DEPARTMENT OF PSYCHOLOGY
ELECTRONIC DESIGNER'S HANDBOOK

A Practical Guide to Transistor Circuit Design

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The newly qualified electronic engineer often finds difficulty in applying his technical knowledge to practical circuit design. Typical stumbling blocks are the choice of suitable operating currents and voltages, the fixing of these regardless of temperature variations which affect transistor behaviour, and conversion of the results of analysis into actual numbers when many of the transistor parameters included are not mentioned in the manufacturer's data sheets.

The object of this book is to explain circuit design methods which enable the engineer to overcome these obstacles and to design practical circuits. Part One describes these basic techniques, and emphasis is placed on designing circuits to operate correctly in spite of ambient temperature variations and spreads in transistor parameters. In the author’s opinion it is more important that the engineer should understand the basic techniques of design than that he should acquire a superficial knowledge of a great number of circuits. Consequently only a few circuits are dealt with, but these are examined in very great detail. When the techniques are understood, the reader should have little difficulty in applying them to any circuit provided its mode of operation is known.

Part Two shows how novel designs can be synthesized and put into practical form; this section will also interest the experienced designer, as it contains several unusual circuits.

Practical difficulties in design and testing of circuits are examined in Part Three, some of the problems discussed rarely being mentioned in standard textbooks.

Approximations used in the text are justified where necessary by derivations given in the Appendixes.

I should like to thank Mr. P. Broderick who helped clarify many obscure points; the management of Marconi Instruments Limited for permission to publish this work; Mr. P. L. Burton, formerly of...
PREFACE

English Electric Company, who first taught me the principles of circuit design; and finally my wife who undertook the exacting task of preparing the typescript.

March, 1966

T.K.H.

Preface to the Second Edition

In this second edition the text has been brought up to date to include new device types now on the market. The opportunity has also been taken to correct several errors which readers have kindly pointed out and additional explanatory matter has been added to the more difficult parts of the book.

February, 1970

T.K.H.
1—Semiconductor diode properties

Like the thermionic diode, the semiconductor diode has a current-voltage relationship which changes with the polarity of the applied source.

FORWARD CONDUCTION

When the anode is made sufficiently positive with respect to the cathode, forward conduction begins, as shown in region (a) of Fig. 1.1. The voltage which has to be applied before appreciable forward current flows depends on the semiconductor. It cannot be given an absolute value unless the forward current which is considered 'appreciable' is more closely defined, but this 'turn-on' voltage, as it is often called, is approximately 0.1 V for germanium and 0.5 V for silicon at 25°C. As the temperature varies, this voltage (and, in fact, the whole voltage scale of the forward characteristic) changes at the rate of -2 to -2.5 mV/degC.

For higher forward voltages the current increases exponentially and is eventually limited only by the capability of the source and the bulk-resistance of the diode, although a lower limit must be adhered to in practice in order to avoid catastrophic failure due to overheating.

The forward incremental resistance of the diode is of importance in the design of circuits where a diode is subjected to signals of varying amplitude. It is defined as the rate of change of voltage with respect to current at a specified point on the characteristic. Because of the exponential law relating voltage and current, this quantity becomes smaller as the current or voltage at which it is specified increases; in fact, to a close approximation the incremental resistance is inversely proportional to the current at the specified operating
point. Its actual value depends on the area and construction of the diode junction, being low for large junctions.

To summarize the properties of the forward characteristic, conduction begins at about 0-1 V for germanium and 0-5 V for silicon, at 25°C; incremental resistance at any current is inversely proportional to the current; the forward characteristic variation with temperature takes the form of a linear shift of -2 to -2-5 mV/degC in the forward voltage at any given current (see Fig. 1.1). Maximum permissible temperatures of operation are 90–100°C for germanium and 125–200°C for silicon, depending on the manufacturer. These are junction temperatures which are the sum of ambient temperature and temperature rise caused by the mean power dissipated in the junction.

**REVERSE CHARACTERISTIC**

When the anode becomes negative to the cathode, the current which flows is only a small proportion of the forward current which would flow if the polarity were reversed. This is shown by region (b) in Fig. 1.1, and the small current which does flow is called the **reverse leakage current**. This current is hardly dependent on applied voltage (until region (c) is reached) but depends greatly on junction temperature. Its variation with temperature is an exponential tending to infinity and is remembered most easily by the law that it doubles every 9 or 10 degC rise.

The semiconductor material and junction area determine the order of leakage. For instance, a germanium diode of small area may have a reverse leakage of 2 μA compared with a silicon diode of the same area having about 20 nA, both at 25°C. Large-area power diodes might have values of 0-5 mA for germanium and 5 μA for silicon.

