0.051 - 0.11 \frac{B}{h_c} \left( \text{for} \frac{B}{h_c} \leq 3.2 \right) \]
- \( F \) = freeboard
- \( R \) = potential runup, as if the seaward slope were infinitely high
- \( B \) = crest width
- \( h_c \) = total height of the crest above the sea bottom
- \( H_i \) = incident wave height

73. Runup can be estimated by a number of methods, but the method developed by Ahrens and McCartney (1975) is particularly useful for analysis by the wave transmission formula above. It is expressed as

\[ \frac{R}{H} = \frac{a \xi}{(1 + b \xi)} \]  

where \( a \) and \( b \) are empirical coefficients associated with the particular type of armor unit in place. In this case, the surf similarity parameters \( \xi \) (Equation 4) is related to the incident wave height, the equivalent deepwater
Figure 11. Conceptual illustration of wave transmission
wave length of the incident wave period, and the slope of the seaward face. Values of $a$ and $b$ have been derived by Seelig (1980a) from monochromatic laboratory data for riprap revetments (graded quarystone) on an impermeable surface, uniform quarystone on both highly permeable and conventional multilayered breakwaters, and for dolosse on conventional multilayered breakwaters. From experiments with conventional multilayered breakwaters, additional values have been fit for this report to monochromatic runup data taken by Jackson (1968b). The values of these runup coefficients are presented below in Table 5 along with the linear correlation coefficient $r$ to the data from which they were derived. The relation of the runup predicted using these units as a function of $\xi$ is illustrated in Figure 12. An important feature to note is that some armor unit types may have runup advantages over other types in that they can be more efficient energy dissipaters with respect to runup. Some of this effect may be due to variations in underlayer material size and porosity that are functions of the primary armor unit weight as well as the total depth of the primary armor. This means that breakwaters built with certain heavier

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>a</th>
<th>b</th>
<th>r</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riprap (revetments)</td>
<td>0.956</td>
<td>0.398</td>
<td>--</td>
<td>Ahrens and McCartney (1975), impermeable base</td>
</tr>
<tr>
<td>Quarystone (breakwaters)</td>
<td>0.692</td>
<td>0.504</td>
<td>--</td>
<td>Hudson (1958), highly permeable core</td>
</tr>
<tr>
<td>Quarystone (breakwaters)</td>
<td>0.775</td>
<td>0.361</td>
<td>--</td>
<td>Gunbak (1976), multilayered</td>
</tr>
<tr>
<td>Modified Cubes (breakwaters)</td>
<td>0.95</td>
<td>0.69</td>
<td>0.91</td>
<td>Jackson (1968a), multilayered</td>
</tr>
<tr>
<td>Tetrapods (breakwaters)</td>
<td>1.01</td>
<td>0.91</td>
<td>0.76</td>
<td>Jackson (1968a), multilayered</td>
</tr>
<tr>
<td>Quadripods (breakwaters)</td>
<td>0.59</td>
<td>0.35</td>
<td>0.83</td>
<td>Jackson (1968a), multilayered</td>
</tr>
<tr>
<td>Hexapods (breakwaters)</td>
<td>0.82</td>
<td>0.63</td>
<td>0.78</td>
<td>Jackson (1968a), multilayered</td>
</tr>
<tr>
<td>Tribars (breakwaters)</td>
<td>1.81</td>
<td>1.57</td>
<td>0.78</td>
<td>Jackson (1968a), multilayered</td>
</tr>
<tr>
<td>Dolosse (breakwaters)</td>
<td>0.988</td>
<td>0.703</td>
<td>--</td>
<td>Bottin, Chatham, and Carver (1976), multilayered</td>
</tr>
</tbody>
</table>
armor units may have lower crest elevations and, in some cases, less overall volume and cost. This aspect of armor unit characteristics has not been very well explored to date.

A variety of interpolation schemes based on other armor unit parameters (including stability coefficient, layer porosity, layer coefficient, and combinations of these parameters) failed to yield results similar to the a and b values directly fit to runup data. The marginal correlation of a and b for some armor units to the Ahrens and McCartney (1975) runup equation is an indication that further carefully controlled runup experiments are badly needed. Runup is difficult to measure precisely with instruments on a rough permeable slope; therefore, the above data were measured primarily by manual means. Losada and Gimenez-Curto (1980) also investigated runup on breakwaters as a function of $\xi$, which is expressed as

$$\frac{R}{H} = A(1 - e^{B\xi})$$

(30)
where \( A \) and \( B \) are empirical coefficients yielding runup trends very similar to those proposed by Ahrens and McCartney (1975). Their regression showed similar trends and correlation, as indicated in Table 6 and Figure 13.

Table 6
Runup Coefficients of Losada and Gimenez-Curto (1980)

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>( A )</th>
<th>( B )</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riprap</td>
<td>1.789</td>
<td>-0.455</td>
<td>0.96</td>
</tr>
<tr>
<td>Quartzite</td>
<td>1.451</td>
<td>-0.523</td>
<td>0.81</td>
</tr>
<tr>
<td>Quartzite</td>
<td>1.370</td>
<td>-0.596</td>
<td>0.61</td>
</tr>
<tr>
<td>Tetrapods</td>
<td>0.934</td>
<td>-0.750</td>
<td>0.74</td>
</tr>
<tr>
<td>Dolosse</td>
<td>1.216</td>
<td>-0.568</td>
<td>0.74</td>
</tr>
<tr>
<td>Quadripods</td>
<td>1.538</td>
<td>-0.248</td>
<td>0.86</td>
</tr>
</tbody>
</table>

\[
\frac{R/H}{\xi} = A \left[1 - e^{B \xi}\right] \\
\xi = \tan \theta/\sqrt{H/L}
\]

Figure 13. Relative runup versus the surf parameter (after Losada and Gimenez-Curto 1980)

75. Irregular runup can be predicted, based on either of the above relations, by applying a joint cpd for the sea state (Longuet-Higgins 1975 and Ochi 1980) to predict the runup of each wave as a function of its steepness \( H/L_0 \). This process applies the principle of equivalence, first proposed by Saville (1962), which assumes that the effects of each wave in an irregular
sea state may be represented by the effects of an equivalent monochromatic wave of the same height and length. Losada and Gimenez-Curto (1980) have applied this principle and Equation 30 to several joint distribution functions for $H$ and $T$ to derive distributions of runup that compare well with experimental data. The SPM (1984) proposes a more expedient method which assumes the runup heights will have a Rayleigh distribution. An alternative expedient method has been proposed by Andrew and Smith (in preparation) which assumes a Rayleigh distribution of wave heights and a constant wave period equal to the period of peak energy density. The resulting distribution of runup heights is not Rayleigh distributed, in keeping with the joint effect of height and period as predicted by the runup formulae above. Interactive programs written in BASIC for microcomputers are available from WES to estimate both runup and wave transmission by overtopping by this technique.* An example of transmitted wave height exceedance probabilities estimated by this method is presented in Figure 14.

76. The principle of equivalence may not remain the key to prediction

![Graph](image)

Figure 14. Predicted transmitted wave height exceedance probabilities

of wave transmission by overtopping since investigators have noted that over-
topping tends to generate waves of much shorter period than the incident wave
(Jensen and Sorensen 1979 and Jensen 1984). Investigations of wave transmis-
sion over a natural reef and associated laboratory experiments by Gerritsen
(1981) resulted in development of a theoretical approach to the redistribution
of energy that occurs with wave breaking on, and spilling over, a low-crested
or submerged reef. The transmitted waves were found to be fairly well repre-
sented as a collection of "solitons" or wave energy packets generated by inci-
dent breaking waves represented as long waves or bores (analogous to hydraulic
jumps). The phenomenon of "surf beat" or wave grouping was found to be criti-
cal to higher levels of energy transfer. The methods of Gerritsen (1981)
might yield useful results if applied to wave transmission by rubble-mound
breakwaters.

Wave Transmission Through Permeable Breakwaters

77. The tendency of wave energy to permeate through the interior of
rubble-mound breakwaters can be important for structures with relatively
coarse core material. Keulegan (1973) performed laboratory experiments of
this phenomenon which led several others to further theoretical and laboratory
investigations. Sollitt and Cross (1976) and Madsen and White (1976) devel-
oped semiempirical techniques to predict wave transmission through permeable
rubble-mound breakwaters. Wave transmission by this mode was assumed by these
authors to be a function of wave steepness $H/L$, structure permeability,
structure width, and the capacity of the structure to reflect wave energy or
to dissipate it in turbulence. The theory of long waves was applied to formu-
late expressions for wave transmission since it was assumed that the waves of
significant consequence would be much longer than the width of the structure.
Laboratory experiments indicate this as a practical assumption for most break-
water sites.

78. Madsen and White (1976) also developed a computer program for
predicting wave transmission through multilayered rubble-mound breakwaters.
This program was refined by Seelig (1980b) who successfully tested its pre-
dictions against an extensive set of laboratory results to account for com-
bined wave transmission from overtopping and permeation. It was further
modified for interactive use (US Army Engineer Waterways Experiment Station,
Coastal Engineering Research Center (WES, CERC) 1984a) and to incorporate the estimation of wave transmission by overtopping for irregular waves as proposed by Andrew and Smith (in preparation). This program, titled "MADSEN," is an extremely useful tool to analytically predict wave transmission for planning purposes where diffraction is not a significant factor.
The Principle of Optimization

79. Optimization is referred to as "trade-off analysis" in some Corps of Engineers planning guidance (Board of Engineers for Rivers and Harbors 1985) in the sense that identification of an optimum plan usually requires one desirable goal to be compromised or "traded off" against one or more other desirable goals. The basic trade-off in public works economics can be stated as a contest between minimum costs versus maximum benefits. The desired effect, such as elimination of damages by wave attack, must be balanced against the desired goal of no cost. To eliminate the remotest likelihood of damages, a structure might be astronomically expensive to build and maintain. A structure in which all but some very remote likelihood of damages is eliminated might be much more affordable. The damages or other economic losses and inefficiencies which are undesirable in their unmitigated state can be associated with a level of cost to those who are suffering the losses. The tangible benefits realized by a public works project are the sum of the incremental reductions in that level of costs directly attributable to the functional performance of the project. The construction and maintenance costs of the project are added costs to the beneficiaries, however. These project costs can be considered as negative benefits, thus the optimum plan is the combination of features which achieves the maximum net benefits. These maximum net benefits must be positive; that is, the benefits must exceed the project costs for federal participation to be possible.

An Idealized Approach

80. Figure 15 illustrates the principles discussed above in an idealized arrangement. The horizontal line labeled "user (total) cost without structure" refers to the expected annual economic losses that exist without any mitigation. This cost may increase over time due to population increase, inflation, or other factors, but it can be represented by an equivalent annual amount for the sake of evaluating project alternatives. The representative amount of economic losses without the project will be the same for each alternative. The ensemble of alternatives will individually reduce these
Figure 15. Idealized cost-effective optimization
economic losses by varying degrees. The line labeled "user cost with structure" represents the economic losses at the reduced level, and its shape in this case indicates that the alternative plans along the x-axis are ordered by increasing benefits. The benefits themselves are the difference between the "without project" and "with-project" conditions, as indicated by the line labeled "user benefits."

81. Project alternatives which are built so soundly as to preclude deterioration of any kind would obviously have a tremendous first cost. Most projects therefore accept some minimal level of predictable deterioration and associated maintenance costs in order to reduce the first costs to an affordable level. A number of authors have treated this problem as an independent matter, taking for granted that a specific level of benefits is to be achieved by all alternatives. This approach overlooks the situation in public works development in which the "user" or beneficiary is the same agency which must pay the life cycle project cost. A true optimum plan must minimize all costs, i.e., the economic losses and the structure life cycle costs.

82. Additionally, Figure 15 shows the hypothetical ensemble of alternatives to be ordered in terms of increasing first cost, as indicated by the shape of the line labeled "structural first cost." The increasing first costs are taken in this idealized representation to correspond to reduced maintenance liability as indicated by the line labeled "structural maintenance cost" which slopes in the opposite direction. The sum of these costs for each alternative is shown as "total structural cost," with a minimum in the vicinity of alternative 5. The sum of the total structural cost and the user cost with structure is shown as the line labeled "total cost with structure." This line dips below the "user cost without structure" line at a point where the benefits first exceed the costs. The region where benefits exceed the costs has been shaded and labeled as "project feasibility." The alternative with the maximum vertical spread in this shaded area has the maximum net benefits, indicated by the optimum point on the line labeled "net benefits." This point corresponds to the point of minimum total cost with structure, somewhere around alternative 9.

83. It is useful to note that the optimum can be identified without knowledge of the without-project condition. Port and harbor projects are often justified in terms of transportation savings over some alternate route or through some other existing port. The user cost with project would in these
cases be compared to the user cost through the alternate route, for definition of project benefits. Whatever economic philosophy or administrative policy is applied, an estimate of tangible economic benefits must be made. This estimate, either with the "user cost with project" or with the benefits themselves, can be applied in the manner of Figure 15 to optimize the major features of the project.

84. The idealized nature of Figure 15 is useful to illustrate the concept of cost-effective optimization, but it is misleading in its implication that a set of alternatives will follow such a smooth comparison of costs and benefits. A typical set of plans could not, in most cases, be ordered by both increasing benefits and first cost. Neither is it the case that increasing first cost always means reduced maintenance. The two types of design criteria--functional performance and structural integrity--are essentially independent of each other, and both have an effect on first cost. Practical applications require that an ensemble of alternatives be compared without reference to the order of their benefits, first cost, and maintenance cost. A systematic approach to criteria development as a means of initially identifying alternatives is important in this respect. An alternative is thus known in the optimization process by its governing design criteria rather than its resultant physical features.

A Practical Approach

85. The analytical and practical aspects of rubble-mound breakwater design have now been reviewed. The discussion above concerning the principles governing optimization indicates that the first cost, maintenance cost, and user cost with project must be estimated for each plan in a set of alternatives. The total costs with project need to range at close intervals from well below to well above the unknown minimum for reliable identification of the optimum alternative. A procedure is proposed below that accomplishes this optimization exercise using information commonly available and already incorporated in most coastal engineering planning and design efforts. Potential future refinements are mentioned where appropriate.

Step 1--define site conditions

86. The physical conditions and other constraints affecting the design of a rubble-mound breakwater, such as water level, tidal currents, foundation
characteristics, and wave climate, must first be quantitatively defined. It is assumed that many of these conditions have been already defined in a master planning effort which identified the tentative need for a breakwater and its most promising alignment. The water level and wave climate are the most critical considerations for this optimization procedure, specifically the estimation of the annual cumulative probability distribution \( F(H_s, T, t, \phi, \sigma, d) \). Current practices typically require planners and designers to estimate \( F(H_s) \) for a limited range of wave directions \( \phi \) affecting the site of the breakwater or, at best, to define a wave rose and then deal with the marginal distribution of wave heights for one sector. Design values of wave period \( T \), storm duration \( t \), directional spreading \( \sigma \), and depth \( d \) associated with a given wave height are typically subjectively determined. The mathematical estimation of \( F(H_s, T_p) \) is becoming more common, however, with the availability of hindcast data bases of wave information (Corson et al. 1981).

87. The Extremal Type I distribution for \( F(H_s) \), based on \( H_s \) values above an extreme threshold value, is recommended in this procedure for designs where hindcast information or other comparably long records of wave data are available. A Weibull distribution of extremes is a workable alternative. Application software program WAVDIST for estimating Extremal Type I and Weibull significant wave height distributions has been documented in a Coastal Engineering Technical Note (CETN),* and an example of its use in a design problem is presented in Andrew, Smith, and McKee (1985).

Step 2--estimate expected economic losses

88. Estimation of losses or "user costs" due to wave attack requires derivation of a site-specific relation in which losses are a function of incident wave height. The typical harbor mooring area or cargo transfer area is unaffected by waves below a certain height \( H_{LO} \) which might be on the order of 1 m. The total disruption of the port or harbor area at the other extreme, by the worst conceivable wave attack, is also possible to estimate as a practical upper limit to losses \( $L_{\text{max}} \). These two values are useful in that they

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* US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, in preparation, "Estimation of Extremal Significant Wave Height Distributions: Computer Program WAVDIST (MACE-17)," Coastal Engineering Technical Note, Vicksburg, Miss.
do not require historical information for their estimation. They can be based on a current engineering and property valuation assessment of the facilities to be protected. Historical information relating specific levels of economic loss (in dollars) to the measured or hindcast wave height of the associated storm can then be used to derive a function of the form, as follows:

\[ \$L(H_s) = \frac{1 - e^{-A(H_s - H_{Lo})}}{\text{max}} \]  

(31)

where \( A \) is a coefficient determined by regression. This function, illustrated in Figure 16, can then be used to estimate the expected annual economic losses, or user costs without project, according to Equations 16 and 17 by

\[ E(\frac{\$L}{\text{yr}}) = \lambda \int \begin{array}{c} \$L(H_s) \left( \frac{dF(H_s)}{dH_s} \right) \end{array} dH_s \]  

(32)

where \( F(H_s) \) is the cumulative probability distribution of significant wave heights derived in Step 1. A joint distribution \( F(H_s, T) \) should be applied where operations and facilities are particularly sensitive to a certain range of periods. Software has been developed to estimate both \( \$L(H_s) \) and \( E(\frac{\$L}{\text{yr}}) \) given an estimate of \( \$L_{\text{max}}, H_{Lo}, \) Extremal Type I \( F(H_s) \) coefficients, and at least one historical data point \( [H_s, \$L(H_s)] \). The program

Figure 16. Economic loss function versus incident wave height
"BWLOSS1" has been documented in a CETN*, and a sample is provided in Appendix A of this report.

Step 3--formulate ensemble of alternatives

89. This step is highly subjective and will control the scope of the overall optimization effort since it determines the number of individual alternative breakwater configurations that must be investigated. The application of practical judgment can reduce this number, but too few alternatives or a conservative bias could also preclude identification of an optimum plan. A proposed method of organizing an ensemble of alternatives is illustrated in Table 7.

Table 7
Selection of Alternative Design Criteria for Return Periods of Storms Causing the Stated Conditions

<table>
<thead>
<tr>
<th>Functional Performance</th>
<th>Structural Integrity</th>
</tr>
</thead>
<tbody>
<tr>
<td>x% ( H_t &gt; H^* )</td>
<td>%D ≤ %D(( H_d ))</td>
</tr>
<tr>
<td>≤ 10</td>
<td>≤30, 40, 50, 60, 70, 80, 90, 100</td>
</tr>
<tr>
<td>20</td>
<td>≤30, 40, 50, 60, 70, 80, 90, 100</td>
</tr>
<tr>
<td>30</td>
<td>≤30, 40, 50, 60, 70, 80, 90, 100</td>
</tr>
<tr>
<td>40</td>
<td>≤30, 40, 50, 60, 70, 80, 90, 100</td>
</tr>
<tr>
<td>50</td>
<td>≤30, 40, 50, 60, 70, 80, 90, 100</td>
</tr>
</tbody>
</table>

90. Table 7 lists a comprehensive set of potential functional and structural design criteria combinations which may be abbreviated by carefully considered subjective judgments. The first column in Table 7, "functional performance," refers to an exceedance value \( x\% \) of transmitted wave heights \( H_t \) greater than some critical wave height \( H^* \). \( H^* \) might conveniently be taken as the \( H_{LO} \) value applied in the loss function of Step 2, but this is not necessary. This column includes a range of functional performance design criteria which could be addressed in terms of wave transmission. A wave height of 1 m, for example, might be a threshold value for damage to vessels.

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* US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, in preparation," Estimation of Expected Annual Economic Losses Due to Wave Attack--Computer Program BWLOSS1 (MACE-15)," Coastal Engineering Technical Note, Vicksburg, Miss.
moored behind the breakwater. The last functional performance criteria would thus be that \( x \% \) of the waves transmitted by the breakwater during a 50-year storm would be in excess of 1 m. The return period convention is in keeping with traditional practice, though the phrase "with 2 percent probability per year" would be a more accurate description of the storm of interest. Estimated probability per year might be a more appropriate increment in terms of providing even steps of cost between alternatives, but either convention will serve. The value of \( x \% \) should relate to some consideration of the actual number of waves of \( H^* \) or greater necessary to cause a measurable effect. A storm whose peak conditions lasted 3 hr with \( T_p = 10 \text{ sec} \) would include roughly 1,080 waves. A small value of \( x \% \) is appropriate, on the order of 1 percent, which for the example condition would include 10 or 11 waves. These waves would not likely occur in sequence, but a few of them might.

91. The shorter return periods of 20 or \( \leq 10 \) years might be too risky for a small boat harbor where relatively fragile vessels and mooring facilities are planned immediately on the lee of the breakwater. These criteria are reasonable, however, when losses due to cargo handling inefficiencies or vessel transit time are all that is at stake. The 50-year storm is, on the other hand, a very conservative criterion for wave transmission. At least four functional performance criteria should be addressed to assure identification of an optimum design.

92. The second column of Table 7 includes choices for structural integrity criteria in terms of the damage to the armor layer, as might be estimated by Equation 14. The \%D(\( H_d \)) value chosen should be consistent with the incipient damage level, as measured in model experiments pertinent to the breakwater design at hand. \( H_d \) is the wave height applied in analytical stability relations. Return periods of 30 years or less for the storm represented by \( H_d \) will plainly involve substantial expected damage and therefore should be investigated only for minor breakwaters where repairs can be easily accomplished or postponed without significant adverse consequences. Long return periods greater than 50 years are important to address, however, since rubble-mound breakwaters require such a tremendous commitment of equipment and materials to repair. The risk of affordable quarrrystone being unavailable 30 or 40 years in the future might be great, even though it may be readily available at present. Repair of breakwaters in remote areas involves high mobilization and demobilization costs, even for small repair efforts.
Another important consideration in favor of addressing these longer return periods is the uncertainty of the future funding capacity of local sponsors for repair efforts.

93. The final choice of alternatives should contain a minimum of 15-20 pairs of functional performance and structural integrity criteria pairs. A single pair of these criteria will define each alternative breakwater configuration throughout the optimization process. Consistency in application of these criteria in analytical design efforts is critical to maximizing the reliability of the procedure. New alternatives should not be added without carrying the new ones through the entire procedure.

Step 4-identify apparent optimum combination of armor size and type, slope, and crest elevation for each alternative

94. Each pair of design criteria will have several combinations of features that will provide the same performance and stability. An acceptable method of choosing an apparent cost-effective combination for each plan is to consider a standard parameterized cross section, as illustrated in Figure 17. The Hudson formula (Equation 1), the relation of armor thickness and crest width to armor weight (Equation 9), and the wave transmission relations (Equations 26 through 29) can then be used to approximate all the dimensions of this standard cross section for a range of armor type and slope combinations. The relative advantages of armor unit hydraulic stability and of runup dissipation are both measured by this approach. The relative cost per unit length of breakwater trunk for each slope and unit type combination can also be estimated by incorporating representative unit prices for each armor type and size.

95. This method does not deal with the variation of reserve stability between armor types which would involve a substantial amount of extra input and computational effort. The question of reserve stability is addressed later in this proposed procedure, but at this stage it is neglected as a time-saving measure. "BWCOMP," an interactive computer program, has been developed to estimate the volume and first cost per unit trunk length of the parameterized cross section of Figure 17, given the two design criteria (as incident $H_s$ and $T_p$ values and an $H^*$ maximum transmitted height) along with the other information discussed above. The program is documented in Appendix B of this report and in a CETN (WES, CERC 1984b).
Step 5--design detailed cross section for each alternative

96. This is the second highly subjective step in the proposed optimization procedure where coastal engineers should, for each pair of design criteria, prepare a cross-section design with all the detailed features appropriate for the site conditions and other constraints. Practical considerations discussed in Part II of this report should be incorporated. All the specialized experience and intuition available should be applied in this step, but it must be applied consistently to each alternative. It is critical that bias be studiously avoided at this stage. An estimate of the construction cost for each alternative detailed cross section should be prepared at the conclusion of this step.

Step 6--estimate wave transmission characteristics of each alternative

97. An analytical procedure should be performed at this point to estimate the wave transmission characteristics as a function of incident waves $H_t(H_i)$ for each alternative. The program MADSEN (Seelig 1980a and WES, CERC 1984a) is useful for this purpose. The program accounts for the relative size and permeability of each layer of the breakwater cross section and the relative runup characteristics of the armor layer. Wave transmission by overtopping (Equations 26 through 29) and permeation (Madsen and White 1976) is estimated. The program is not as well verified for concrete armor units as for quarystone, but it serves well at this stage for comparative purposes. A range of incident wave conditions should be simulated to obtain a substantial set of $H_t(H_i)$ points, including several more severe than the design condition. The incident wave conditions need to correspond to height and period combinations predicted for the site in Step 1. Wave period is a sensitive factor for wave transmission, as applied in the program MADSEN. An appropriate wave period (such as the peak spectral period $T_p$) must therefore be associated with each (significant) incident wave height, as suggested in Step 1. Transmitted waves are not Rayleigh distributed, as discussed in Part IV and Andrew and Smith (in preparation), but can be represented by a single height such as the root mean square wave height $H_{rms}$ or $H_{13.5}$. MADSEN predicts the $H_{rms}$ of waves transmitted by the combined effects of both permeation and overtopping.
Step 7—estimate economic losses with the breakwater for each alternative

98. The climate of transmitted waves behind the breakwater can now be approximated as a cumulative probability distribution \( F(H_t) \) given a set of \( H_t(H_i) \) points from Step 6 and the cumulative distribution of incident waves \( F(H_i) \) from Step 1. The loss function estimated in Step 2 can be used to estimate the expected annual economic losses \( E\{\$L'/yr\} \) for each alternative by

\[
E\{\$L'/yr\} = \lambda \int \$L(H_t) \frac{dF(H_t)}{dH_t} dH_t
\]

(33)

"BWLOSS2," a computer program, has been developed to perform these computations. It has been documented in a CETN*, and it is included in Appendix C of this report.

Step 8—estimate expected annual breakwater damages for each alternative

99. The methods discussed in Part III can be applied to relate a damage function \( \%D(H/H_d) \) to each alternative. The incident wave climate defined by \( F(H) \) from Step 1 can in turn be applied to estimate the expected annual damages \( E\{\$D/yr\} \) given representative unit repair prices \( \$/vol \) and the volume of the armor layer \( Vol \) by adapting Equation 18 as follows:

\[
E\{\$D/yr\} = Vol \frac{\$}{vol} \lambda \int \%D(H/H_d) \left[ \frac{dF(H)}{dH} \right] dH
\]

(34)

100. This quantity is useful for comparative purposes, but it does not relate directly to a programmed cash flow for repairs. It is better that Equation 18 be applied to each alternative in its unmodified form to predict the expected annual \( \%D \) in order to make some judgment if and when a repair project should be scheduled. The average time to reach a threshold level of unacceptable damage \( \%D^* \) can be estimated by simply dividing that value by \( E\{\%D/yr\} \). The return period of \( \%D^* \) could also be estimated by solving for \( H(\%D^*) \) in the damage function (Equation 14) and applying Equation 21 to determine the associated return for that particular storm intensity. A computer program titled

* US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, in preparation, "Estimation of Expected Annual Economic Losses from Waves Transmitted by a Breakwater--Computer Program BWLOSS2 (MACE-16)," Coastal Engineering Technical Note, Vicksburg, Miss.
"BWDAMAGE" has been developed which applies Equation 14 and the information of Table 3 to estimate $E\{\$D/yr\}$, $E\{\$D/yr\}$, and the repair interval by both methods discussed above. This program has been documented in a CETN,* and it is presented in Appendix D of this report. Once a repair interval and the associated extent of repairs have been estimated for an alternative, discounted cash flow methods can be used to estimate the equivalent annual amount which can be substituted for $E\{\$D/yr\}$. The damage functions, as stated in Part III, are currently the least reliable of the analytical tools available for rubble-mound breakwater design and should be used with circumspection.

Step 9--tabulate and sum costs for each alternative

101. This is the final analytical step of the proposed procedure, followed only by laboratory verification of the analytical predictions. The minimum sum of the three costs identifies the cost-effective optimum alternative, as indicated in the following equation:

$$E\left\{\frac{\$Total}{yr}\right\} = E\left\{\frac{\$L'}{yr}\right\} + E\left\{\frac{\$1^{st}}{yr}\right\} + E\left\{\frac{\$D}{yr}\right\}$$  \hspace{1cm} (35)

The first cost must be transformed from a present worth value to an equivalent annual amount $E\{\$1^{st}/yr\}$ by discounting prior to the summation. Incremental benefits $E\{\$B/yr\}$ can be estimated by subtracting $E\{\$L'/yr\}$ from $E\{\$L/yr\}$:

$$E\left\{\frac{\$B}{yr}\right\} = E\left\{\frac{\$L}{yr}\right\} - E\left\{\frac{\$L'}{yr}\right\}$$  \hspace{1cm} (36)

Net benefits $E\{\$B_{net}/yr\}$ can in turn be estimated by subtracting $E\{\$1^{st}/yr\}$ and $E\{\$D/yr\}$ from $E\{\$B/yr\}$ as follows:

$$E\left\{\frac{\$B_{net}}{yr}\right\} = E\left\{\frac{\$B}{yr}\right\} - E\left\{\frac{\$1^{st}}{yr}\right\} - E\left\{\frac{\$D}{yr}\right\}$$  \hspace{1cm} (37)

This method of estimating benefits may not be appropriate for some projects, however, as discussed at the beginning of Part V.

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* US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, in preparation, "Estimation of Rubble-Mound Breakwater Expected Damages--Computer Program BWDAMAGE (MACE-18)," Coastal Engineering Technical Note, Vicksburg, Miss.
Step 10--verify by physical modeling of damages and wave transmission of apparent optimum alternative

102. This step is necessary to assure that all the compounded assumptions and analytical inaccuracies are within acceptable limits. This is the case with any analytical design procedure for rubble-mound breakwaters since the empirical relations have been shown to all have limited confidence. Each laboratory test of analytical assumptions applied to a specific design will narrow the confidence limits and improve the reliability of future analytical efforts. A simple proof test with monochromatic waves of varying period constitutes a minimum effort in this direction, but it is inadequate to test the accuracy of an optimization procedure such as that proposed above.

103. The damage function \( \%D(H/H_d) \) must be verified by model testing, including simulation of conditions for incipient motion and a range of more severe conditions. The design conditions should be simulated as accurately as possible in order to include the effects of the numerous physical parameters not explicit in the analytical stability formula that was applied. Wave period, wave groupiness, storm duration, and static stability, among other factors, should be considered. The static friction factor \( \mu \) from the Iribarren formula (Equation 2) should be measured by sliding tests, as proposed by Price (1979) and Graveson, Jensen, and Sorensen (1980).

104. The fully described incident wave conditions cannot be simulated with monochromatic waves. Either an average (for example JONSWAP) spectral shape or one adjusted to be similar to measured spectra for extreme storms near the site can be applied in flume tests of the apparent optimum cross section. Simulation of a gradual rise to peak conditions, then 1,000 waves or more at the peak (stability criterion) condition, followed by a gradual decrease of wave energy, would be most useful for tests to verify damage functions. A test or tests at the design condition should be followed by tests at more extreme conditions related to the extremal distribution of wave heights (and periods) derived in Step 1. Enough \( \%D(H/H_d) \) points must be measured to verify or refine the \( \%D(H/H_d) \) analytical function that was applied in Step 8. A minimum of three tests would be useful, including the \( \%D(H_d) \) point and at least two more severe conditions. More stability tests should be conducted if agreement with the predicted damage function is not good. Techniques to detect gross rocking motion should be applied in identifying
incipient motion. Actual damage should be measured by before and after soundings on a fine grid, but some judgment must be made as to the additional damage that might have occurred in prototype from armor unit breakage.

105. Wave transmission characteristics of the apparent optimum cross section must also be verified. Tests of these design conditions simulating the fully described forecast conditions at the site as accurately as possible should be performed for the functional performance criteria and a number of more extreme conditions. Again, these extremes should relate to the \( F(H_s, T_p) \) derived in Step 1. At least three \( H_{st}(H_{si}) \) points should be measured in order to verify or refine the economic loss function derived in Step 7. Operational techniques should include efforts to accurately model reflection and wave transmission by both overtopping and permeation. Transmitted waves should be measured as time series comparable to time series measured of incident waves. Coherence and cross-correlation analyses should be performed for the incident and transmitted time series along with computation of more common spectral parameters. Individual runs of 100 or more waves are recommended for the wave transmission tests in keeping with the widely accepted assumption of stationarity in natural sea states.

106. The measured \( \%D(H/H_d) \) data and \( H_t(H_i) \) data should be applied in Steps 6 through 9 for the apparent optimum cross section. All its associated costs should then be adjusted according to the revised expected damages and economic losses with the breakwater in place. Model tests often make significant refinements to a design cross section obvious, and any such refinements should be incorporated. Drastic changes to the original apparent optimum cross section may require similar changes to be made to all the alternatives and for Steps 5 through 9 to be repeated for these cross sections as well. If the original apparent optimum is still indicated as the optimum cross section, then no further model testing will be necessary. A new apparent optimum should have its \( \%D(H/H_d) \) and \( H_t(H_i) \) functions verified in as thorough a manner as the first.
PART VI: SUMMARY AND CONCLUSIONS

Summary

Optimization

107. Optimization has been demonstrated as a systematic process of maximizing net tangible economic benefits or of minimizing the total costs (including economic losses) to the beneficiaries of a public works project. Optimization of rubble-mound breakwaters addresses the incremental net benefits of these structures which are often major features of a larger coastal development. Federal laws and policies currently require that incremental net benefits be positive for all major features of projects proposed for federal funding. Furthermore, cost sharing policies have placed a substantial burden for financing these projects on local and regional governments. Financeability of civil works projects is now an important question outside that of positive net benefits. Rubble-mound breakwaters must achieve the maximum benefits for the least cost in order to be affordable as well as economically feasible. Arbitrary conservatism in design of rubble-mound breakwaters is no longer affordable, and coastal engineers must use all the tools and information available to assure the optimum alternative has been proposed.

Design criteria

108. Alternatives for rubble-mound breakwaters should be optimized according to two criteria: functional performance and structural integrity. The functional performance criterion refers to the structure's effectiveness as a wave barrier as measured by its wave transmission characteristics. The structural integrity criterion refers to the structure's ability to survive an extreme storm without significant damage and the rate it suffers damage from storms more extreme (less probable) than the structural design event.

Analytical design and laboratory verification

109. The analytical tools available to designers of rubble-mound breakwaters have been reviewed in some detail. They have all been shown to be the products of a finite set of laboratory experiments, with very little quantitative prototype verification. Current research continues to refine the precision of these empirical relations, but this precision is not yet sufficient to warrant construction of rubble-mound breakwaters without verification of
analytical predictions by scale model tests. Nevertheless, analytical procedures are available for prediction of armor unit hydraulic stability (resistance to displacement by waves), armor layer damage rates, and breakwater wave transmission characteristics. These tools, with laboratory verification, can be used to systematically select an optimum alternative.

The proposed procedure

A systematic optimization procedure has been proposed which makes use of the analytical tools currently available to coastal engineers for rubble-mound breakwater design. The procedure begins with definition of the site conditions and formulation of an ensemble of alternative design criteria pairs. These steps are followed by estimates of first costs, maintenance costs, and user costs with the breakwater in place for each alternative. The concept of statistical expectation is applied to measure the costs of all alternatives on the same basis. The process is concluded by physical model tests to verify the analytical predictions for structural stability and wave transmission characteristics of the apparent optimum alternative. The entire procedure is summarized in Table 8, with references to pertinent formulae, software, and documentation.

Table 8
Summary of Optimization Procedure

<table>
<thead>
<tr>
<th>Step</th>
<th>Procedure</th>
<th>Pertinent Equations and Tables</th>
<th>Available Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Define site conditions</td>
<td>Equations 19* or 22*</td>
<td>WAVDIST (WES, CERC (in preparation))</td>
</tr>
<tr>
<td>2</td>
<td>Estimate economic losses without breakwater</td>
<td>Equations 31* and 32*</td>
<td>BWLOSS1 (WES, CERC (in preparation) and Appendix A)</td>
</tr>
<tr>
<td>3</td>
<td>Formulate an ensemble of alternative functional and structural criteria pairs</td>
<td>Table 6</td>
<td>--</td>
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</table>

(Continued)

Note: * indicates the equations which are applied in the referenced software.
Table 7 (Concluded)

<table>
<thead>
<tr>
<th>Step</th>
<th>Procedure</th>
<th>Pertinent Equations and Tables</th>
<th>Available Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Identify optimum armor, type W cot 0, and crest elevation for each alternative</td>
<td>Equations 1* (or 2-8), 29* (or 30), and Table 5* or 6</td>
<td>BWCOMP (WES, CERC 1984b and Appendix B)</td>
</tr>
<tr>
<td>5</td>
<td>Design detailed cross section for each alternative</td>
<td>Equations 1-9</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>Estimate wave transmission characteristics of each alternative</td>
<td>Equations 19 or 22, 26*, 27*, 28* and 29* (or 30) and Table 5* or 6</td>
<td>MADSEN (Seelig 1980a and WES, CERC 1984a)</td>
</tr>
<tr>
<td>7</td>
<td>Estimate economic losses with breakwater for each alternative</td>
<td>Equation 19* or 22, 31*, and 33</td>
<td>BWLOSS2 (WES, CERC (in preparation) and Appendix C)</td>
</tr>
<tr>
<td>8</td>
<td>Estimate breakwater damages for each alternative</td>
<td>Equations 19* or 22, 14*, (or 5, 6, 7, and 8 or 23 and 24), 18*, and 34* and Table 3*</td>
<td>BWDAMAGE (WES, CERC (in preparation) and Appendix D)</td>
</tr>
<tr>
<td>9</td>
<td>Tabulate expected costs for each alternative and identify apparent optimum</td>
<td>Equations 35, 36, and 37</td>
<td>--</td>
</tr>
<tr>
<td>10</td>
<td>Verify predicted damage and wave transmission by scale modeling</td>
<td>Equations 1-8, 10-14, and 26-29</td>
<td>--</td>
</tr>
</tbody>
</table>

Conclusions

111. The investigation which was conducted in order to develop the above optimization procedure led to the following conclusions regarding rubble-mound breakwater design:

a. A systematic optimization procedure should be applied in any rubble-mound breakwater design to assure that an alternative with maximum cost effectiveness is proposed.
Rubble-mound breakwater designs should not be constructed without physical model testing of some kind due to the limited confidence of available analytical methods.

The confidence of the key analytical tools for rubble-mound breakwater design would be improved if current research were continuously concentrated in the following specific areas with probabilistic applications in mind:

1. **Site conditions**—Estimation of the long-term joint probability distribution \( F(H, T, t, d, \phi, \sigma) \) for a site should be developed for application in estimating expected breakwater damages and the long-term distribution of transmitted wave characteristics.