Because many circuits involve currents of less than 1 mA, leakage currents often cause changes of circuit operation when changes of ambient temperature occur. For this reason it is important when using diodes to know the maximum possible value of leakage current at the maximum expected junction temperature.

Often the required figures are not available from the manufacturer’s data, since even if a high temperature value is quoted this rarely corresponds to the desired maximum temperature of operation. Fortunately, by applying the ‘doubling every 9 or 10 degC rise’ rule it is simple to transform the data, e.g. a leakage of 100 μA at 100°C will be 50 μA at 90°C, 25 at 80°C, 12-5 at 70°C, and so on.

Some manufacturers give only a low temperature figure and although it might be thought just as easy to apply the rule to obtain values for higher temperatures, this will usually give a highly pessimistic figure since, strictly, not all the leakage current is subject to the ‘doubling’ law. Leakage caused by surface effects and not by semiconductor action remains more or less constant with temperature. The error caused by extrapolating the total leakage is usually negligible for germanium when using the 25°C value, but is often large for silicon. This is particularly true of silicon planar types, where semiconductor leakage is very low compared with surface effects at 25°C. There is no way of correcting for this, in the absence of further information from the manufacturer. The non-varying component of leakage values at 100°C or more can generally be assumed to be negligible; and, in general, the higher the temperature for which the leakage applies, the less will be the error in applying the ‘doubling’ law.

**Breakdown, Zener, and Avalanche Phenomena**

When the applied reverse voltage is increased in magnitude, the leakage current eventually rises and tends to a very high value limited (like the forward current) only by the capability of the source and the diode bulk-resistance (see region (c) in Fig. 1.1).

Depending on the material and construction of the junction, this effect may occur at voltages between about 2-5 V and several thousand volts. By appropriate doping of the semiconductor the voltage at which this current increase takes place can be controlled within close limits. Since the critical voltage is found to be constant throughout the life of the diode the effect, at first sight a defect of the semiconductor diode, has proved to be very useful in practical circuits.

The critical voltage is often referred to as the ‘breakdown voltage’; in spite of this name the diode is not damaged, provided the product of this voltage and the current which is allowed to flow does not represent sufficient power to overheat the junction.

**Zener diodes**

Diodes deliberately designed to exhibit this effect at particular voltages are called Zener diodes. They are specified in several respects: the voltage corresponding to a certain current in the region
(c) (d)—Fig. 1.1; the temperature coefficient of this voltage; the slope between (c) and (d) expressed as a resistance; and the permissible maximum current or power at various temperatures. At present they are invariably silicon diodes.

As will be seen later, the most common uses for the Zener diode are in obtaining stable supplies, in coupling between stages where d.c. levels differ and in clipping circuits. Since the Zener diode is one of the most useful circuit elements, the orders of magnitude of the parameters should be remembered by the designer. This is fortunately easy, because all Zener diodes of a particular power rating have similar characteristics, regardless of the manufacturer.

The relevant facts are:

1. The incremental resistance tends to be less as the nominal Zener voltage increases. A 5-6 V unit will have a resistance of about 50 Ω at the optimum operating current; a 3-3 V unit 100 Ω and a 10 V unit 20 Ω. Tolerance on this resistance is wide, e.g. ±50 per cent.

2. The incremental resistance is a minimum at a particular operating current which depends on junction size; for low-power Zener diodes (300 mW at 25°C) this current usually lies between 5 and 10 mA and is not critical.

3. The temperature coefficient depends on the nominal Zener voltage. Below about 5-6 V the coefficient is negative, and above 5-6 it is positive. At 5-6 V the coefficient is roughly zero. Even if the tolerance on nominal Zener voltage is ignored, the critical voltage for zero coefficient is not exact, and for a nominal 5-6 V unit the usual coefficient is ±0-02 per cent per degree C. A 3-3 V unit is from −0-05 to +0-09 per cent per degree C and a 10 V unit is from +0-05 to +0-09 per cent per degree C. It is worth remembering that 3-3 and 6-8 V units have an actual voltage change with temperature of similar magnitude to that of a transistor or diode junction, namely from 2 to 2-5 mV/degC. This is often useful in circuits where the coefficients of Zener diodes and transistors can be made to add or subtract to give some degree of compensation.