2. **Armor stability**—Standardized methods should be developed for scale model testing of rubble-mound stability in natural irregular sea states. Improved analytical stability prediction should be the goal of tests conducted by these methods, explicitly including the effect of wave period, storm duration, and other factors. Prototype verification of analytical predictions should be attempted also, particularly for new constructions where the design assumptions are most thoroughly documented.

3. **Mechanical strength of armor units**—Prediction of armor unit breakage by scale model tests should be developed in order that both incipient damage and reserve stability can be more accurately defined.

4. **Breakwater damage prediction**—The reserve stability of a wide range of rubble-mound breakwater configurations should be comprehensively tested by methods similar to those developed to detect incipient damage. Improved analytical prediction of reserve stability should be the goal of these tests.

5. **Runup on rubble-mound breakwaters**—Improved instrumentation and testing methods need development for measurement of irregular runup on rough permeable slopes. A concerted effort should be made to define runup coefficients for Equations 31 and 32 while concurrently investigating means for improved analytical prediction of irregular runup. The possibility of armor units designed both for enhanced hydraulic stability and for efficient attenuation of runup should be explored.

6. **Wave transmission**—The characteristics of irregular waves transmitted by rubble-mound breakwaters should be investigated. Improved analytical prediction of transmitted wave characteristics as a function of incident irregular wave characteristics should be the goal of this research.


Bruun, P., and Gunbak, A. 1978 (Mar). "Stability of Sloping Structures in Relation to \( \xi = \tan \alpha \left( \frac{H}{L_0} \right)^{1/2} \)," Coastal Engineering, Elsevier Scientific Publishing Co., Amsterdam, Vol 1, pp 287-322.


APPENDIX A: COMPUTER PROGRAM BWLOSS1

Estimation of Economic Losses as a Function of Wave Height

Program purpose
1. The program BWLOSS1 is intended to aid planners of coastal structures which provide protection from wave attack by deriving an empirical mathematical expression relating a given level of economic losses to the responsible incident significant wave height. This loss function can be used to define the "without-project" condition with respect to the incremental economic benefits provided by artificial wave protection. The program optionally provides an estimate of expected annual economic losses due to wave attack, given the coefficients of an Extremal Type I cumulative probability distribution function of significant wave heights for the site.

Program capabilities
2. BWLOSS1 is written in FORTRAN IV as implemented on the Honeywell DPS-8 mainframe system at the US Army Engineer Waterways Experiment Station (WES). A BASIC version written for the IBM PC is also available. The least squares method is applied to historical data on economic losses associated with the significant wave heights of the storms that caused the losses. A loss function is derived from the following form:

$$ L(H_S) = L_{\text{max}} \left[ 1 - e^{A(H_S - H_{L_0})} \right] $$

where

- $L(H_S)$ = economic losses as a function of significant wave height $H_S$
- $L_{\text{max}}$ = maximum conceivable economic loss from wave attack (at any intensity)
- $A$ = site-specific coefficient derived by regression
- $H_{L_0}$ = maximum significant wave height for which economic losses are negligible

3. The regression requires at least one point for $H_S$, $L(H_S)$, but it can deal with up to 100. The coefficient $A$ is presented along with the nonlinear correlation coefficient and the sum of the square residuals. A table of residuals is optionally presented. Losses can be optionally predicted, given a specific significant wave height.
corresponding to a given level of losses can be predicted. The form of this function is illustrated in Figure 16 in the main text. The program will also apply an Extremal Type I cumulative probability distribution of significant wave heights as follows:

$$F(H_s) = e^{-\left[e^{-\frac{H_s}{\phi}}\right]}$$  \hspace{1cm} (A2)

where

$$F(H_s) = \text{cumulative probability distribution of events where } -H_s < H_s$$

$$\epsilon$$ and $$\phi$$ = site-specific coefficients derived by regression of historical wave data

to estimate the expected annual economic losses by

$$E\left\{\left[\frac{L_{\text{yr}}}{\text{yr}}\right]\right\} = \lambda \int_{H_{LO}}^{H_{LO}} \left[\frac{dF(H_s)}{dH_s}\right] dH_s$$  \hspace{1cm} (A3)

where

$$\lambda = \text{the average number of extreme events per year above the threshold } H_s \text{ value originally used to derive } \epsilon \text{ and } \phi$$  

(must be input by the user)

$$H_{LO}^{\infty} = \text{a practical upper limit taken as the } H_s \text{ value whose probability of exceedance is } 0.0000001$$

4. This formulation assumes that the number of extreme events per year is random and can be represented by a mean value and is independent of the significant wave heights representing the intensity of the individual storms. The lower limit of integration is $$H_{LO}$$, below which the expected losses are taken as zero. Extrapolation of $$F(H_s)$$ to $$H_s$$ values below the threshold value applied to data used to originally derive $$\epsilon$$ and $$\phi$$ is probably conservative, but this question will be the subject of further study. A threshold $$H_s$$ value set equal to $$H_{LO}$$ would presumably resolve any problems if adequate statistical confidence can be maintained. The integration is accomplished by a numerical application of Simpson's Rule with 100 intervals.

5. The majority of the expected losses statistically occur during storms whose $$H_s$$ is just above $$H_{LO}$$ where the probability density is substantial. The higher $$H_s$$ values occur on the tail of the probability density function and may even be precluded by depth limitations. The program does not deal with depth limitations and assumes the Extremal Type I function fully
represents the wave climate at the site. A potential improvement of BWLOSS1 is the incorporation of period and depth effects for an estimate of $L(H_s, T_p, d)$ given the joint probability distribution $F(H_s, T_p, d)$. A further improvement would also incorporate the storm duration $t$ for an estimate of $L(H_s, T_p, t, d)$, given $F(H_s, T_p, t, d)$. These enhancements will involve a much more rigorous computation than is now performed by BWLOSS1. The program is now completely interactive and easily adaptable to execution by microcomputer systems.

Sample Execution and Output

6. Below is a sample execution and output for computer program "BWLOSS1."

INPUT THE MAXIMUM CONCEIVABLE LOSS IN MILLIONS OF DOLLARS
=20.
INPUT THE MAXIMUM SIGNIFICANT WAVE HEIGHT FOR WHICH
LOSSES ARE NEGLIGIBLE - USE CONSISTENT UNITS
=2.
HOW MANY SIGNIFICANT WAVE HEIGHT VS LOSS
DATA POINTS DO YOU HAVE?
=4
ENTER SIGNIFICANT WAVE HT., COMMA, LOSS IN MILLIONS OF DOLLARS
AND RETURN FOR EACH POINT
=3., 5
=4.5, 1.
=6., 2.5
=12., 7.5

DATA ON EXPONENTIAL CURVE...
CURVE HAS FORM: $L(H_s) = L_{max}*(1-exp(A*(H_s-HLo)))$
$L_{max} = 20.0000000$
$HLo = 2.0000$
$A = -0.0437137$
$L(Hs) = LOSSES$
$Hs = SIGNIFICANT WAVE HEIGHT$
NON-LINEAR CORRELATION IS 0.9735292
SUM SQR RESIDUALS..... 1.9485341
PRINT RESIDUAL TABLE (Y/N)?
=Y

<table>
<thead>
<tr>
<th>XVALUE</th>
<th>YVALUE</th>
<th>YEST</th>
<th>DIFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3.0000</td>
<td>0.5000</td>
<td>0.8554</td>
<td>0.3554</td>
</tr>
<tr>
<td>4.5000</td>
<td>1.0000</td>
<td>2.0705</td>
<td>1.0705</td>
</tr>
<tr>
<td>6.0000</td>
<td>2.5000</td>
<td>3.2084</td>
<td>0.7084</td>
</tr>
<tr>
<td>12.0000</td>
<td>7.5000</td>
<td>7.0823</td>
<td>0.4177</td>
</tr>
</tbody>
</table>

DO YOU WANT TO MAKE SOME LOSS PREDICTIONS FROM SIGNIFICANT WAVE HEIGHT DATA (Y/N)?
=Y
INPUT SIGNIFICANT WAVE HEIGHT
=10.
PREDICTED LOSS IN MILLIONS OF DOLLARS IS 5.90
DO YOU WISH TO MAKE ANOTHER PREDICTION (Y/N)?
=N

DO YOU WANT TO PREDICT SIGNIFICANT WAVE HEIGHTS FROM LOSS DATA (Y/N)?
=Y
INPUT LOSS IN MILLIONS OF DOLLARS
=15.
PREDICTED SIGNIFICANT WAVE HEIGHT IS 33.71
DO YOU WISH TO MAKE ANOTHER PREDICTION (Y/N)?
=N

DO YOU WANT TO PREDICT EXPECTED ANNUAL LOSSES (Y/N)?
=Y
SELECT A DISTRIBUTION...
EXTREMAL TYPE I...1
WEIBULL.............2
LOG-EXTREMAL.......3
SELECT 1, 2, OR 3
=1

INPUT EXTREMAL TYPE I EPSILON, AND PHI
=-2.27, 3.216

INPUT AVERAGE NUMBER OF EXTREMAL EVENTS PER YEAR, THE POISSON 'LAMBDA' PARAMETER
=4

EXPECTED ANNUAL LOSS IN MILLIONS OF DOLLARS IS 2.4141522
7. Below is a program listing for computer program BWLOSS1 (FORTRAN version).

10C PROGRAM "BWLOSS1". 11/85 VERSION
20C DESIGN BRANCH-COASTAL ENGINEERING RESEARCH CENTER
30C U.S. ARMY ENGINEERS WATERWAY EXPERIMENT STATION
40C P. O. BOX 631
50C VICKSBURG, MS 39180-0631
60C FOR FURTHER INFORMATION CONCERNING THE APPLICATION
70C OF "BWLOSS1", CALL:
80C ORSON P. SMITH (601)-634-2013 FTS:542-2013 OR
90C ROBERT B. LUND (601)-634-2068 FTS:542-2068 OR
100C DOYLE L. JONES (601)-634-2069 FTS:542-2069
110C
120C FORTRAN 4 HONEYWELL DPS-8
130C REF: "COMPUTER PROGRAM WAVDIST" CETN-I-
140C REF: "PROBABILITY AND STATISTICS" BY MORRIS DEGROOT
150C REF: "COST EFFECTIVE OPTIMIZATION OF RUBBLE-MOUND BREAKWATER
160C CROSS-SECTIONS" BY ORSON P. SMITH
170C REF: "EXTREMAL STATISTICS IN WAVE CLIMATOLOGY" BY BORGMAN AND RESIO
180C
190C N = THE NUMBER OF DATA POINTS
200C X = THE ARRAY OF SIGNIFICANT WAVE HEIGHTS
210C YH = THE ARRAY OF LOSSES CORRESPONDING TO EACH SIGNIFICANT WAVE HEIGHT
220C Y = THE TRANSFORMED Y ARRAY USED IN THE METHOD OF LEAST SQUARES
230C VI = H10, THE MAXIMUM WAVE HEIGHT FOR WHICH LOSSES ARE NEGLIGIBLE
240C W = $Lmax, THE MAXIMUM CONCEIVABLE LOSS IN MILLIONS OF DOLLARS
250C V2 = A, THE REGRESSION COEFFICIENT A<0
260C CORR = THE NON-LINEAR CORRELATION OF THE LOSS FUNCTION
270C ST = THE SUM OF THE SQUARE RESIDUALS
280C D1 = THE ARRAY THAT CONTAINS THE RESIDUAL FOR EACH DATA POINT
290C Z1 = THE LOSSES AS ESTIMATED BY THE LOSS CURVE
300C PDF(X) = THE PROBABILITY DENSITY FUNCTION OF EXTREMAL WAVES
310C CDF(X) = THE CUMULATIVE DISTRIBUTION FUNCTION OF EXTREMAL WAVES
320C G(X) = THE LOSS FUNCTION
330C C = SIMPSON'S RULE COEFFICIENTS
340C
350C INITIALIZE VARIABLES, STRINGS, AND FUNCTIONS
360C DIMENSION X(101),Y(101),YH(101)
370C COMMON X,Y,YH
380C CHARACTER*1 L
390C CHARACTER*60 ST(20)
400C G(X)=W*(1-EXP(V2*(X-V1)))
410C ST(1)="***************"  "BWLOSS1" IS A PROGRAM WHICH FITS AN EXPONENTIAL CURVE TO HISTORICAL INFORMATION ON ECONOMIC LOSSES CAUSED BY WAVE ATTACK. EACH STORM CAUSED BY WAVE ATTACK, THE MAXIMUM SIGNIFICANT WAVE HEIGHT FOR WHICH LOSSES CAN BE NEGLECTED AND AT LEAST ONE HISTORICAL LOSS WITH ASSOCIATED WAVE HEIGHT. THE EXPONENTIAL CURVE IS COMPUTED BY
THE LEAST SQUARES METHOD. ITS PARAMETERS AND NON-LINEAR CORRELATION ARE PRINTED.
THE PROGRAM WILL ALSO ESTIMATE EXPECTED ANNUAL LOSSES GIVEN THE PARAMETERS FOR THE LONG-TERM CUMULATIVE PROBABILITY DISTRIBUTION OF SIGNIFICANT WAVE HEIGHTS AT THE SITE. THE PROGRAM WILL ACCEPT THREE DIFFERENT DISTRIBUTIONS: (1) EXTREMAL
TYPE I; (2) WEIBULL; AND (3) LOG EXTREMAL.

DO 50 I=1,20
WRITE(6,407) ST(I)
407 FORMAT(1X,A60)
CONTINUE
C GET THE FACTS
WRITE(6,408)
408 FORMAT(///)
WRITE(6,101) FORMATDx, "INPUT THE MAXIMUM CONCEIVABLE LOSS IN MILLIONS OF DOLLARS")
READ,W
IF(W.LE.0) 60 TO 1
WRITE(6,201) FORMATDx, "INPUT THE MAXIMUM SIGNIFICANT WAVE HEIGHT FOR WHICH LOSSES ARE NEGLIGIBLE - USE CONSISTENT UNITS")
READ,X(1)
IF(X(1).LT.0) 60 TO 2
YH(1)=0
Y(1)=0
WRITE(6,102) FORMATDx, "HOW MANY SIGNIFICANT WAVE HEIGHT VS LOSS DATA POINTS DO YOU HAVE?")
READ,N
IF(N.LE.1) GO TO 4
IF( N.GT.100) PRINT, '100 POINTS IS MAXIMUM-REINPUT' GO TO 4
WRITE(6,104)
FORMATDx, "ENTER SIGNIFICANT WAVE HT., COMMA, LOSS IN MILLIONS OF DOLLARS", AND RETURN FOR EACH POINT")
I=2
READ,X(I),YH(I)
IF( YH(I).GT. W ) GO TO 17
Y(I)=ALOG(1-YH(I)/W)
IF(I .EQ. (N+1)) GO TO 18
I=I+1
GO TO 15
WRITE(6,105) FORMATDx, "ERROR-YOUR INPUT LOSS IS MORE THAN YOUR MAXIMUM"
& " LOSSES. RE-INPUT POINT")
GO TO 15
N=N+1
C FIT CURVE TO INPUT DATA
CALL LOG(N,W,V1,V2)
WRITE(6,110)

CALL ANS(L)

IF(L .EQ. 'N') GO TO 75

PRINT,'INPUT SIGNIFICANT WAVE HEIGHT'

READ,H

IF(H .LT. 0) 60 TO 45

SL06=G(H)

IF(H .LE. X(1)) SLOG=0

WRITE(6,114) SLOG

FORMAT(dX,"PREDICTED LOSS IN MILLIONS OF DOLLARS IS ",F7.2)

PRINT,'DO YOU WISH TO MAKE ANOTHER PREDICTION(Y/N)?'

CALL ANS(L).

IF(L .EQ. 'Y') GO TO 45

CALL LOG(N,W,V1,V2)

SUBROUTINE LOG TO FIT EXPONENTIAL CURVE TO INPUT DATA

SUBROUTINE LOG(N,W,V1,V2)

COMMON X,Y,YH

DIMENSION X(101),Y(101),YH(101),D1(101),Z1(101)

G(X)=W*(1.0-EXP(V2*(X-V1)))

CALCULATE PARAMETERS V1 AND V2 BY THE LEAST SQUARES METHOD

YSUM=0
DT=0
DB=0
DO 20 K=2,N
YSUM=YSUM+YH(K)
DT=DT+Y(K) *(X(K)-X(1))
DB=DB+(X(K)-X(1)) **2
V1=X(1)
V2=DT/DB
CALCULATE CORRELATION COEFFICIENT (R**2)

YAV6=YSUM/N
ST=0
SB=0
DO 70 I=1,N
Z1(I)=G(X(I))
D1(I)=(YH(I)-Z1(I))**2
D2=(YH(I)-YAV6)**2
ST=ST+D1(I)
SB=SB+D2
IF((1-ST/SB).LT.0) CORR=1-ST/SB

CORR=SQRT(1.0-ST/SB)
PRINT OUT PARAMETERS AND OTHER DATA
WRITE(6,80)
80 FORMAT(1X, "DATA ON EXPONENTIAL CURVE...", /, 1X, & "CURVE HAS FORM: \$L(Hs)=Lmax{1-exp[A*(Hs-HLo)]}\$"
WRITE(6,82) W, V1, V2
WRITE(6,84) CORR
84 FORMAT(1X, "NON-LINEAR CORRELATION IS", F9.7)
WRITE(6,37) ST
37 FORMAT(1X, "SUM SQR RESIDUALS.....", 4X, F11.7)
CALL ANS(L)
IF(L.EQ.'Y') K=1
IF(L.NE.'N').AND. L.NE.'Y') GO TO 35
IF(K.EQ.1) WRITE(6,45)
45 FORMAT(1X, "PRINT RESIDUAL TABLE (Y/N)?")
DO 60 I=1,N
60 FORMAT(1X, "XVALUE YVALUE YEST DIFF")
DO 60 I=1,N
60 FORMAT(F11.4, F11.4, F11.4, F11.4)
CONTINUE
RETURN
END

SUBROUTINE EXPCK(W, V2, CUT)
DOUBLE PRECISION W
REAL LAMBDA
FD1(X)=-(ALOG(-ALOG(X)))*PHI+EPSI
FD2(X)=((-ALOG(1-X))**(1/A1))*B1
FD3(X)=B2/((-ALOG(X))**(1/A2))
PDF1(X)=EXP(-EXP(-(X-EPSI)/PHI))*EXP(-(X-EPSI)/PHI)/PHI
PDF2(X)=A1*(X**(A1-1))*EXP(-(X/B1)**A1)/(B1**A1)
PDF3(X)=A2*(B2**A2)*EXP(-(B2/X)**A2)/((X**(A2+1))
CDF1(X)=EXP(-EXP((EPSI-X)/PHI))
CDF2(X)=1.0-EXP(-(X/B1)**A1)
CDF3(X)=EXP(-(B2/X)*A2)

G(X)=W**(1.0-EXP(V2**(X-CUT)))

PRINT, 'SELECT A DISTRIBUTION...
PRINT, 'EXTREMAL TYPE I... 1'
PRINT, 'WEIBULL .......... 2'
PRINT, 'LOG-EXTREMAL..... 3'
PRINT, 'SELECT 1, 2, OR 3'
READ, ID
IF(ID.LT.1.OR.ID.GT.3) 60 TO 70
IF(ID.EQ.1) WRITE(6,104)
IF(ID.EQ.2) WRITE(6,114)
IF(ID.EQ.3) WRITE(6,124)
WRITE(6,5)
IF(ID.EQ.1) FAC1=PDF1(XV)
IF(ID.EQ.2) FAC1=PDF2(XV)
IF(ID.EQ.3) FAC1=PDF3(XV)
FAC2=6(XV)
IF(XV.LT.CUT) FAC2=0
SUM=SUM+C*FAC1*FAC2
CONTINUE
2620      SUM=SUM/300.0*D
2630      WRITE(6,14) SUM*LAMBDA
2640 14   FORMAT(/,1X,"EXPECTED ANNUAL LOSS IN MILLIONS OF DOLLARS IS",F14.7)
2650      RETURN
2660      END
2670
2680C SUBROUTINE TO ANSWER YES/NO QUESTIONS
2700      SUBROUTINE ANS(L)
2710      CHARACTER*1 L
2720      GO TO 25
2730 30   PRINT,'REINPUT RESPONSE'
2740 25   READ(5,10) L
2750 10   FORMAT(A1)
2760      IF( L.NE. 'Y'.AND. L.NE. 'N' ) GO TO 30
2770      RETURN
2780      END

*
APPENDIX B: COMPUTER PROGRAM BWCOMP

Comparison of Breakwater Volumes and Costs

Program purpose
1. The program BWCOMP calculates breakwater volumes and costs demonstrating the effect of varying breakwater slopes on wave transmission, the choice of armor size and shape, and overall volume and cost.

Background
2. Systematic comparison of the relative cost of rubble-mound breakwaters designed with varying combinations of slope is tedious and awkward to present in project reports, yet it must be accomplished to assure that a cost-effective cross section is chosen. Wave transmission by overtopping during a given wave condition is a function of a breakwater's seaward slope, its crest elevation and width, and its surface roughness. A breakwater's stability is a function of its seaward slope, its leeward slope, the size and shape of its armor units, and other factors which affect the overall cost to a lesser degree. The cost of armor units varies with size and shape. The problem of cost comparison is further complicated by the interrelation of most of these factors.

3. The program BWCOMP was designed to make the task of comparing variations of these factors easier to accomplish and present. The following simplifying assumptions make the program economical to use:

   a. Wave transmission by permeation through the structure is typically much smaller than transmission by overtopping and can be neglected for this comparative analysis.

   b. The unit price ($ per unit volume or unit weight) and availability of primary armor units tend to be most critical to the overall breakwater cost and constructibility, as compared to unit price variations and availability of secondary armor, underlayers, filter material, or core material. A single average unit price can therefore be derived, for the purposes of this comparative analysis, to include all materials except the primary armor.

   c. Most rubble-mound breakwaters intended primarily as wave barriers for harbors or ports must be designed for some overtopping during extreme events, with primary armor extending down the leeward slope below the water line. Final designs of rubble-mound breakwaters may have complex features in detail, but the above assumptions allow adoption of a standard parameterized cross section, as shown in Figure 17 of the main text.
modification to this cross section is required for jetties oriented straight into oncoming waves or breakwaters with monolithic superstructures.

4. The program BWCOMP uses the above assumptions to make cost comparisons of alternate armor material and slope combinations with accuracy appropriate for the earliest stages of planning or for Step 4 of the optimization procedure proposed in the main text. Final design should involve all the detailed considerations recommended in the Shore Protection Manual (SPM) (1984)* and other guidance available.

Program Input

5. The program is fully interactive in its present form and accommodates either English or metric units. It is written in FORTRAN as implemented on the US Army Engineer Waterways Experiment Station (WES) Honeywell DPS-8 mainframe computer system. A BASIC version is also available for use on microcomputers. The interactive input required is demonstrated by the example interactive session included in this appendix. The associated output is shown (in part) in the example output included in this appendix. Two wave conditions must be specified: one for determining armor size and the other for determining crest elevation as a function of a specified maximum transmitted wave height. A percentage exceedance must be associated with the specified maximum transmitted height such that x% of the transmitted waves can exceed the maximum height during the sea state represented by the second specified wave condition. The input unit prices (cost per volume) should be average values for the materials (rock or concrete) and armor unit types. This is a comparative analysis, so fine precision in these estimates is not necessary, but consistency is important. Prices that vary with the weight or volume of the individual armor units may require successive runs of BWCOMP since an estimate of these individual unit weights is necessary to input the appropriate unit price.

* References cited in this appendix are included in the References at the end of the main text.
Computations

6. The sequence of computations is summarized in the BWCOMP flowchart in Figure B1. The narrative below describes the assumptions and equations applied in this sequence.

7. The program performs all computations for each of seven pairs of seaward and leeward slopes: 1:1.5/1:1.5, 1:2.0/1:1.5, 1:2.0/1:2.0, 1:2.5/1:1.5, 1:2.5/1:2.0, 1:3.0/1:1.5, and 1:3.0/1:2.0. Identical computations are performed for each of 10 armor units for each of these slope combinations. The stability, geometry, and runup coefficients which are assumed for each armor unit are specified in DATA statements at the beginning of the program listing, as summarized in Table B1. The crest elevation is first assumed as 0.3 m then increased in 0.3-m increments until the estimated transmitted wave height is less than the specified maximum. The computed dimensions and costs for all 10 armor units are then printed in a table for each slope combination (i.e. in seven tables).

8. The wave conditions are checked for breaking or nonbreaking conditions by Goda's breaker index formula (Goda 1975) assuming a horizontal bottom. The stability or transmission incident heights are set equal to the breaker height at the specified depth if the breaker height is smaller. The stability coefficient $K_d$ for Hudson's formula (Equation 1 in the main text) is chosen accordingly for each armor unit type. The weight computed by Hudson's formula is then applied to compute the armor thickness and minimum crest width by Equation 9 (main text) by assuming "$n$" values of 2 and 3, respectively. The crest elevation derived from wave transmission computations then allows all dimensions of the parameterized cross section (Figure 17, main text) and the corresponding volumes and costs per unit trunk length to be estimated. Specifications from Figures 7-109 through 7-115 in the SPM (1984) are applied to estimate the armor thickness and number of individual armor units per unit trunk length.

9. The crest elevation is determined by first assuming a crest elevation (initially 1 ft* above the still-water level) and then estimating the transmitted wave for the specified incident wave condition and the current breakwater geometry. The estimated transmitted wave height is compared to the

* To convert feet to metres, use a conversion factor of 0.3048.
Figure B1. BWCOMP flowchart
Table B1

Armor Unit Parameters* Applied in Computer Program "BWCOMP"

<table>
<thead>
<tr>
<th>Armor Unit Type</th>
<th>Stability Coefficients</th>
<th>Runup Coefficients</th>
<th>Geometric Parameters</th>
<th>Armor Thickness$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_{db}$**</td>
<td>$K_{dnbr}$†</td>
<td>$a^{tt}$</td>
<td>$b^{tt}$</td>
</tr>
<tr>
<td>Quarrystone§§ (uniform)</td>
<td>2.0</td>
<td>4.0</td>
<td>0.775</td>
<td>0.361</td>
</tr>
<tr>
<td>Quarrystone§§ (graded riprap)</td>
<td>2.2</td>
<td>2.5</td>
<td>0.956</td>
<td>0.398</td>
</tr>
<tr>
<td>Plain cubes</td>
<td>3.5</td>
<td>4.0</td>
<td>0.775#</td>
<td>0.361#</td>
</tr>
<tr>
<td>Modified cubes</td>
<td>6.5</td>
<td>7.5</td>
<td>0.95</td>
<td>0.69</td>
</tr>
<tr>
<td>Tetrapods</td>
<td>7.0</td>
<td>8.0</td>
<td>1.01</td>
<td>0.91</td>
</tr>
<tr>
<td>Quadripods</td>
<td>7.0</td>
<td>8.0</td>
<td>0.59</td>
<td>0.35</td>
</tr>
<tr>
<td>Hexapods</td>
<td>8.0</td>
<td>9.5</td>
<td>0.82</td>
<td>0.63</td>
</tr>
<tr>
<td>Tribars</td>
<td>9.0</td>
<td>10.0</td>
<td>1.81</td>
<td>1.57</td>
</tr>
<tr>
<td>Toskanes</td>
<td>11.0</td>
<td>22.0</td>
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<td>0.703#</td>
</tr>
<tr>
<td>Dolosse</td>
<td>15.0</td>
<td>31.0</td>
<td>0.988</td>
<td>0.703</td>
</tr>
</tbody>
</table>

* All parameters assume random placement on a breakwater trunk with permeable underlayers.
** Breaking wave stability coefficients for Hudson's formula.
† Nonbreaking wave stability coefficients for Hudson's formula.
$^{tt}$ Coefficients for runup formula (Ahrens and McCartney 1975): $R/H = a\xi/(1 + b\xi) ; \xi = \tan \theta/\sqrt{H/L}$. 
$^+$ Layer coefficient.
$^\#$ Porosity.
§ $W$ is the weight of an armor unit and $\rho_r$ is the mass density.
$^{§§}$ Rough angular quarrystone.
# Coefficients assumed same as most similarly shaped units.
specified maximum, and the crest is increased 1 ft for another round of transmission computations if the condition is not satisfied. The computations apply to Equations 26-29 and the armor unit data from Table 5 in the main text.

Sample Interactive Session

10. Below is a sample interactive session for program "BWCOMP" (FORTTRAN version).

BWCOMP IS AN INTERACTIVE PROGRAM WHICH COMPUTES BREAKWATER VOLUMES AND COSTS FOR A SIMPLE PARAMETERIZED CROSS SECTION FOR THE PURPOSE OF COMPARING THE EFFECT OF CHANGING THE SEAWARD AND LEENWARD SLOPES ON THE SIZE AND RELATIVE COST OF A RANGE OF ARMOR UNIT TYPES. THE OUTPUT OF BWCOMP SHOULD NOT BE USED AS A COST ESTIMATING TOOL IN ANY STAGE OF A PLANNING OR DESIGN PROJECT. THE ASSUMPTIONS APPLIED ARE INTENDED TO TENTATIVELY IDENTIFY THE OPTIMUM COMBINATION OF SLOPES AND ARMOR UNIT FOR A GIVEN SET OF DESIGN CRITERIA.

TITLE OF THIS RUN? (UP TO 78 CHARACTERS)
EXAMPLE EXECUTION OF PROGRAM BWCOMP

ENGLISH(0) OR METRIC(1) UNITS?
=0

SIG. WAVE HEIGHT, FT(F4.1)?
=15.
PEAK WAVE PERIOD, SEC(F4.1)?
=12.
DO YOU WANT TO SPECIFY A SEPARATE WAVE HEIGHT AND PERIOD FOR WAVE TRANSMISSION COMPUTATIONS(Y OR N)?
=Y
Sigs. (FOR TRANSMISSION) IN FT =?
=12.
PEAK PERIOD (FOR TRANSMISSION) IN SECONDS =?
=10.

MAX. ALLOWABLE TRANSMITTED WAVE HEIGHT, FT(F4.1)?
=2.

%EXCEEDANCE OF TRANSMITTED HEIGHT IN SEA STATE?
=1.

DO YOU WANT TO ALLOW FOR INCREASED RUNUP AND OVERTOPPING CAUSED BY ONSHORE WIND(Y OR N)?
=Y

ONSHORE WIND VELOCITY, MPH (F5.1)?
=35.
WATER DEPTH AT TOE OF STRUCTURE, FT(F4.1)?
=19.
FRESH WATER(0) OR SALT WATER(1)?
UNIT WEIGHT OF ROCK IS ASSUMED TO BE 165 LBS/ CUFT (2643 KG/ CUM). DO YOU WISH TO ENTER AN ALTERNATE VALUE? (Y OR N)

UNIT WEIGHT OF CONCRETE IS ASSUMED TO BE 149.5 LBS/ CUFT (2423 KG/ CUM). DO YOU WISH TO ENTER AN ALTERNATE VALUE? (Y OR N)

INPUT UNIT PRICE OF UNIFORM ARMOR ROCK, IN PLACE ($/TONS) = 50.

INPUT UNIT PRICE OF GRADED ARMOR ROCK, IN PLACE ($/TONS) = 45.

INPUT UNIT PRICES ($/CY, IN PLACE) FOR THESE ARMOR UNITS:
PLAIN CUBE, MOD. CUBE, TETRAPOD, QUADROPOD, HEXAPOD, TRIBAR, TOSKANE, & DOLOS

FORMAT (8F7.2); SEPARATE PRICES BY COMMAS
= 60., 95., 100., 100., 100., 105., 110., 115.

INPUT UNIT PRICE FOR CORE MATERIAL (GRADED ROCK), $/TONS = 35.
11. Below is a sample output for computer program BWCOMP.

EXAMPLE EXECUTION OF PROGRAM BWCOMP

**TABLE OF COMPARATIVE BREAKWATER QUANTITIES AND COSTS**

SEAWARD SLOPE = 1:1.5 LEENWARD SLOPE = 1:1.5 WATER DEPTH = 19 FT
INCIDENT WAVE : SIG HEIGHT = 15 FT PEAK PERIOD = 12 SEC
(DEPTH LIMITED WAVE HEIGHT = 14.3331 FT)
INCIDENT WAVE (FOR TRANSMISSION COMPUTATIONS):
SIG. HEIGHT = 12 FT PEAK PERIOD = 10 SEC
MAX TRANSMITTED WAVE HEIGHT = 2 FT (1 % EXCEEDANCE)

**RELATIVE VALUES PER UNIT TRUNK LENGTH**

<table>
<thead>
<tr>
<th>ARMOR UNIT</th>
<th>SIZE</th>
<th>CREST HEIGHT</th>
<th>CREST WIDTH</th>
<th>NO. ARMOR</th>
<th>ARMOR VOLUME</th>
<th>CORE VOLUME</th>
<th>ARMOR COST</th>
<th>CORE COST</th>
<th>TOTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TONS</td>
<td>FT</td>
<td>FT</td>
<td>UNITS</td>
<td>CY</td>
<td>CY</td>
<td>($)</td>
<td>($)</td>
<td>($)</td>
</tr>
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<td>ROCK, UNIF</td>
<td>20.6</td>
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<td>18.5</td>
<td>4.9</td>
<td>71.4</td>
<td>66.4</td>
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<td>5129</td>
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<tr>
<td>ROCK, GRAD</td>
<td>18.7</td>
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<td>18.0</td>
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<td>75.3</td>
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<td>3835</td>
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<td>6.2</td>
<td>28.9</td>
<td>80.3</td>
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<td>8.8</td>
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<td>15.0</td>
<td>5.4</td>
<td>23.6</td>
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<td>2452</td>
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<td>13.7</td>
<td>5.1</td>
<td>22.4</td>
<td>55.8</td>
<td>2240</td>
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<td>2341</td>
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<tr>
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<td>15.9</td>
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<td>51.2</td>
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<td>2835</td>
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<td>13.6</td>
<td>6.0</td>
<td>20.5</td>
<td>60.7</td>
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<td>110</td>
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<tr>
<td>DOLOS</td>
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<td>10.6</td>
<td>7.5</td>
<td>15.3</td>
<td>69.9</td>
<td>1763</td>
<td>127</td>
<td>1891</td>
</tr>
</tbody>
</table>

**NOTES:**
1. CREST HEIGHT IS ABOVE STILL WATER LEVEL
2. ARMOR VOLUME FOR ROCK IS THE TOTAL CROSS-SECTION VOLUME
3. ARMOR VOLUME FOR ARTIFICIAL UNITS IS THE CONCRETE VOLUME
**EXAMPLE EXECUTION OF PROGRAM BWCOMP**

### Table of Comparative Breakwater Quantities and Costs

Seaward Slope = 1 : 3  Leeward Slope = 1 : 1.5  Water Depth = 19 FT
Incident Wave: Sig Height = 15 FT  Peak Period = 12 SEC
(Depth Limited Wave Height = 14.3331 FT)

Incident Wave (for Transmission Computations):
Sig. Height = 12 FT  Peak Period = 10 SEC
Max Transmitted Wave Height = 2 FT (1 % Exceedance)

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>Size (Tons)</th>
<th>Crest Height (FT)</th>
<th>Crest Width (FT)</th>
<th>No. Armor Units</th>
<th>Armor Core Height (FT)</th>
<th>Armor Core Width (FT)</th>
<th>Total Cost (CY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock, UNIF</td>
<td>10.3</td>
<td>15.0</td>
<td>14.8</td>
<td>8.1</td>
<td>59.1</td>
<td>55.8</td>
<td>4147</td>
</tr>
<tr>
<td>Rock, Grad</td>
<td>9.4</td>
<td>17.0</td>
<td>14.3</td>
<td>9.1</td>
<td>60.6</td>
<td>66.4</td>
<td>3828</td>
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<tr>
<td>Cube</td>
<td>6.6</td>
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<td>8.8</td>
<td>38.4</td>
<td>49.9</td>
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<tr>
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<td>2428</td>
</tr>
<tr>
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<td>12.0</td>
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<td>21.4</td>
<td>51.3</td>
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</tr>
<tr>
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<td>69.9</td>
<td>1586</td>
</tr>
</tbody>
</table>

**Notes:**
1. Crest Height is above Still Water Level
2. Armor Volume for Rock is the Total Cross-Section Volume
3. Armor Volume for Artificial Units is the Concrete Volume

---

**EXAMPLE EXECUTION OF PROGRAM BWCOMP**

Seaward Slope = 1 : 2  Leeward Slope = 1 : 2  Water Depth = 19 FT
Incident Wave: Sig Height = 15 FT  Peak Period = 12 SEC
(Depth Limited Wave Height = 14.3331 FT)

Incident Wave (for Transmission Computations):
Sig. Height = 12 FT  Peak Period = 10 SEC
Max Transmitted Wave Height = 2 FT (1 % Exceedance)

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>Size (Tons)</th>
<th>Crest Height (FT)</th>
<th>Crest Width (FT)</th>
<th>No. Armor Units</th>
<th>Armor Core Height (FT)</th>
<th>Armor Core Width (FT)</th>
<th>Total Cost (CY)</th>
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</thead>
<tbody>
<tr>
<td>Rock, UNIF</td>
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<td>12.5</td>
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<td>10.2</td>
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<td>1786</td>
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</tbody>
</table>

**Notes:**
1. Crest Height is above Still Water Level
2. Armor Volume for Rock is the Total Cross-Section Volume
3. Armor Volume for Artificial Units is the Concrete Volume
### Example Execution of Program BWCOMP

**Table of Comparative Breakwater Quantities and Costs**

Seaward Slope = 1:2  
Leeward Slope = 1:1.5  
Water Depth = 19 ft  
Incident Wave: Sig. Height = 15 ft  
Peak Period = 12 sec  
(Depth Limited Wave Height = 14.3331 ft)

Incident Wave (for Transmission Computations):  
Sig. Height = 12 ft  
Peak Period = 10 sec  
Max Transmitted Wave Height = 2 ft (1% Exceedance)

#### Relative Values per Unit Trunk Length

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>Size</th>
<th>CREST HEIGHT</th>
<th>CREST WIDTH</th>
<th>No. Armor</th>
<th>Armor Core</th>
<th>Armor Cost</th>
<th>Core Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TONS</td>
<td>FT</td>
<td>FT</td>
<td>UNITS</td>
<td>VOL</td>
<td>CY</td>
<td>CY</td>
<td>($)</td>
</tr>
<tr>
<td>Rock, Unif</td>
<td>15.5</td>
<td>20.0</td>
<td>16.9</td>
<td>5.8</td>
<td>63.7</td>
<td>59.2</td>
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</tr>
<tr>
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</tbody>
</table>

**Notes:**  
1. CREST HEIGHT IS ABOVE STILL WATER LEVEL  
2. ARMOR VOLUME FOR ROCK IS THE TOTAL CROSS-SECTION VOLUME  
3. ARMOR VOLUME FOR ARTIFICIAL UNITS IS THE CONCRETE VOLUME