Most manufacturers will supply specially selected units with a coefficient of ±0-002 or even ±0-0005 per cent per degree C. There are three methods by which selection is done. The simplest is to pick 'good' diodes of nominal voltage 5-6 V from a batch by measuring the coefficients at a given current. This is not usually satisfactory for ultra-stable circuits, since the coefficient of such a unit invariably changes with the actual temperature, e.g. ±0-005 per cent at 25°C, −0-01 per cent at −10°C, +0-01 per cent at 60°C, and changes in operating current alter the temperature for which the coefficient is zero. A second method is to carry out the above tests, but at two temperatures, e.g. 0 and 60°C, then choose units which when put in series yield ±0-005 per cent per degree C at both temperatures (one may be −0-01 at 0°C and +0-02 at 60°C, the other +0-01 and −0-025). This is a better method but is still not entirely satisfactory, because at other than the two selected temperatures the coefficient is uncontrolled (and tends to be >0-01 at −20°C) and the series addition naturally results in twice the incremental resistance, the nominal voltage being 11-2 V.

The third method consists in adding normal diodes in series with a Zener diode so that the temperature coefficients cancel. Thus a 6-8 or 8-2 V Zener diode and one or two diodes in series can by careful selection yield a very small temperature coefficient. This gives the best results of the three methods when the units are to maintain a low coefficient over a very wide range of temperatures, because the coefficients which are being cancelled are more or less constant with temperature (only for units near 5-6 V nominal is this not so).
Moreover, the incremental resistance is generally no worse than that of a single 5·6 V Zener diode. The resulting voltage of such a compound unit is in the 8–10 V region.

A similar idea is to cancel the Zener coefficient against the base–emitter coefficient of a transistor. This arrangement is particularly useful in stabilized power supplies where the transistor acts as a reference amplifier (see Chapter 9).

‘Reference’ and ‘regulator’ Zener diodes. The low-power Zener diode of about 300 mW maximum dissipation at 25°C is often called a ‘reference diode’ since one of its main uses is for producing a reference voltage in a stabilized supply. Higher-power Zener diodes of 1 W, 10 W, and even higher 25°C ratings are often used as simple direct regulators, that is the load is connected directly to the Zener diode. These high-power units are therefore known as regulator (Zener) diodes. Because of their large junction area, values of impedance are much lower than the typical reference diode values and currents are much higher, e.g. a 6·8 V, 10 W unit could be run at 200 mA giving an impedance of about 5 Ω.

High-frequency Effects
At high frequencies the junction capacitance of a semiconductor diode becomes significant. Its actual value depends on the construction used, on the area of the device, and also on the applied voltage if reverse-biased. Capacitance is always less at high reverse voltages and its law of variation is given by \( C = K/V_R^n \), where \( n \) lies between \( 1/2 \) and \( 1/3 \).

This dependence of \( C \) on \( V_R \) is useful in enabling resonant circuits to be tuned by variation of applied voltage, the diode being used as a capacitive tuning element. Since the diode is reverse-biased, little current is taken from the voltage source and the arrangement is therefore suitable for remote control, there being negligible voltage drop even in long leads.

One snag is that the diode capacitance is lossy at high frequencies, so that circuit \( Q \)-values are reduced, but much improved results are being obtained with diodes specially designed for this application.

Hole storage
The phenomenon of hole storage occurs when a diode which has been conducting in the forward direction is rapidly reverse-biased. Instead of cutting off and passing a normal reverse circuit, a semiconductor diode will remain conducting for a time \( \tau \) (just as if its connections had been reversed at the same time as the current reversal) and then will suddenly cut off. The value of \( \tau \) depends on the type of diode junction and its area (being longer for large-area devices), and is proportional to the forward current which had been flowing prior to the reversal and inversely proportional to the current flowing immediately after reversal.

Manufacturers’ data gives a value ‘\( Q \)’ which means stored charge (no connection with circuit quality factor \( Q \)) for stated conditions; knowing the current \( I_R \) flowing after reversal, the time \( \tau \) before conduction ceases is given simply by \( \tau = Q/I_R \), provided \( Q \) is the value for conditions just before reversal.

**DIODE APPLICATIONS**

Diodes are often divided into the two classes, signal diodes and rectifier (or power) diodes. This is an arbitrary division and is based on the application in which the manufacturer expects a particular device to be most useful. A large-area device intended for use as a power rectifier will normally be of little use for rectifying signals of 100 kHz, because its capacitance is likely to be high and its hole storage large and unspecified. Naturally, many borderline types exist which are useful for relatively low-power rectification and also for many audio-signal applications (e.g. Mullard OA202, Texas Instruments IS121 and IS922).

There are many applications of diodes which will be readily invented by the designer as required. For a simple example, suppose a triangular voltage waveform has been generated and it is required to lessen its rate of rise whenever its voltage exceeds a certain level \( V_1 \). Figure 1.2 shows a simple solution. \( V_{out} \) will follow \( V_{in} \) until its
potential reaches $V_1$ plus the 'turn-on' level for $D_1$. Above this voltage alternation occurs, so that the waveshape given in the diagram is obtained.