---

**Table of Comparative Breakwater Quantities and Costs**

Seaward Slope = 1:2.5  
Leeward Slope = 1:1.5  
Water Depth = 19 ft  
Incident Wave: Sig. Height = 15 ft  
Peak Period = 12 sec  
(Depth Limited Wave Height = 14.3331 ft)

Incident Wave (for Transmission Computations):  
Sig. Height = 12 ft  
Peak Period = 10 sec  
Max Transmitted Wave Height = 2 ft (1% Exceedance)

#### Relative Values per Unit Trunk Length

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>Size</th>
<th>CREST HEIGHT</th>
<th>CREST WIDTH</th>
<th>No. Armor</th>
<th>Armor Core</th>
<th>Armor Cost</th>
<th>Core Cost</th>
<th>Total Cost</th>
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</thead>
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<tr>
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<td>FT</td>
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<td>VOL</td>
<td>CY</td>
<td>CY</td>
<td>($)</td>
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<tr>
<td>Rock, Unif</td>
<td>12.4</td>
<td>17.0</td>
<td>15.7</td>
<td>6.9</td>
<td>60.4</td>
<td>56.5</td>
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<td>Rock, Grad</td>
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</table>

**Notes:**  
1. CREST HEIGHT IS ABOVE STILL WATER LEVEL  
2. ARMOR VOLUME FOR ROCK IS THE TOTAL CROSS-SECTION VOLUME  
3. ARMOR VOLUME FOR ARTIFICIAL UNITS IS THE CONCRETE VOLUME
Table of Comparative Breakwater Quantities and Costs

Seaward Slope = 1 : 2.5  Leeward Slope = 1 : 2  Water Depth = 19 ft
Incident Wave: Sig Height = 15 ft  Peak Period = 12 sec
(Depth Limited Wave Height = 14.3331 ft)
Incident Wave (for transmission computations):
Sig. Height = 12 ft  Peak Period = 10 sec
Max Transmitted Wave Height = 2 ft (1% Exceedance)

Relative Values per Unit Trunk Length

<table>
<thead>
<tr>
<th>Armor Unit</th>
<th>Size (Tons)</th>
<th>Crest Height (ft)</th>
<th>Crest Width (ft)</th>
<th>No. Armor Units</th>
<th>Armor Core Vol (CY)</th>
<th>Core Vol (CY)</th>
<th>Armor Cost ($)</th>
<th>Core Cost ($)</th>
<th>Total Cost ($)</th>
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<tr>
<td>Rock, Unif</td>
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</tbody>
</table>

Notes: 1. Crest Height is above still water level
2. Armor Volume for Rock is the total cross-section volume
3. Armor Volume for artificial units is the concrete volume

Table of Comparative Breakwater Quantities and Costs

Seaward Slope = 1 : 3  Leeward Slope = 1 : 2  Water Depth = 19 ft
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<th>Total Cost ($)</th>
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<td>78.1</td>
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<td>142</td>
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</tbody>
</table>

Notes: 1. Crest Height is above still water level
2. Armor Volume for Rock is the total cross-section volume
3. Armor Volume for artificial units is the concrete volume

B11
Sample Program Listing

12. The program listing for BWCOMP (FORTRAN version) is as follows:

10C ****************************
20C * PROGRAM BWCOMP - VERSION 9/85 *
30C * CONTACT FOR QUESTIONS ON PROGRAM USE: *
40C * ORSON SMITH *
50C * COASTAL DESIGN BRANCH *
60C * COASTAL ENGINEERING RESEARCH CENTER *
70C * U. S. ARMY ENGINEER WATERWAYS EXPERIMENT *
80C * STATION, F. O. BOX 631 *
90C * VICKSBURG, MS 39180 *
100C *******************************
110 CHARACTER ANW*2,ANSW*2,WINDU*3, ANSWIND*2
120 CHARACTER ANW*2,LU*2,RHOU*8,VOLU*2,UNIT*10,WU*4
130 CHARACTER TITLE*80
140 REAL KT,NO,KD,LO
150 DIMENSION XKDELTA(10),H(2),T(2),RA(10),RB(10)
160 DIMENSION XKDBR(10),XKDNR(10),P(10),UNITC(11)
170C * DATA IS FOR ARMOR UNITS: (1) UNIFORM QUARRYSTONE, (2) GRADED *
180C * RIPRAP, (3) PLAIN CUBES, (4) MODIFIED CUBES, (5) TETRAPODS *
190C * (6) QUADRIPODS, (7) HEXAPODS, (8) TRIBARS, (9) TOSKANES, *
200C * (10) DOLOSSE - ALL FOR RANDOM PLACEMENT ON TRUNK *
210C 220C ** STABILITY COEFFICIENTS - BR: BREAKING; NBR: NON-BREAKING **
230C DATA XKDBR/2.0,2.2,3.5,5.5,7.0,7.0,8.0,9.0,11.0,15.0/
240C DATA XKDNBR/4.0,2.5,4.0,7.5,8.0,8.0,9.5,10.0,22.0,31.0/
250C ** LAYER COEFFICIENTS **
260C DATA XKDELTA/1.0,1.0,1.1,1.1,1.04,.95,1.15,1.02,1.03,.94/
270C ** POROSITIES **
280C DATA P/4.,37.,43.,47.,5,49.,47.,54.,52.,56/
290C ** RUNUP COEFFICIENTS A & B(AHRENS & MCCARTNEY, 1975) **
300C DATA RA/,.775,7.75,7.75,.95,1.01,59,.82,1.61,.988,.988/
310C DATA RB/,.361,.398,.361,.69,.91,.35,.63,1.57,.703,.703/
320C *******************************
330C 340C ***BEGIN INTERACTIVE INPUT OF DATA***
350C 360C WRITE(6,10)
370 10 FORMAT(1H1,,/,,5X,"BWCOMP IS AN INTERACTIVE PROGRAM WHICH
compiles breakwater ",/","VOLUMES AND COSTS FOR A SIMPLE
parameterized cross section for ",/","THE PURPOSE OF COM
paring the effect of changing the seaward ",/","AND LEEWARD
SLOPES ON THE SIZE AND RELATIVE COST OF A RANGE OF
"",/","ARMOR UNIT TYPES. THE OUTPUT OF BWCOMP SHOULD NOT
be used ",/","AS A COST ESTIMATING TOOL IN ANY STAGE OF A
planning or design ",/","PROJECT. THE ASSUMPTIONS APPLIED
are intended to tentatively ",/","IDENTIFY THE OPTIMUM
COMBINATION OF SLOPES AND ARMOR UNIT ",/","FOR A GIVEN
SET OF DESIGN CRITERIA.")
380 390 400 410 420 430 440 450 460 470 480 490 500 15 WRITE(6,20)
500 20 FORMAT(,,5X,"TITLE OF THIS RUN? (UP TO 78 CHARACTERS)")
510C READ(5,21)TITLE
510  21  FORMAT(A80)
520  WRITE(6,300)
530  300  FORMAT(/,5X,"ENGLISH(0) OR METRIC(1) UNITS?")
540  READ(5,301)IUNITS
550  301  FORMAT(I2)
560  23  WRITE(6,45)
570  24  FORMAT(V)
580  25  IF(IUNITS) 302,302,303
590     302  LU="FT"
600     303  GO TO 304
610     304  WRITE(6,35) LU
620     35  FORMAT(/,3X,"SIG. WAVE HEIGHT, ",A2,"(F4.1)?")
630     36  READ(5,25)H(1)
640     37  WRITE(6,46)
650     45  IF(IUNITS.EQ.0) GO TO 119
660     46  WRITE(6,47) LU
670     47  FORMAT(/,3X,"MAX. ALLOWABLE TRANSMITTED WAVE HEIGHT, ",
680          ",A2","(F4.1)?")
690     48  READ(5,25) HTMAX
700     49  WRITE(6,122)
710     122  FORMAT(/,3X,"EXCEEDANCE OF TRANSMITTED HEIGHT IN SEA STATE?")
720     123  READ(5,51) EXC
730     124  EXCP=EXC
740     125  WRITE(6,123)
750     126  IF(IUNITS.EQ.1) GO TO 124
760     127  IF(IUNITS.EQ.0) WINDU="MPH"
770     128  IF(IUNITS.EQ.1) WINDU="M/S"
780     129  WRITE(6,125) WINDU
790     130  IF(IUNITS.EQ.0) GO TO 124
800     131  WRITE(6,302) LU="FT"
810     302  GO TO 304
820     303  WRITE(6,35) LU
830     35  FORMAT(/,3X,"SIG. WAVE HEIGHT, ",A2,"(F4.1)?")
840     36  READ(5,25)H(1)
850     37  WRITE(6,46)
860     45  IF(IUNITS.EQ.0) GO TO 119
870     46  WRITE(6,47) LU
880     47  FORMAT(/,3X,"MAX. ALLOWABLE TRANSMITTED WAVE HEIGHT, ",
890          ",A2","(F4.1)?")
900     48  READ(5,25) HTMAX
910     49  WRITE(6,122)
920     122  FORMAT(/,3X,"EXCEEDANCE OF TRANSMITTED HEIGHT IN SEA STATE?")
930     123  READ(5,51) EXC
940     124  EXCP=EXC
950     125  WRITE(6,123)
960     126  IF(IUNITS.EQ.1) GO TO 124
970     127  IF(IUNITS.EQ.0) WINDU="MPH"
980     128  IF(IUNITS.EQ.1) WINDU="M/S"
990     129  WRITE(6,125) WINDU
READ(5,126) WIND
120 FORMAT(F5.1)
124 WRITE(6,120) LU
20 FORMAT(3X,"WATER DEPTH AT TOE OF STRUCTURE, ",A2,"(F4.1)?")
25 READ(5,25) D
30 WRITE(6,130)
40 FORMAT(3X,"FRESH WATER(0) OR SALT WATER(1)??")
50 READ(5,24)
60 C
70 CHECK FOR BREAKING BY GODA'S FORMULA FOR HORIZONTAL_BOTTOM C
80 DO 60 I=1,2
90 HI=H(I)
100 IF(IUNITS.EQ.0) LO=5.12*T(I)*T(I)
110 IF(IUNITS.EQ.1) LO=1.56*T(I)*T(I)
120 HB=0.17*LOG10(1.-EXP(-4.71239*D/LO))
130 IF(HB.LT.H(I)) HI=HB
140 IF(I.EQ.1) H1=HI
150 IF(I.EQ.2) H2=HI
160 CONTINUE
170 H2P=H2*SQRT((-ALOG(166)))/2.
180 WRITE(6,135)
190 FORMAT(3X,"UNIT WEIGHT OF ROCK IS ASSUMED TO BE",/,"165 LBS/CUFT(2643 KG/CUM). DO YOU WISH TO ENTER AN ",/
200 "ALTERNATE VALUE? (Y OR N) ")
210 READ(5,23) ANS
220 IF(ANS.EQ."Y") GO TO 55
230 IF(ANS.EQ."N") GO TO 77
240 55 IF(IUNITS) 305,305,306
250 305 RHOU="LBS/CUFT"
260 GO TO 307
270 306 RHOU="KG/CUM"
280 307 WRITE(6,145) RHOU
290 145 FORMAT(3X,"ENTER UNIT WEIGHT OF ROCK, ",A8,"(F6.1)"")
300 READ(5,25) RHODR
310 GO TO 88
320 88 CONTINUE
330 WRITE(6,335)
340 335 FORMAT(3X,"UNIT WEIGHT OF CONCRETE IS ASSUMED TO BE",/
350 "149.5 LBS/CUFT(2423 KG/CUM). DO YOU WISH TO ENTER AN ",/
360 "ALTERNATE VALUE? (Y OR N) ")
370 READ(5,23) ANS
380 IF(ANS.EQ."Y") GO TO 555
390 IF(ANS.EQ."N") GO TO 777
400 555 WRITE(6,345) RHOUC
410 345 FORMAT(3X,"ENTER UNIT WEIGHT OF CONCRETE, ",A8,"(F6.1)"")
420 READ(5,25) RHOC
430 GO TO 888
440 888 CONTINUE
450 WRITE(6,355)
460 355 FORMAT(3X,"UNIT WEIGHT OF CONCRETE IS ASSUMED TO BE",/
470 "149.5 LBS/CUFT(2423 KG/CUM). DO YOU WISH TO ENTER AN ",/
480 "ALTERNATE VALUE? (Y OR N) ")
490 READ(5,23) ANS
500 IF(ANS.EQ."Y") GO TO 555
510 IF(ANS.EQ."N") GO TO 777
520 555 WRITE(6,345) RHOU
1570 778 RHOC=149.5  
1580   GO TO 888  
1590 779 RHOC=2423. 
1600 888 CONTINUE  
1610   IF (UNITS) 80, 80, 83  
1620  80 IF (WATER) 81, 81, 82  
1630  81 RHOW=62.4  
1640   GO TO 86  
1650  82 RHOW=64.  
1660   GO TO 86  
1670  83 IF (WATER) 84, 84, 85  
1680  84 RHOW=1000.  
1690   GO TO 86  
1700  85 RHOW=1025.6  
1710  86 CONTINUE  
1720   IF (UNITS.EQ.0) VOLU="CY"  
1730   IF (UNITS.EQ.1) VOLU="CM"  
1740   IF (UNITS.EQ.0) WU="TONS"  
1750   IF (UNITS.EQ.1) WU="MT"  
1760   WRITE(6,700) WU  
1770  700 FORMAT(/,3X,"INPUT UNIT PRICE OF UNIFORM ARMOR ROCK, 
1780   & IN PLACE("$,A4,")")  
1790   READ(5,710) UNITC(1)  
1800  710 FORMAT(F7.2)  
1810   WRITE(6,720) WU  
1820  720 FORMAT(/,3X,"INPUT UNIT PRICE OF GRADED ARMOR ROCK, 
1830   & IN PLACE("$,A4,")")  
1840   READ(5,710) UNITC(2)  
1850   WRITE(6,800) VOLU  
1860  800 FORMAT(/,3X,"INPUT UNIT PRICES("$,A2," IN PLACE) 
1870   & FOR THESE ARMOR UNITS:"))  
1880   WRITE(6,810)  
1890  810 FORMAT(/,3X,"PLAIN CUBE,MOD.CUBE, 
1900   & TETRAPOD, QUADRIPOD, HEXAPOD, TRIBAR, TOSKANE, & DOLOS")  
1910   WRITE(6,820)  
1920  820 FORMAT(/,3X,"FORMAT(8F7.2); SEPARATE PRICES BY COMMAS")  
1930   READ(5,830) (UNITC(I), I=3,10)  
1940  830 FORMAT(BF7.2)  
1950   WRITE(6,840) WU  
1960  840 FORMAT(/,3X,"INPUT UNIT PRICE FOR CORE MATERIAL 
1970   & (GRADED ROCK), "$",A4)  
1980   READ(5,850) UNITC(11)  
1990  850 FORMAT(F7.2)  
2000C  
2010C * * SET COT LEEWARD SLOPE * *  
2020C  
2030   DO 999 II=1,2  
2040   IF (II.EQ.1) COTP=1.5  
2050   IF (II.EQ.2) COTP=2.0  
2060C  
2070C * * SET COT SEAWARD SLOPE * *  
2080C  
2090   DO 998 JJ=1,4  

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2100 IF(II.EQ.2 .AND. JJ.EQ.1) GO TO 998
2110 IF(JJ.EQ.1) CO=1.5
2120 IF(JJ.EQ.2) CO=2.0
2130 IF(JJ.EQ.3) CO=2.5
2140 IF(JJ.EQ.4) CO=3.0
2150 THETA=ATAN(1./CO)
2160 THETAP=ATAN(1./COP)
2170 PRINT HEADINGS FOR OUTPUT TABLE * *
2180 WRITE(6,851) TITLE
2190 851 FORMAT(1H1,11X,* " ,A80," *")
2200 WRITE(6,920) TITLE
2210 920 FORMAT(///,30X,"TABLE OF COMPARATIVE BREAKWATER"
2220 & QUANTITIES AND COSTS")
2230 WRITE(6,930) TITLE
2240 930 FORMAT(/,32X,"SEAWARD SLOPE = 1: "F3.1,
2250 & LEEWARD SLOPE = 1: "F3.1)
2260 WRITE(6,940) TITLE
2270 940 FORMAT(/,32X,"WATER DEPTH = "F4.1,1X,A2)
2280 WRITE(6,950) TITLE
2290 950 FORMAT(/,32X,"INCIDENT WAVE: SIG. HEIGHT = "F4.1,1X,A2,
2300 & PEAK PERIOD = ",F4.1,"SEC")
2310 IF(H1.LT.H1) WRITE(6,945) H1,LU
2320 945 FORMAT(32X,"DEPTH LIMITED WAVE HEIGHT = "F4.1,1X,A2," ")
2330 IF(ANSW.EQ."Y") WRITE(6,951) H2,LU,T(2)
2340 951 FORMAT(32X,"INCIDENT WAVE FOR TRANSMISSION COMPUTATIONS:",
2350 &/",32X,"SIG. HEIGHT = ",F4.1,1X,A2," PEAK PERIOD = ",F4.1,
2360 &"SEC")
2370 IF(ANSW.EQ."Y") AND.H2.LT.H(2) WRITE(6,945) H2,LU
2380 945 FORMAT(/,32X,"MAX TRANSMITTED WAVE HEIGHT = "F4.1,1X,A2)
2390 WRITE(6,952) EXCP
2400 952 FORMAT(32X,"% EXCEEDANCE")
2410 WRITE(6,960) TITLE
2420 960 FORMAT(/,53X,"* * * RELATIVE VALUES PER UNIT"
2430 & TRUNK LENGTH * * *)
2440 WRITE(6,970) TITLE
2450 970 FORMAT(/,11X,"ARMOR UNIT","2X","SIZE","2X","CREST","2X","
2460 &2X","CREST","2X","NO.ARMOR","2X","ARMOR","2X","CORE","2X","
2470 &2X","ARMOR","2X","CORE","2X","TOTAL")
2480 WRITE(6,980) WU,LU,LU,VLU,VLU
2490 980 FORMAT(23X,"","A4","","2X","HT.","A2","","2X","WDTH","A2","","2X","
2500 &","UNITS","3X","VOL","A2","","2X","VOL","A2","","2X","COST","$",""
2510 &","COST","$","2X","COST","$")
2520 SET ARMOR UNIT TYPE * *
2530 DO 997 NN=1,10
2540 ICHECK=0
2550 IF(NN.EQ.1) RHO=RHOR
2560 IF(NN.EQ.2) RHO=RHOC
2570 IF(NN.EQ.3) RHO=RHOC
2580 IF(NN.EQ.4) RHO=RHOC
2590 IF(NN.EQ.5) RHO=RHOC
2600 IF(NN.EQ.6) RHO=RHOC
2610 IF(NN.EQ.7) RHO=RHOC
2620 IF(NN.EQ.8) RHO=RHOC
2630 IF(NN.EQ.9) RHO=RHOC
2640 IF(NN.EQ.10) RHO=RHOC

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2630 IF(ABS(H1-H(1)) .LT. 0.000000001) KD=XKDNBR(NN)
2640 IF(H1.LT.H(1)) KD=XKDRBR(NN)
2650 W=RHO/(H1*3)/(KD*COT*(((RHO/RH0)-1.)*3))
2660 AT=2.*((W/RHO)**0.33)
2670 IF(NN.EQ.3) AT=2.*((W/RHO)**0.33)
2680 IF(NN.EQ.4) AT=1.03*((W/(0.78*RHO)**0.33)
2690 IF(NN.EQ.5) AT=1.361*((W/(0.16*RHO)**0.33)
2700 IF(NN.EQ.6) AT=1.03*((W/(0.16*RHO)**0.33)
2710 IF(NN.EQ.7) AT=1.03*((W/(0.16*RHO)**0.33)
2720 IF(NN.EQ.8) AT=1.03*((W/(0.16*RHO)**0.33)
2730 IF(NN.EQ.9) AT=1.03*((W/(0.16*RHO)**0.33)
2740 IF(NN.EQ.10) AT=1.03*((W/(0.16*RHO)**0.33)
2750 TOPW=3.0*XKDELTA(NN)#((W/RHO)**0.33)
2760 IF(DUNITS.EQ.2) C=1.
2770 IF(DUNITS.EQ.1) C=.3
2780C * * INCREASE CREST ELEVATION (C) TO SATISFY HTMAX * *
2790C 2800 IF(ICHECK.EQ.0) GO TO 500
2810 IF(INUNITS.EQ.0) C=C+1.
2820 IF(INUNITS.EQ.1) C=C+.3
2830 500 HS=C+D
2840 505 TANB=1./COT
2850 IF(INUNITS.EQ.0) SURF=TANB/SDRT(H2P/(5.12*T(2)*T(2)))
2860 IF(INUNITS.EQ.1) SURF=TANB/SDRT(H2P/(1.56*T(2)*T(2)))
2870 550 RH=RA(NN)*SURF/(1.*RB(NN)*SURF)
2880 R=H2P*RH
2890 IF(ANSWIND.EQ."N") GO TO 551
2900 IF(INUNITS.EQ.1) WIND=WIND/447
2910 551 WF=1.+(WF*(C/R+.1)*SIN(THETA))
2920 R=R*WF
2930 551 FR=C/R
2940 CR=0.51 -0.11*TOPW/HS
2950 KT=CR*(1.-FR)
2960 HT=H2P*KT
3000C * * CHECK IF HTMAX CRITERIA MET * *
3010C 3020C 604 IF(H.T.LE.HTMAX) GO TO 605
3030 605 ICHECK=1
3040 606 GO TO 996
3050C * * VOLUME AND COST COMPUTATIONS * *
3060C 3070C 605 CONTINUE
3080C 3090C 3100 TVOL=(TOPW*HS)+((HS**2)*COT/2.)+(HS**2)*((CDTP/2.)
3110 AH=C+(1.5*H1)
3120 IF(AH.GT.HS) AH=HS
3130 SAH=C+(0.5*H1)
3140 AVOL=(TOPW*AT)+(AT*(AH/SIN(THETA)))+(AT*(SAH/SIN(THETAP)))
3150 CVOL=TVOL-AVOL

B17
AA = TDPW + (AH/SIN(THETA)) + (SAH/SIN(THETAP))

IF (IUNITS.EQ.0) CVOL = CVOL/27.