This circuit is simple to understand, but note the slight error caused by $V_{D1}$, the forward drop before significant conduction begins, and note also that the reverse leakage current $I_R$ for $D_1$ must be such that $R_1I_R$ is negligible in comparison with $V_{in}$. At high frequencies this circuit will exhibit two faults: first, the capacitance of $D_1$ will load $R_2$, and, secondly, hole storage will cause the waveform to change slope at a higher point when rising than when falling. The first effect can be ignored if $C_D(R_1 + R_2)$ is small compared with the triangle period; the second is more complicated as it depends on the $Q$ for the diode and on $R_1$ and $R_2$, since these determine the conditions prior to $D_1$ being reversed.

![Fig. 1.3 Logic 'AND' gate](image)

Difficulties will therefore be experienced if high-frequency operation is attempted; 'high-frequency' is a relative term and implies here a cyclic period so small that $r_s$ and $C_D R_2$ are a significant fraction of the period.

As indicated above, this type of circuit can be invented as required, and the same is true of the many diode logic gate circuits, of which one example is given in Fig. 1.3.

This is known as an 'AND' gate because if $V_{in1}$ and $V_{in2}$ can each have only the values 0 or $+V_1$, $V_{out}$ remains at 0 unless both $V_{in1}$ and $V_{in2}$ are at $+V_1$. In this case $V_{out}$ rises to

$$V_{out} = V_{in1} + V_{in2}$$

This and many similar gates are often used in computer circuits, and their basic operation is obvious. The difficulty in designing this kind of circuit is mainly concerned with high-speed operation.

**Rectifier Circuits**

Much more subtle in operation are the rectifier and d.c. restorer circuits shown in Figs. 1.4 and 1.5. Although in very common use and apparently simple in function, the complete analysis of these circuits is difficult.

In the rectifier circuit (Fig. 1.4) diode $D_1$ conducts only when $V_{in}$ is more positive than $V_{out}$ so that with either pulse or sine-wave input

![Fig. 1.4 Rectifier circuit](image)

waveform conduction occurs as soon as $V_{in}$ rises towards $V_{in}$. As a result, $C$ charges through $R_s$ and $D_1$ and may or may not reach $+V_{in}$ before $V_{in}$ descends towards zero. As soon as $V_{in}$ has descended below $V_{out}$, $C$ discharges through $R$ towards zero until $V_{in}$ rises again.

When used for power rectification, the object being to obtain d.c. from a.c. with the minimum of added ripple, the time constant $CR$ is so large compared with the signal period that little discharge takes place. If $R_s$ is small, then $V_{out}$ (d.c.) is approximately equal to the positive input peak voltage for any input waveform; a circuit designed to operate in this way is known as a peak rectifier. In practice the source and diode possess some resistance, so that perfect peak rectification is impossible.

![Fig. 1.5 Restorer circuit, d.c.](image)
When $R_s$ is so large that $CR_s \gg T$, and $RC$ is also $\gg T$, $V_{out}$ depends only on the positive average value of $V_{in}$ and the circuit is then known as an average rectifier.

As illustrated, Fig. 1.4 is called a half-wave rectifier since $D_1$ completely ignores one half-cycle of the input signal; a full-wave rectifier uses two diodes so arranged that both half-cycles contribute to the d.c. output (Fig. 1.7). This has the advantage that for similar ripple performance to the half-wave circuit, $C$ need only be half the value because it discharges through $R$ for only half the signal period before being recharged. Also, the ripple frequency is twice the input frequency, which is advantageous since any additional ripple-

removing circuits which may be required become less bulky as the ripple frequency increases. Practical snags are that a push–pull input is required (achieved by a transformer in Fig. 1.7) and that unequal values of $R_{a1}$ and $R_{a2}$, or unequal input signals $e_1$ and $e_2$, cause the output contribution from two paths to be different, once more producing a ripple component at the input frequency.

Another full-wave circuit, the bridge rectifier (Fig. 1.8), uses four diodes, but no push–pull input is required. This arrangement possesses the advantages of full-wave rectification and avoids the problems of unequal input voltages and source resistance except for different diode characteristics. It also has a very definite advantage when the input has to be coupled through a transformer for reasons of isolation or voltage change. This is discussed more fully in Appendix 1, but the important result is that for a given $V_{out}$ and $V_{in}$

the transformer core can be smaller if a bridge rectifier is used. The designer must therefore balance the cost of extra diodes against the size and cost of the input transformer before making his decision.