P(1) = 0.37

NO = 2.*AA*XKDELTA(NN)*(1.-P(NN))*((RHO/W)**0.67)

IF (IUNITS.EQ.0) CC = (UNITC(11)**0.63*CVOL*RHOR)/2000.

IF (IUNITS.EQ.1) CC = (UNITC(11)**0.63*CVOL*RHOR)/1000.

IF (NN.GE.3) AVOL = (W/RHOC)*NO

IF (IUNITS.EQ.0) AVOL = AVOL/27.


IF (IUNITS.EQ.1) W = W/1000.

AC = AVOL*UNITC(NN)

IF (NN.LE.2) AC = NO*W*UNITC(NN)

TC = AC + CC

IF (NN.EQ.1) UNIT = "ROCK, UNIF."

IF (NN.EQ.2) UNIT = "ROCK, BRAZ."

IF (NN.EQ.3) UNIT = "CUBE"

IF (NN.EQ.4) UNIT = "MOD. CUBE"

IF (NN.EQ.5) UNIT = "TETRAPOD"

IF (NN.EQ.6) UNIT = "QUADRIPOD"

IF (NN.EQ.7) UNIT = "HEXAPOD"

IF (NN.EQ.8) UNIT = "TRIBAR"

IF (NN.EQ.9) UNIT = "TOSKANE"

IF (NN.EQ.10) UNIT = "DOLOS"

* * TABLE OUTPUT * *

WRITE(6,900) UNIT, W, C, TDPW, NO, AVOL, CVOL, AC, CC, TC

900 FORMAT(/,11X,A10,F8.1,2X,8(2X,FB.1))

CONTINUE

982 FORMAT(/,11X,"NOTES: 1. CREST HEIGHT IS ABOVE STILL"

& WATER LEVEL",/19X,"2. ARMOR VOLUME FOR ROCK IS THE"

& TOTAL CROSS-SECTION VOLUME",/19X,"3. ARMOR VOLUME"

& FOR ARTIFICIAL UNITS IS THE CONCRETE VOLUME")

CONTINUE

998 CONTINUE

999 CONTINUE

WRITE(6,985)

985 FORMAT(/,3X,"DO YOU WISH TO MAKE ANOTHER RUN?"

READ(5,986) ANW

986 FORMAT(A2)

IF (ANW.EQ."Y") GO TO 15

IF (ANW.EQ."N") GO TO 14

14 STOP

END
APPENDIX C: COMPUTER PROGRAM BWLOSS2

Estimation of Economic Losses from Transmitted Waves

Program purpose

1. BWLOSS2 fits an Extremal Type I long-term probability distribution to transmitted wave height data and estimates expected annual economic losses due to wave attack after a protective breakwater has been built.

Program capabilities

2. Estimation of incremental economic benefits directly related to a rubble-mound breakwater built for wave protection requires that the costs to the beneficiaries with the breakwater in place be determined. A rubble-mound breakwater built so high that no waves are transmitted during the worst conceivable conditions is seldom affordable. It is often more cost effective to accept a small amount of risk that waves from a very severe storm will be transmitted and cause an estimable degree of economic loss. The damage caused by these transmitted waves is assumed to follow a previously derived economic loss function of wave height.

\[
L(H_s) = L_{max} \left( 1 - e^{A(H_s - H_{Lo})} \right)
\]

where

- \( L(H_s) \) = the economic losses caused by a storm of significant wave height \( H_s \)
- \( L_{max} \) = the maximum conceivable loss due to wave attack
- \( H_{Lo} \) = the maximum wave height for which losses can be neglected
- \( A \) = a coefficient determined by regression of historical \( H_s \), \( L(H_s) \) information

The computer program BWLOSS1 (Appendix A) is available to derive this economic loss function from property valuations, coastal engineering data on wave climate, and historical economic loss data. The program BWLOSS2 requires that \( L_{max} \), \( H_{Lo} \), and \( A \) be input, along with the \( \epsilon \) and \( \phi \) parameters of the Extremal Type I cumulative probability distribution of significant wave heights \( H_s \) which defines the wave climate incident on the seaward side of the breakwater as follows:
\[ F(H_s) = e^{-e^{\frac{(e-H_s)\phi}{\phi}}} \]  \hspace{1cm} (C2)

3. At least two data points of transmitted wave height versus a return period of the associated incident wave height are also required to transform the incident probability distribution to an Extremal Type I cumulative probability distribution of transmitted waves \( F(H_t) \). The transformation is accomplished by a least squares fit of the Extremal Type I function above to the \( H_t \) points, given the cumulative probability of the corresponding incident \( H_s \) value as represented by the traditional return period. The nonlinear coefficient of correlation and sum of least squares are computed to indicate the goodness of the fit. A table of residuals is optionally provided.

4. The transmitted wave heights during any storm represented by \( H_s \) are probably not Rayleigh distributed, but the transmitted wave height associated with a given incident significant wave height is assumed to be at the 13.5 percent exceedance level among all transmitted waves (including those of zero height). This is the same exceedance level as the significant wave height in a Rayleigh distributed sea state. The methods of Andrew and Smith (in preparation) can be applied to estimate the transmitted wave heights at other exceedance levels.

5. The program BWLOSS2, in a manner similar to its sister program BWLOSS1, computes an expected, or long-term, average annual economic loss due to transmitted waves by the following formulation:

\[ E\{\frac{\$L'}{yr}\} = \lambda \int_{H_{LO}}^{H_{S\infty}} \frac{dF(H_t)}{dH_t} dh_t \]  \hspace{1cm} (C3)

where \( \frac{dF(H_t)}{dH_t} \) is the probability density function \( f(H_t) \) associated with \( F(H_t) \). \( \lambda \) is the Poisson parameter, or average number of extreme events per year, as defined to derive \( F(H_s) \). The assumption of a random number of extreme events per year which can be represented by a mean value independent of individual \( H_s \) (or \( H_t \)) values, is critical to the above definition of \( E\{\frac{\$L'}{yr}\} \). \( \$L(H_s) \) is taken to describe also \( \$L(H_t) \) since the transmitted waves are now the incident waves to the facilities and operations incurring losses. \( H_t \) thus represents the intensity of the transmitted wave.
climate associated with an incident wave climate of intensity \(H_S\) and return period defined as

\[
RT(H_S) = \frac{1}{\lambda \left[ 1 - F(H_S) \right]}
\]

(C4)

6. The upper limit of integration \(H^\infty\) is taken as the \(H_L\) value corresponding to a probability of exceedance in any year of 0.0000001. The lower limit of integration is the \(H_{LO}\) value below which losses are assumed as zero. The extrapolation of \(F(H_S)\) to values of \(H_S\) below that originally used as a threshold for data to derive the \(\epsilon\) and \(\phi\) parameters is probably conservative, but this question will be the subject of further study. The choice of the threshold \(H_S\) as equal to \(H_{LO}\) would resolve any problems, however. The integration between \(H_{LO}\) and \(H^\infty\) is approximated numerically by an application of Simpson's rule with 100 intervals.

Program input

7. The program BWLOSS2 is completely interactive in its present form. It is written in FORTRAN IV as implemented on the US Army Engineer Waterways Experiment Station (WES) Honeywell DPS-8 mainframe system, but it is easily adaptable to microcomputer systems.

Sample Interactive Session

8. The following sample interactive session demonstrates the required user input:

```
INPUT $Lmax FOR LOSS CURVE: $L(H_S)=Lmax*exp(-A(H_S-HLo))
=20.0
INPUT HLo, THE MAXIMUM WAVE HEIGHT FOR WHICH LOSSES ARE NEGLIGIBLE
=2.0
INPUT A, THE REGRESSION COEFFICIENT
=-.086
INPUT AVG NUMBER OF EXTREMAL EVENTS PER YEAR, THE POISSON "LAMBD" PARAMETER
=4.0
INPUT THE NUMBER OF TRANSMITTED WAVE HEIGHT, RETURN PERIOD DATA POINTS YOU HAVE.
=3
INPUT THE DATA POINTS-ONE AT A TIME.
INPUT TRANSMITTED WAVE HEIGHT, COMMA, THEN RETURN PERIOD IN YEARS
=2.0,20
=2.7,50
=4.0,100
PRINT RESIDUAL TABLES Y/N?
=Y
```
LEAST SQUARES RESULTS

EXTREMAL TYPE I

\[ F(h_5) = \Pr(H_5 < h_5) = \exp\left(-\exp\left(-\frac{h_5 - \text{E}PSI}{\text{PHI}}\right)\right) \]

\[ \text{E}PSI = -3.856 \]
\[ \text{PHI} = 1.294 \]
\[ \text{MEAN} = -3.189 \]
\[ \text{VARIANCE} = 2.755 \]

<table>
<thead>
<tr>
<th>XVALUE</th>
<th>YVALUE</th>
<th>YEST</th>
<th>DIFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0000</td>
<td>0.9875</td>
<td>0.9892</td>
<td>0.0017</td>
</tr>
<tr>
<td>2.7000</td>
<td>0.9950</td>
<td>0.9937</td>
<td>0.0013</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.9975</td>
<td>0.9977</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

NON-LINEAR CORRELATION IS 0.91370593
SUM SQR RESIDUALS IS 0.000000

RETURN PERIOD TABLE

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>10.00</td>
<td>0.90</td>
</tr>
<tr>
<td>25.00</td>
<td>2.10</td>
</tr>
<tr>
<td>50.00</td>
<td>3.00</td>
</tr>
<tr>
<td>100.00</td>
<td>3.90</td>
</tr>
</tbody>
</table>

WEIBULL

\[ F(h_s) = \Pr(H_s < h_s) = 1 - \exp\left(-\frac{h_s - BETA}{ALPHA}\right) \]

\[ ALPHA = 0.4443 \]
\[ BETA = 0.069 \]
\[ MEAN = 0.175 \]
\[ VARIANCE = 0.217 \]

<table>
<thead>
<tr>
<th>XVALUE</th>
<th>YVALUE</th>
<th>YEST</th>
<th>DIFF</th>
</tr>
</thead>
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<tr>
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<td>0.9875</td>
<td>0.9886</td>
<td>0.0011</td>
</tr>
<tr>
<td>2.7000</td>
<td>0.9950</td>
<td>0.9940</td>
<td>0.0010</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.9975</td>
<td>0.9977</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

NON-LINEAR CORRELATION IS 0.95775742
SUM SQR RESIDUALS IS 0.000000

RETURN PERIOD TABLE

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>0.01</td>
</tr>
<tr>
<td>10.00</td>
<td>1.30</td>
</tr>
<tr>
<td>25.00</td>
<td>2.14</td>
</tr>
<tr>
<td>50.00</td>
<td>2.93</td>
</tr>
<tr>
<td>100.00</td>
<td>3.86</td>
</tr>
</tbody>
</table>
LOG EXTREMAL

\[ F(hs) = \Pr(Hs) = \exp(-((\text{BETA}/hs)^\text{ALPHA})) \]

\[
\begin{align*}
\text{ALPHA} &= 2.3009 \\
\text{BETA} &= 0.280 \\
\text{MEAN} &= 0.453 \\
\text{VARIANCE} &= 0.391 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>XVALUE</th>
<th>YVALUE</th>
<th>YEST</th>
<th>DIFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0000</td>
<td>0.9875</td>
<td>0.9885</td>
<td>0.0010</td>
</tr>
<tr>
<td>2.7000</td>
<td>0.9950</td>
<td>0.9942</td>
<td>0.0008</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.9975</td>
<td>0.9977</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

NON-LINEAR CORRELATION IS 0.97002341

SUM SQR RESIDUALS IS 0.000000

RETURN PERIOD TABLE

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>1.05</td>
</tr>
<tr>
<td>10.00</td>
<td>1.42</td>
</tr>
<tr>
<td>25.00</td>
<td>2.13</td>
</tr>
<tr>
<td>50.00</td>
<td>2.00</td>
</tr>
<tr>
<td>100.00</td>
<td>3.89</td>
</tr>
</tbody>
</table>

SELECT A DISTRIBUTION...

EXTREMAL TYPE 1...1
WEIBULL.......2
LOG-EXTREMAL....3
SELECT 1, 2, OR 3
=1

EXPECTED ANNUAL LOSS IN MILLIONS OF DOLLARS IS 0.0865405

*
A sample program listing for BWLOSS2 is given below:

```fortran
10C PROGRAM *BWLOSS2*. 9/85 VERSION
20C DESIGN BRANCH-COASTAL ENGINEERING RESEARCH CENTER
30C U.S. ARMY ENGINEERS WATERWAY EXPERIMENT STATION
40C VICKSBURG, MS 39180
50C FOR FURTHER INFORMATION CONCERNING THE APPLICATION
60C OF *BWLOSS2*, CALL
70C ORSON P. SMITH (681)-634-2013 FTS:542-2013 OR
80C ROBERT B. LUND (681)-634-2068 FTS:542-2068
90C
100C INITIALIZE STRINGS, VARIABLES, AND FUNCTIONS
110 DIMENSION X(200),Y(200),D1(200),Y2(200),Z1(200)
120 DIMENSION ALPHA(3),BETA(3)
130 REAL LAMBDA
140 CHARACTER*6 WORD(14)
150 CHARACTER*1 LG
160 WORD(1)="******************************************************************************'
170 WORD(2)="BWLOSS2 IS A PROGRAM WHICH FITS LONG-TERM CUMULATIVE "
180 WORD(3)="TIVE PROBABILITY DISTRIBUTIONS TO TRANSMITTED SIGNIFICANT "
190 WORD(4)="IFICANT WAVE HEIGHT DATA TO ESTIMATE EXPECTED "
200 WORD(5)="ANNUAL ECONOMIC LOSSES DUE TO WAVE ATTACK AFTER A "
210 WORD(6)=" BREAKWATER HAS BEEN BUILT. THE PROGRAM REQUIRES "
220 WORD(7)=" DATA ON A PREVIOUSLY DEFINED LOSS VS. SIGNIFICANT "
230 WORD(8)=" WAVE HEIGHT FUNCTION (REF. PROGRAM BWLOSS1) AND AT "
240 WORD(9)=" LEAST 2 POINTS OF TRANSMITTED WAVE HEIGHT, RETURN "
250 WORD(10)=" PERIOD DATA. THE DISTRIBUTIONS OF TRANSMITTED WAVE "
260 WORD(11)=" HEIGHTS ARE CALCULATED BY THE METHOD OF LEAST "
270 WORD(12)=" SQUARES. A RESIDUAL TABLE CAN BE OPTIONALLY "
280 WORD(13)=" PRINTED."
290 WORD(14)=WORD(1)
300 DO 3 I=1,14
310 WRITE(6,4) WORD(I)
320 CONTINUE
330 4 FORMAT(A60)
340 3 CONTINUE
350 PRINT,''
360 PRINT,''
370 PRINT,'GET THE FACTS'
380 5 PRINT,'INPUT $\text{max}$ FOR LOSS CURVE: $L(H_s) = 1 - \exp(A(H_s-H_{Lo}))'
390 READ,C1
400 IF(C1 .LE. 0) GO TO 5
410 PRINT,'INPUT $H_{Lo}$, THE MAXIMUM WAVE HEIGHT FOR WHICH LOSSES ARE NEGLIGIBLE'
420 READ,C3
430 IF(C3 .LE. 0) GO TO 10
440 PRINT,'INPUT $A$, THE REGRESSION COEFFICIENT'
450 READ,C2
460 IF(C2 .GT. 0) GO TO 15
470 PRINT,'INPUT AVERAGE NUMBER OF EXTREMAL EVENTS PER YEAR, '
480 PRINT,'THE POISSON "$\lambda$" PARAMETER'
490 READ,LAMBDA
500
```

Sample Program Listing
510 IF( LAMBDA .LE. 0 ) GO TO 17
520 20 PRINT,'INPUT THE NUMBER OF TRANSMITTED WAVE HEIGHT,'
530 PRINT,'RETURN PERIOD DATA POINTS YOU HAVE.'
540 READ,N2
550 IF( N2 .LT. 2 ) PRINT,'YOU NEED AT LEAST TWO POINTS'
560 IF( N2 .LT. 2 ) GO TO 20
570 IF( N2 .GT. 100 ) PRINT,'TOO MANY POINTS-100 IS MAXIMUM'
580 IF( N2 .GT. 100 ) GO TO 20
590 PRINT,'INPUT THE DATA POINTS-ONE AT A TIME.'
600 PRINT,'INPUT TRANSMITTED WAVE HEIGHT,COMMA,THEN RETURN PERIOD IN YEARS '
610 DO 30 I=1,N2
620 READ,I(I),Y(I)
630 30 Y(I)=1.0-1.0/(LAMBDA-Y(I))
640
650C PUT TRANSMITTED WAVE HEIGHTS IN ORDER
660 CALL ORDERIN2,X,Y)
670 CALL WAVTRNS(X,Y,ALPHA,BETA,LAMBDA,N2)
680 CALL EXPCT(C1,C2,C3,ALPHA,BETA,LAMBDA)
690 STOP
700 END
710
720C SUBROUTINE TO PUT DATA IN ORDER-LOWEST TO HIGHEST
730 SUBROUTINE ORDERIN2,X,Y)
740 DIMENSION X(100),Y(100)
750 DO 20 K=2,N2
760 J=N2-K+2
770 DO 10 I=1,J-1
780 IF( X(I) ,LT. X(I+1)) GO TO 10
790 T1=X(I)
800 T2=Y(I)
810 X(I)=X(I+1)
820 Y(I)=Y(I+1)
830 X(I+1)=T1
840 Y(I+1)=T2
850 10 CONTINUE
860 20 CONTINUE
870 RETURN
880 END
890
900C SUBROUTINE WAVTRNS(HS,Y,ALPHA,BETA,LAMBDA,N)
910 DIMENSION YACT(200,3),YEST(200,3),DUM1(200),DUM2(200),HS(200),Y(200)
920 DIMENSION YAV6(3),CORR(3),A(3),B(3),ST(3),SB(3),ALPHA(3),BETA(3),VAR(3)
930 DIMENSION RET(5),CHS(5,3)
940 REAL MEAN(3)
950 REAL LAMBDA
960 F1(X)=EXP(-EXP(-X-EPSI)/PHI))
970 F2(X)=1.0-EXP(-1/(X/SIGMA)**2))
980 F3(X)=EXP(-((SIGMA2/X)**2))
990 CHARACTER*20 IFLAG(3)
1000 CHARACTER*17 DEF
1010 CHARACTER*34 FORM(3)
1020 CHARACTER*26 TITLE(1)
1030 CHARACTER*1 LOGIC

C7
INITIALIZATION OF STRINGS AND CONSTANTS

IFLAG(1) = 'EXTREME TYPE I'
IFLAG(2) = 'WEIBULL'
IFLAG(3) = 'LOG EXTREMA'

DEF = 'F(hs) = Pr(Hs < hs) = '

FORM(1) = 'EXP(-EXP(-(hs-EPsi)/PHI))'
FORM(2) = '1-EXP(-(hs/BETA)**ALPHA))'
FORM(3) = 'EXP(-(BETA/hs)**ALPHA))'

TITLE = 'LEAST SQUARES RESULTS'
DATA RET = 0.0, 10.0, 25.0, 50.0, 100.0/
EULER = .5772156649
C2 = .7796968

PRINT, 'PRINT RESIDUAL TABLES(Y/N)?'
READ(5,17) LOGIC

FORMAT(A1)

IF(LOGIC .NE. 'N') .AND. LOGIC .NE. 'Y') GO TO 16
DO 25 I=1,N
DO 30 K=1,3
YACT(I,K) = YACT(I,1)
CONTINUE
CONTINUE

DO 40 J=1,N
SX = SX + HS(J)
SY = SY + YACT(J,1)
SXX = SXX + HS(J)**2
SLX = SLX + AL06(HS(J))
SLXX = SLXX + (AL0G(HS(J)))**2
SLLY = SLLY - AL0G(AL06(YACT(J,1)))
SLLQY = SLLQY + AL06(AL06(1.0-YACT(J,1)))
SXLLY = SXLLY - HS(J)**AL0G(AL06(YACT(J,1)))
SXXLLY = SXXLLY - AL06(AL0G(HS(J)))*AL06(AL0G(YACT(J,1)))
TOOBIG = TOOBIG + AL06(AL0G(HS(J)))*AL06(AL0G(1.0-YACT(J,1)))
CONTINUE

A(1) = (N*SXLLY-SX*SLLY)/(N*SXI-SI)**2
A(2) = (N*TOOBIG-SLX*SLLQY)/(N*SLXX-SLX)**2
A(3) = (N*SXLLY-SLX*SLLY)/(N*SLXX-SLX)**2
1570 \( B(1) = (SIX*SLLY-SXLLY*SIX)/(N*SIX-SI*X^2) \)
1580 \( B(2) = (SLIX*SLLY-TOOBIG*SLIX)/(N*SLIX-SLIX^2) \)
1590 \( B(3) = (SLIX*SLLY-SLILY*SLIX)/(N*SLILY-SLIX^2) \)

1600C CALCULATE PARAMETERS OF EACH DISTRIBUTION FROM SLOPE AND INTERCEPT DATA

1610 \( PHI = 1.0/A(1) \)
1620 \( EPSI = -B(1)/A(1) \)
1630 \( C = A(2) \)
1640 \( SIGMA = EXP(-B(2)/A(2)) \)
1650 \( U = A(3) \)
1660 \( SIGMA2 = EXP(-B(3)/A(3)) \)
1670

1680C ASSIGN ARRAYS ALPHA AND BETA THE PARAMETERS OF EACH DISTRIBUTION

1690C FOR EASY PRINT OUT OF DATA

1700 \( ALPHA(1) = EPSI \)
1710 \( BETA(1) = PHI \)
1720 \( ALPHA(2) = C \)
1730 \( BETA(2) = SIGMA \)
1740 \( ALPHA(3) = 0 \)
1750 \( BETA(3) = SIGMA2 \)

1760C CALCULATE APPROXIMATE PROBABILITY AS ESTIMATED BY DISTRIBUTION

1770 DO 100 J=1,N
1780 \( YEST(J,1) = F1(HS(J)) \)
1790 \( YEST(J,2) = F2(HS(J)) \)
1800 \( YEST(J,3) = F3(HS(J)) \)
1810 100 CONTINUE

1820

1830C CALCULATE AVERAGE PROBABILITY AND CORRELATION COEFFICIENTS

1840 DO 110 K=1,3
1850 \( YAVG(K) = SY/FLDAT(N) \)
1860 \( ST(K) = 0 \)
1870 \( SB(K) = 0 \)
1880 110 CONTINUE
1890

1900 DO 120 K=1,3
1910 DO 130 I=1,N
1920 \( ST(K) = ST(K) + (YACT(I,K) - YEST(I,K))^2 \)
1930 \( SB(K) = SB(K) + (YACT(I,K) - YAVG(K))^2 \)
1940 130 CONTINUE
1950 CORR(K) = 1.0 - ST(K)/SB(K)
1960 120 CONTINUE
1970

1980C CALCULATE DATA FOR RETURN PERIOD TABLES

1990 DO 57 J=1,5
2000 \( PROB = 1.0 - 1.0/(LAMBOA*RET(J)) \)
2010 IF (PROB .LE. 0) PROB = .0000001
2020 \( CHS(J,1) = -ALOG(-ALOG(PROB)) * PHI * EPSI \)
2030 \( CHS(J,2) = (-ALOG(1.0-PROB)) * (1.0/C) * SIGMA \)
2040 \( CHS(J,3) = SIGMA2 / ((-ALOG(PROB)) * (1.0/U)) \)
2050 57 CONTINUE
2060

2070C CALCULATE MEAN AND VARIANCE FOR EACH DISTRIBUTION

2080 \( MEAN(1) = EPSI + EULER * PHI \)
2090 \( VAR(1) = 1.6449341 * PHI^2 \)
PARA = 1.0 + 1.0/C
CALL GAMMA(PARA, MEAN)
MEAN(2) = SIGMA + MEAN
FAC1 = SIGMA**2 + MEAN**2
PARA = 1.0 + 2.0/C
CALL GAMMA(PARA, HV2)
FAC2 = SIGMA**2 + HV2
VAR(2) = FAC2 - FAC1
PARA = 1.0 - 1.0/U
CALL GAMMA(PARA, HPC)
MEAN(3) = SIGMA**2 + HPC
PARA = 1.0 - 2.0/U
CALL GAMMA(PARA, HPD)
VAR(3) = SIGMA**2 + HPD - MEAN(3)**2
WRITE OUT THE DATA FOR EACH DISTRIBUTION
WRITE(6, 135) TITLE(1)
FORMAT(/, 20X, A26, //)
DO 150 K = 1, 3
KTEMP = K
WRITE(6, 160) IFLAG(K), DEF, FORM(K)
FORMAT(15X, A30, //, 11X, A34)
IF (K .EQ. 1) WRITE(6, 159) EPSI, PHI
FORMAT(1X, "EPSI = " & F10.3, //, 1X, "PHI = " & F10.3)
IF (K .GT. 1) WRITE(6, 161) ALPHA(K), BETA(K)
WRITE(6, 162) MEAN(K), VAR(K)
FORMAT(1X, "ALPHA = " & F10.4, //, 11X, "BETA = " & F10.3)
WRITE(6, 163) MEAN(K), VAR(K)
FORMAT(1X, "MEAN = " & F10.3, //, 1X, "VARIANCE = " & F10.3)
IF (LOGIC .EQ. 'N') GO TO 207
DO 170 I = 1, N
DUM1(I) = YACT(1, K)
DUM2(I) = YEST(1, K)
HOLD1 = CORR(K)
L2 = N
CONTINUE
CALL RESIDUAL(HS, DUM1, DUM2, HOLD1, L2)
WRITE(6, 208) FORMAT(7X, "RETURN PERIOD TABLE", //, 6X, "YEAR", 13X, "HS")
DO 211 J = 1, 5
WRITE(6, 212) RET(J), CHS(J, K)
FORMAT(1X, F9.2, 8X, F9.2)
CONTINUE
WRITE(6, 165) FORMAT(/)
CONTINUE
RETURN
END
SUBROUTINE TO HELP PRINT OUT DATA
SUBROUTINE RESIDUAL(X, YACT, YEST, CORR, N)
DIMENSION X(N), YACT(N), YEST(N), DIFF(200)
SSR = 0
DO 10 I = 1, N
DIFF(I) = (YACT(I) - YEST(I))^2
SSR = SSR + DIFF(I)

DO 25 I = 1, N
   FORMAT(//,1X,"IVALUE YVALUE YEST DIFF ",/,)
   WRITE(6,15) X(I), YACT(I), YEST(I), SQRT(DIFF(I))
   CONTINUE
   WRITE(6,40) CORR, SSR

RETURN
END

SUBROUTINE TO EVALUATE THE GAMMA FUNCTION

PROGRAM ADJUSTS ALPHA TO BE BETWEEN 1.0 AND 2.0
AND THEN MULTIPLIES BY GF TO COMPENSATE

SUBROUTINE GAMMA(ALPHA, AREA)
DOUBLE PRECISION C(25), SUM
GF = 1.0
IF(ALPHA .LT. 1, 2, 3) THEN
   PRINT, 'TROUBLE IN GAMMA'
   AREA = 1.0
   GO TO 200
END IF

FOR GAMMA OF A NEGATIVE NUMBER
M = INT(ALPHA)
EPSI = ALPHA - FLOAT(M)
IF(M .EQ. 0) GF = GF / ALPHA
IF(M .EQ. 0) ALPHA = ALPHA + 1.0
IF(M .EQ. 0) GO TO 100
IF(M .EQ. 1) GF = 1.0
IF(M .EQ. 1) GO TO 100
DO 10 I = 2, M
   J = M + (I - 1)
   GF = GF / (EPSI + FLOAT(J))
   ALPHA = EPSI + 2.0
   GO TO 100
10 CONTINUE

FOR GAMMA OF A POSITIVE NUMBER
M = INT(ALPHA)
EPSI = ALPHA - FLOAT(M)
IF(M .EQ. 0) GF = 1.0 / (EPSI * (EPSI + 1.0))
IF(M .EQ. 0) ALPHA = EPSI + 2.0
IF(M .EQ. 0) GO TO 100
DO 20 I = 1, 2 - M
   J = M + (I - 1)
   GF = GF / (EPSI + FLOAT(J))
   ALPHA = EPSI + 2.0
20 CONTINUE

COEFFICIENTS FOR SERIES EXPANSION OF THE GAMMA INTEGRAL

SEE HANDBOOK OF MATHEMATICAL FUNCTIONS BY ABRAMOWITZ AND SEGUN
C(2) = 0.5772156649015329
C(3) = -0.5587887152823538
C(4) = 0.042026350480952
C(5) = 1.16658613922915
C(6) = -0.041977345555443
C(7) = 0.08961971527807
C(8) = 0.0721843246663
C(9) = -0.011651675918591
C(10) = 0.0002152416741149
C(11) = -0.00128050823802
C(12) = -0.00020134867928
C(13) = -0.00012504934821
C(14) = -0.00011330272320
C(15) = -0.000082056330417
C(16) = 6.116995E-09
C(17) = 5.0020075E-09
C(18) = 1.812746E-09
C(19) = 1.043427E-10
C(20) = 7.7823E-12
C(21) = -3.69680E-12
C(22) = 5.1E-13
C(23) = -2.06E-14
C(24) = -5.4E-15
C(25) = 1.4E-15

340 C SUM SERIES
3410 SUM = 0.0
3420 DD 50 K = 1,25
3430 SUM = SUM + C(K) * (ALPHA**K)
3440 50 AREA = 6F/SUM
3450 200 RETURN
3460 END
3470
3490 SUBROUTINE EXPCT(M, V2, CUT, ALPHA, BETA, LAMBDA)
3500 DIMENSION ALPHA(3), BETA(3)
3510 DOUBLE PRECISION BU
3520 REAL LAMBDAA
3530 FD1(X) = -(ALOG(ALOG(X))*PHI) + EPSI
3540 FD2(X) = ((-ALOG(1-X))**(1/A1)) * B1
3550 FD3(X) = B2 / ((ALOG(X))**(1/A2))
3560 PDF1(X) = EXP(-EXP(-X-EPSI)/PHI) + EXP(-EPI-X-EPSI)/PHI ) / PHI
3580 PDF3(X) = A2 * (B2**(A2)) * EXP(-B2/X) / X**(A2+1)
3590 CDF1(X) = EXP(-EXP((EPSI-X)/PHI))
3600 CDF2(X) = 1.0 - EXP(-X/B1)**A1
3610 CDF3(X) = EXP(-B2/X)**A2
3620 G(X) = 1.0 - EXP(-V2**(X-CUT))
3630 70 PRINT, 'SELECT A DISTRIBUTION...'
3640 PRINT, 'EXTREMAL TYPE I...1'
3650 PRINT, 'WEIBULL............2'
3660 PRINT, 'LOG-EXTREMAL......3'
3670 PRINT, 'SELECT 1, 2, OR 3'
3680 READ, ID
IF(ID .LT. 1 .OR. ID .GT. 3) GO TO 70
3700 EPSI=ALPHA(1)
3710 PHI=BETA(1)
3720 A1=ALPHA(2)
3730 B1=BETA(2)
3740 A2=ALPHA(3)
3750 B2=BETA(3)
3760 BL=CUT
3770 IF(ID .EQ. 1) BU=FD1(1-.0000001)
3780 IF(ID .EQ. 2) BU=FD2(1-.0000001)
3790 IF(ID .EQ. 3) BU=FD3(1-.0000001)
3800 SUM=0
3810 D=BU-BL
3820 K=-1
3830 DO 10 I=1,101
3840 K=-K
3850 IF( K .LT. 0) C=4
3860 IF( K .GT. 0 ) C=2
3870 IF( I .EQ. 1 .OR. I .EQ. 101) C=1
3880 ADD=FLOAT(I-1)*D/100.0
3890 XV=BL+ADD
3900 IF(ID .EQ. 1 .AND. EXP(-(XV-EPSI)/PHI) .GT. 0.2) GO TO 10
3910 IF(ID .EQ. 2 .AND. ((XV/B1)**A1) .GT. 0.2) GO TO 10
3920 IF(ID .EQ. 3 .AND. ((B2/XV)**A2) .GT. 0.2) GO TO 10
3930 IF( ID .EQ. 1 ) FAC1=PDF1(XV)
3940 IF( ID .EQ. 2 ) FAC1=PDF2(XV)
3950 IF( ID .EQ. 3 ) FAC1=PDF3(XV)
3960 FAC2=6(XV)
3970 IF(XV .LT. CUT) FAC2=0
3980 SUM=SUM+C*FAC1*FAC2
3990 10 CONTINUE
4000 SUM=SUM/300.0*D
4010 WRITE(6,14) SUM+LAMBD
4020 14 FORMAT(/,IX,"EXPECTED ANNUAL LOSS IN MILLIONS OF DOLLARS IS",F14.7)
4030 RETURN
4040 END
APPENDIX D: COMPUTER PROGRAM BWDAMAGE

Estimation of Rubble-Mound Breakwater Armor Layer Expected Damages

Program purposes
1. The computer program BWDAMAGE estimates the expected annual damage to a rubble-mound breakwater, both in cost and percentage of the armor layer displaced. It also estimates the interval that repairs of a specified damage level could be scheduled. This information is vital to breakwater optimization analyses which require an estimate of expected repair costs to fully define the life cycle costs of the structure.

Program capabilities
2. The program assumes that the damage to a particular breakwater configuration is primarily a function of the type of armor unit that has been placed. The breakwater of interest has been designed to suffer a minimum displacement (erosion) of the primary armor during a design event represented by a significant wave height $H_d$. The level of damage acceptable at this level of incident wave energy is in current practice defined by the resolution of scale model tests and is on the order of 1-5 percent. BWDAMAGE assumes that the damage caused by more severe incident events represented by a significant wave height $H_s$ is predictable by a function of the following form:

$$ \%D\left( \frac{H_s}{H_d} \right) = \%D(H_d) e^{S_r(H/H_d - 1)} \tag{D1} $$

where $S_r$ is the "reserve stability factor." $S_r$ is a relative measure of the rate at which breakwaters built with a particular type of armor unit will suffer damage with increasing wave heights. A higher $S_r$ value implies a higher rate of damage. The program applies the $\%D(H_s = H_d)$ and $S_r$ values presented in Table 3 of the main text. The values in Table 3 were determined with monochromatic scale model tests which made no account of rocking and the probable associated armor unit breakage. The predictions using the above formula should not be used in final design decisions without careful verification in laboratory tests with irregular waves and attention to armor unit rocking. The armor units in the program's library include quarystone (rough, uniform), quadripods, tribars, and dolosse. The program also allows input of an armor unit title, $\%D(H_d)$, and $S_r$ for any other type of unit.
3. The expected annual, or long-term average annual, damages are estimated by the following formula, given the parameters which define the incident wave climate by an annual cumulative probability distribution of significant wave heights \( F(H_S) \) as follows:

\[
E\{\%D/yr\} = \lambda \int \%D(H_S/H_d) \left[ \frac{dF(H_S)}{dH_S} \right] dH_S \tag{D2}
\]

where

\[
E\{\%D/yr\} = \text{the expected annual damage}
\]

\[
\lambda = \text{the Poisson parameter or average number of extreme events per year}
\]

\[
\left[ \frac{dF(H_S)}{dH_S} \right] = \text{the probability density function corresponding to } F(H_S)
\]

4. The relation above implicitly assumes that \( H_d \) is a significant wave height representing some design sea state since its probability of exceedance in any year and that of all higher values of \( H_S \) in \( \%D(H_S/H_d) \) are determined by a distribution of significant wave heights. It further assumes that the number of extreme events per year is a random variable which can be represented by a mean value and is independent of the individual significant wave heights representing the intensity of these storms. BWDAMAGE requires the user to input \( \lambda \) and the \( \epsilon \) and \( \phi \) parameters of the Extremal Type I cumulative probability distribution of incident significant wave heights where

\[
F(H_S) = e^{-\left[ (\epsilon-H_S)/\phi \right]}
\]

\( \tag{D3} \)

The associated probability density function is thus

\[
\frac{dF(H_S)}{dH_S} = \frac{F(H_S)}{\phi} e^{-\left[ (\epsilon-H_S)/\phi \right]}
\]

\( \tag{D4} \)

Input of the design significant wave height \( H_d \) and \( \lambda \) the average number of extreme events per year \( \lambda \) are required so the expected annual damages can be estimated as

\[
E\{\%D/yr\} = \lambda \int \%D(H_S/H_d) \left[ \frac{dF(H_S)}{dH_S} \right] dH_S \tag{D5}
\]

D2
where $H_{S_{\infty}}$ is the significant wave height with an annual probability of exceedance of 0.0000001 computed by the program using $F(H_S)$. The program also requires that a representative repair cost per unit volume of armor layer and the total volume of the armor layer per unit trunk length by input. This allows the expected annual cost of repairs $E\{D/yr\}$ to be estimated as

$$E\{\frac{D}{yr}\} = \frac{\text{vol}}{\text{vol}} E\{\frac{D}{yr}\}$$

(D6)

5. The interval at which a specified level of damage will occur is estimated in two ways. The first simply divides the specified level of percent damage by the expected annual amount to give an average repair interval in years. This is considered to be much more appropriate since it includes an account of all the events addressed in the computation of expectation and indirectly measures the real world accumulation of damage from successive storms. The other method involves solving the $%D(H_s/H_d)$ function for the wave height $H_s$ which would cause the specified level of damage. The specified Extremal Type I $F(H_s)$ is then applied to determine the return period $RT$ of this $H_s$ as

$$RT(H_s) = \frac{1}{\lambda [1 - F(H_s)]}$$

(D7)

This second method is much less conservative and will predict an interval on the order of $\lambda$ times as long as that predicted by the first method.

Program input

6. Written in FORTRAN IV as implemented on the US Army Engineer Waterways Experiment Station (WES) Honeywell DPS-8 mainframe system, BWDAMAGE is completely interactive in its present form. A BASIC version written for the IBM PC is also available.

Sample Interactive Session

7. The following sample interactive session demonstrates the required and optional user input:
PROGRAM BWDAMAGE 12/85 VERSION

******************************************************************************
* "BWDAMAGE" ESTIMATES THE EXPECTED ANNUAL DAMAGE TO THE ARMOR LAYER OF A RUBBLEMOUND BREAKWATER, *
* BOTH IN COST AND PERCENTAGE DISPLAYED. IT ALSO ESTIMATES THE INTERVAL THAT REPAIRS OF A SPECI- *
* FIED DAMAGE LEVEL COULD BE SCHEDULED. *
******************************************************************************

ENGLISH OR METRIC UNITS (E/M)?

=E

1....QUARRYSTONE (NON-BREAKING WAVES)
2....QUARRYSTONE (BREAKING WAVES)
3....QUADROPODS (NON-BREAKING WAVES)
4....TRIBARS (NON-BREAKING WAVES)
5....DOLOSSE (NON-BREAKING WAVES)
6....DOLOSSE (BREAKING WAVES)
7....OTHER

SELECT NUMBER OF ARMOR UNIT

=1

INPUT HEIGHT OF DESIGN WAVE IN FEET

=15.0

SELECT A DISTRIBUTION...
EXTREMAL TYPE I ... 1
WIEBULL .............. 2
LOG-EXTREMAL ........ 3
SELECT 1, 2, OR 3

=1

INPUT EXTREMAL TYPE I EPSILON, AND PHI

=-2.27, 3.216

INPUT AVERAGE NUMBER OF EXTREMAL EVENTS PER YEAR, THE POISSON 'LAMBDA' PARAMETER

=4

INPUT THE VOLUME OF ARMOR LAYER IN CUBIC FEET PER LINEAR FOOT

=321.3

INPUT THE COST OF THE ARMOR IN DOLLARS PER CUBIC FOOT

=3.90

1....QUARRYSTONE (NON-BREAKING WAVES)

EXPECTED DAMAGE PER LINEAR FOOT PER YEAR IS $0.42

EXPECTED REPAIR COST PER LINEAR FOOT PER YEAR IS $5.31

DO YOU WANT TO PREDICT A REPAIR INTERVAL (Y OR N)?

=Y

INPUT % DAMAGE TO ARMOR LAYER AT TIME OF REPAIRS

=5.0

THE AVERAGE REPAIR INTERVAL FOR 5.00% DAMAGE BASED ON 0.42% PER YEAR EXPECTED DAMAGE IS 11.8 YEARS

THE RETURN PERIOD OF THE STORM CAUSING 5.00% DAMAGE IS 75.81 YEARS

ANOTHER RUN (Y/N)?

=N
8. A sample program listing for BWDAMAGE (FORTRAN version) is given below:

```
10C PROGRAM "BWDAMAGE" 12/85 VERSION
20C DESIGN BRANCH-COASTAL ENGINEERING RESEARCH CENTER
30C U.S. ARMY ENGINEERS WATERWAYS EXPERIMENT STATION
40C VICKSBURG, MS 39180-0631
50C FOR FURTHER INFORMATION CONCERNING THE APPLICATION
60C OF "BWDAMAGE", CALL.............
70C ORSON P. SMITH 601-634-2013 FTS:542-2013 OR
80C ROBERT B. LUND 601-634-2060 FTS:542-2060 OR
90C DOYLE L. JONES 601-634-2069 FTS:542-2069

100 FORTRAN 4 HONEYWELL DPS-B
120C REF: "COMPUTER PROGRAM WAVDIST" CETN-I-
130C REF: "EXTREMAL STATISTICS IN WAVE CLIMATOLOGY" BY BORGMAN AND RESIO
140C REF: "SHORE PROTECTION MANUAL" 1984 4TH ED., 2 VOLS.
150C REF: "COST EFFECTIVE OPTIMIZATION OF RUBBLEMOUND BREAKWATER" BY O. SMITH

160
170
180C A11 = REGRESSION COEFFICIENT FOR DAMAGE EQUATION
190C A22 = REGRESSION COEFFICIENT FOR DAMAGE EQUATION
200C EPSI = EXTREMAL TYPE 1 LOCATION PARAMETER
210C PHI = EXTREMAL TYPE 1 SCALE PARAMETER
220C A1 = WEIBULL LOCATION PARAMETER
230C B1 = WEIBULL SCALE PARAMETER
240C A2 = LOG-EXTREMAL LOCATION PARAMETER
250C B2 = LOG-EXTREMAL SCALE PARAMETER
260C ID = IDENTIFIES DISTRIBUTIONS
270C LAMBDA= POISSON 'LAMBDA' PARAMETER
280C BL = HEIGHT OF DESIGN WAVE
290C BU = UPPER LIMIT OF INTEGRATION
300C L = E OR M, ENGLISH OR METRIC UNITS
310C K = NUMBER OF ARMOR UNIT USED
320C V1 = VOLUME OF ARMOR LAYER PER LINEAR FOOT (OR METER)
330C V2 = COST OF ARMOR IN DOLLARS PER CUBIC FEET (OR METERS)
340C DP = PERCENTAGE OF DAMAGE TO ARMOR LAYER AT TIME OF REPAIRS
350C
360C INITIALIZE VARIABLES,STRINGS,AND FUNCTIONS
370C DIMENSION A11(7),A22(7)
380C COMMON EPSI,PHI,A1,B1,A2,B2,ID,LAMBDA
390C REAL LAMBDA
400C CHARACTER*1 L,ANS
410C CHARACTER*6 SL(9)
420C CHARACTER*37 DUM,ST(7)
430C CHARACTER*6 UNIT(2)
440C CHARACTER*12 VOL(2)
450C CHARACTER*5 LEN(2)
460C DOUBLE PRECISION BL,BU
470C F(X)=T1*EXP(T2*(X-1))
480C DATA A11/3.,2.,3.,3.,2.,2.,0./
490C DATA A22/6.95,3.65,6.,4.07,1.68,3.55,0./
500C UNIT(1)="METERS"
510C UNIT(2)="FEET"
```
SL(1)='************** BWDAMAGE ESTIMATES THE EXPECTED ANNUAL DAMAGE*
SL(2)='* TO THE ARMOR LAYER OF A RUBBLEMOUND BREAKWATER, *
SL(4)='* BOTH IN COST AND PERCENTAGE DISPLAYED. IT ALSO *
SL(5)='* ESTIMATES THE INTERVAL THAT REPAIRS OF A SPECI-*
SL(6)='* FIED DAMAGE LEVEL COULD BE SCHEDULED. *
SL(7)=SL(1)
SL(9)=
DUM='INPUT THE VOLUME OF ARMOR LAYER IN
ST(1)='1....QUARRYSTONE (NON-BREAKING WAVES)'
ST(2)='2....QUARRYSTONE (BREAKING WAVES)'
ST(3)='3....QUADRIPods (NON-BREAKING WAVES)'
ST(4)='4....TRIBARS (NON-BREAKING WAVES)'
ST(5)='5....DOLOsSE (NON-BREAKING WAVES)'
ST(6)='6....DOLOsSE (BREAKING WAVES)'
ST(7)='7...OTHER'
WRITE(6,43)
FORMAT///,IX,"PROGRAM BWDAMAGE 12/85 VERSION",///)
IU=1
DO 50 I=1,9
WRITE(6,52) SL(I)
FORMAT(IX,A60)
CONTINUE
READ(S,7) L
FORMAT(A1)
IF(L .NE. 'E'.AND. L .NE. 'M') GO TO 9
IF(L .EQ. 'E') IU=2
PRINT,
PRINT,'SELECT ARMOR UNIT
DO 55 I=1,7
WRITE(6,4) ST(I)
FORMAT(IX,A37)
CONTINUE
PRINT,'SELECT NUMBER OF ARMOR UNIT
READ,K
IF( K .LT. 1 .OR. K .GT. 7) GOTO 3
IF( K .NE. 7) GO TO 41
PRINT,'INPUT NAME OF ARMOR UNIT
READ(S,7) ST(7)
FORMAT(A20)
PRINT,'INPUT COEFFICIENTS FOR DAMAGE EQUATION-
PRINT,'y.D(Hd) AND Sr FOR y.D(H/Hd) = yD(Hd)exp[Sr(H/Hd - 1)]
READ, A11(7),A22(7)
PRINT,
PRINT,'INPUT HEIGHT OF DESIGN WAVE IN ',UNIT(IU)
LESE
IB&B
IF(IB .LE. 0) GO TO 41
PRINT,''
INITIALIZE VARIABLES FOR INTEGRATION
T1=A11(K)
T2=A22(K)
CALL EXPCT(T1,T2,BL,BU,SUM,D)
PRINT,''
CONVERT FRACTIONAL DAMAGE TO DOLLARS DAMAGE
IF(DU.EQ.1)PRINT,'INPUT THE VOLUME OF ARMOR LAYER IN CUBIC METERS'
T1=A11(K)
T2=A22(K)
CALL EXPCT(T1,T2,BL,BU,SUM,D)
CONVERT FRACTIONAL DAMAGE TO DOLLARS DAMAGE
IF(DU.EQ.2)PRINT,'INPUT THE VOLUME OF ARMOR LAYER IN CUBIC FEET'
T1=A11(K)
T2=A22(K)
CALL EXPCT(T1,T2,BL,BU,SUM,D)
EXPECTED DAMAGE PER LINEAR METER IS
IF(SUM.LE.8)GO TO 31
PRINT,'DO YOU WANT TO PREDICT A REPAIR INTERVAL(Y OR N)'
READ(5,33)
ANS
IF(ANS.EQ.'N') GO TO 31
INPUT '/DAMAGE TO ARMOR LAYER AT TIME OF REPAIRS'
READ,DP
IF(DP.LT.0.OR.DP.GT.100.0) GO TO 83
HDP=BL*(1+ALOG(DP/T1)/T2))
RT=1./(LAMBDA*(1.-HDP))
REPAIR=DP/D
THE AVERAGE REPAIR INTERVAL FOR DAMAGE IS F6.2,'X' YEARS'
THE RETURN PERIOD OF THE STORM CAUSING DAMAGE IS F6.2,'X' YEARS'
FORMAT(/,1X,"ANOTHER RUN (Y/N)?")
READ(5,35) L

IF (L .EQ. 'Y') GO TO 1
IF (L .NE. 'N') GO TO 31
STOP
END

SUBROUTINE EXPCT(T1,T2,BL,BU,SUM,D2)
COMMON EPSI,PHI,A1,B1,A2,B2,ID,LAMBDA
DOUBLE PRECISION BL,BU
REAL LAMBDA

FD1(X)=-(ALOG(-ALOG(X))*PHI)+EPSI
FD2(X)=((-AL0G(1-X))**(1/A1))*EXP(-((X/B1)**A1)/(B1*A1))
FD3(X)=B2/((-AL0G(X))**(1/A2))
FDL(X)=EXP(-EXP(-(X-EPSI)/PHI))/EXP(-(X-EPSI)/PHI)
PDF1(X)=AL(T(X)**(A1-1))*EXP(-(X/Bl)**A1)/(Bl**A1)
PDF2(X)=A2/(B2**(A2+1))
PDF3(X)=A2**(B2/((A2+1))
CDF1(X)=EXP(-EXP((EPSI-X)/PHI))
CDF2(X)=1.0-EXP(-(X/B1)**A1)
CDF3(X)=EXP(-(B2/X)**A2)
G(X)=T1*EXP(T2*(X-1))

PRINT, 'SELECT A DISTRIBUTION...'
PRINT, 'EXTREMAL TYPE I...' 
PRINT, 'WEIBULL 2'
PRINT, 'LOG-EXTREMAL 3'
READ, ID
IFM. LT. 1 .OR. ID .GT. 3) GO TO 70
IF (ID .EQ. 1) WRITE(6,104)
FORMAT(/,IX,"INPUT EXTREMAL TYPE I EPSILON, AND PHI")
IF (ID .EQ. 1) READ, EPSI, PHI
IF (ID .EQ. 2) WRITE(6,114)
FORMAT(/,IX,"INPUT WEIBULL ALPHA AND BETA")
IF (ID .EQ. 2) READ, A1, B1
IF (ID .EQ. 3) WRITE(6,124)
FORMAT(/,IX,"INPUT LOG-EXTREMAL ALPHA AND BETA")
IF (ID .EQ. 3) READ, A2, B2
WRITE(6,5)
FORMAT(/,IX,"INPUT AVERAGE NUMBER OF EXTREMAL EVENTS PER YEAR,", 
& /,1X,"THE POISSON 'LAMBDA' PARAMETER")
READ, LAMBDA

IF (ID .EQ. 1) BU=FD1(1-.0000001)
IF (ID .EQ. 2) BU=FD2(1-.0000001)
IF (ID .EQ. 3) BU=FD3(1-.0000001)
SUM=0
D=BU-BL

2000 DO 10 I=1,101
2000 K=1
2000 DO 10 I=1,101
2000 K=K
2100 IF (K .LT. 0) C=4
2110 IF (K .GT. 0) C=2
2120 IF (I .EQ. 1 .OR. 1.EQ. 101) C=1
2130 ADD=FLOAT(I-1)*D/LAMBDAT
2140 XV=BL*ADD
2150 IF (ID .EQ. 1 .AND. EXP(-(XV-EPSI)/PHI) .GT. B2.0) GO TO 10
2160 IF (ID .EQ. 2 .AND. ((XV/B1)**A1) .GT. B2.0) GO TO 10
2170 IF (ID .EQ. 3 .AND. ((B2/X)**A2) .GT. B2.0) GO TO 10
2180 IF (ID .EQ. 1) FAC1=PDF1(XV)
2190 IF (ID .EQ. 2) FAC1=PDF2(XV)
2190 IF (ID .EQ. 3) FAC1=PDF3(XV)
2200 FAC2=G(XV/BL)/100.0
2210 CONTINUE
2220 IF (FAC2 .LT. 0) FAC2=0
2230 IF (FAC2 .GT. 1) FAC2=1
2240 SUM=SUM*C*FAC1*FAC2
2260 10 CONTINUE
2270 11 SUM=SUM/300.0*D*LAMBDA
2280 D=SUM/100.0
2290 RETURN
END
APPENDIX E: NOTATION

a  Empirical runup coefficient
A  Empirical runup coefficient
   Empirical coefficient for prediction of economic losses
due to wave attack
A_d  Average eroded cross sectional breakwater area
b  Empirical runup coefficient
B  Breakwater crest width
   Empirical runup coefficient
C  A parameter of the Weibull probability distribution
   Empirical coefficient for wave transmission by overtopping
D'  Dimensionless breakwater damage
D_r  Breakwater damage rate (armor displacement per unit time)
D_15  Sediment size from a graded sample for which 15 percent is finer
   by weight
D_85  Sediment size from a graded sample for which 85 percent is finer
   by weight
D_n50  "Nominal" or equivalent cube dimension of the mean stone weight
   of a graded sample based on size gradation analysis
E{[k]}  Mathematical expectation
f(x)  Statistical probability density function of x
F  Freeboard
F(x)  Cumulative probability density function of x
g  Acceleration of gravity
h_c  Eroded breakwater crest height
h'_c  Original breakwater crest height
H  Wave height
H_*  Critical wave height regarding wave transmission
H_d  Design wave for armor unit sizing
H_i  Incident wave height
H_Lo  Maximum wave height below which economic losses can be neglected
due to wave attack
H_S  Significant wave height
H_t  Transmitted wave height
K_d  Empirical armor stability coefficient, as applied in the Hudson
    formula
\( K_o \) Armor stability coefficient, as applied in the DHI-Iribarren formula

\( K_{to} \) Coefficient of wave transmission by overtopping

\( K_{A} \) Layer coefficient

\( L_p \) Wave length corresponding to the period of peak energy density

\( n \) Number of armor units comprising the layer thickness

\( N \) Armor stability number, as applied in the original Iribarren formula

Number of waves in a specific model test

\( r \) Layer thickness

Statistical correlation coefficient

\( R \) Runup height above still-water level

\( S_r \) Reserve stability coefficient

\( S_2 \) Dimensionless damage, equivalent to the number of \( D_{n50} \) cubes eroded from a cross sectional width of \( D_{n50} \)

\( t \) Time duration of wave exposure

\( T_p \) Wave period of peak energy density

\( T_z \) Average wave period, derived from a count of "zero crossings" with respect to an arbitrary mean

\( T \) Wave period

\( v \) Flow velocity

\( V_{ol} \) Total armor layer volume per unit breakwater trunk length

\( W \) Weight of a primary armor unit

\( W_{50} \) The mean stone weight of a graded sample

\( \Delta \) Armor material density relative to sea water = \( (\rho_r - \rho_w)/\rho_w \)

\( \varepsilon \) A parameter of the Extremal Type I and Weibull probability distributions

\( \theta \) Angle from horizontal of the seaward slope of a breakwater

\( \lambda \) Poisson parameter, denoting the mean number of extreme storms per year

\( \mu \) Coefficient of static friction

\( \nu \) Kinematic fluid viscosity

\( \xi \) Surf similarity parameter

\( \rho_r \) Mass density of the armor material

\( \rho_w \) Mass density of sea water

\( \sigma \) Wave directional spreading parameter
\( \phi \) Natural angle of repose of armor material

Wave angle

A parameter of the Extremal Type I and Weibull probability distribution

\( B_{\text{net}} \) Net incremental benefits

\( B_{\text{Benefits}} \) Total incremental benefits

\( D \) Cost of breakwater repair

\( 1_{\text{st}} \) Construction cost

\( L \) Economic losses due to wave attack

\( L' \) Economic losses due to attack by transmitted waves

\( L_{\text{max}} \) Maximum conceivable economic losses due to wave attack

\( \text{Total} \) Total incremental project feature (breakwater) cost, including associated economic losses

\( D \) Percent of breakwater armor displaced

\( D' \) A critical value of breakwater damage by displacement

\( \$/\text{Vol} \) Cost per unit volume per unit breakwater trunk length