The above description of the principle of peak and average rectification is useful in showing the limit cases where $R_s$ is in one case zero and in the other very large. For intermediate values, which are inevitable in real designs, a more complete analysis is required. The rough consequences of finite $R_s$ and $R$ are, first, that $V_{out}$ could never reach $V_{in}$, since during $D_1$ conduction $C$ could only charge to $V_{in}R/(R + R_s)$; and, secondly, that if $CR_sR/(R + R_s)$ (i.e. the charge time constant) is comparable with the time $\tau$ of $D_1$ conduction, $C$ will charge even less than the above value. For the pulse input case

$\tau$ is known and the analysis turns out to be simple once the procedure is known; for sine-wave input $\tau$ is initially unknown, so that the calculations are more involved (see Appendix 1).

The most straightforward method for analysing such circuits is to assume the capacitor carries no charge and then apply the input, noting instantaneous waveforms throughout the circuit. After many cycles an equilibrium state is eventually reached in which the charge gained and the charge lost by the capacitor in one cycle are equal. Knowing the magnitudes of charge lost and gained, equating the two leads to the steady (d.c.) component of voltage or charge which is present on the capacitor in this equilibrium state.
It is useful before demonstrating this technique to recall a few facts about charging capacitors.

(1) The current required to charge a capacitor at a rate of \( \frac{dV}{dt} \) \( \text{V/sec} \) is given by \( i = \frac{dV}{dt} \), where \( C \) is in farads and \( i \) is in amperes (equally correct and often more useful units are microfarads and microamps).

(2) From (1) it is clear that to charge a capacitor to any voltage in zero time would require infinite current, so that in analysis, if one plate of a capacitor is moved instantaneously, its other plate must move the same number of volts at the same time.

(3) From (1) it is also clear that a finite but instantaneous step of current into a capacitor causes the voltage to rise from zero at a finite rate of \( \frac{i}{C} \) \( \text{V/sec} \), there being no initial step of voltage. The rate of rise continues indefinitely provided the current step remains.

(4) Again from (1), charging a capacitor with a fixed current produces a linear rate of rise of voltage, and no other form of charging will achieve this (though there are many methods which will produce this constant current).

(5) Regarding a capacitor as a reactance of \( \frac{1}{j\omega C} \) is correct only in linear circuits with pure sine-wave input. Any non-linearity (such as a diode) produces non-sinusoidal waves and makes the reactance concept invalid, since its value is different at the fundamental and harmonic frequencies.

(6) If a circuit is receiving a constant frequency input and an equilibrium state is reached when each cycle of operation produces exactly the same waveforms (d.c. and a.c.) as the last, then the charge received by each capacitor in the circuit during one cycle must be equal. If not, the charge would change between successive cycles and waveforms would be different.

Consider now Fig. 1.4, where \( D_1 \) is assumed to be a perfect diode in that its turn-on voltage is zero, its forward resistance is zero, and its reverse leakage is zero. Assume also that the time constant \( CR \) is many times the input period.

The action of the circuit at the moment \( V_{in} \) is applied depends on which part of the \( V_{in} \) waveform is occurring at that instant. For the moment let \( V_{in} \) be at zero and be just beginning to rise.

Since \( C \) has zero charge, \( V_{out} \) is initially at zero and \( D_1 \) is just beginning to conduct. As \( V_{in} \) rises, \( D_1 \) conducts and \( C \) charges through \( R_s \), following but not equaling \( V_{in} \). When \( V_{in} \) reaches the end of its positive peak, \( V_{out} \) is approaching its maximum possible value of \( V_{in} R/(R + R_s) \), but may not reach this since \( V_{in} \) now begins to descend. As \( V_{in} \) descends, \( V_{out} \) tends to fall, not because of the fall in \( V_{in} \) which cannot itself pull \( V_{out} \) downwards (because reverse diode current cannot flow), but because of the discharge path through \( R \). Under the assumed conditions where \( RC \) is large compared with a period of \( V_{in} \), \( V_{out} \) now falls towards zero at a much slower rate than the descent of \( V_{in} \).

This continues until \( V_{in} \) again rises to a level equal to \( V_{out} \), when \( D_1 \) conducts and \( C \) again charges towards \( V_{in} \). The voltage waveform across \( C \) and the current, \( i_s \), taken from the source are therefore as shown in Fig. 1.6.

Since \( R \) discharges \( C \) only slightly during one cycle, \( V_{out} \) and \( i_s \) are almost constant. Hence, the current always tending to discharge \( C \) is \( V_{out}/R \) and the charge lost by \( C \) in one cycle is \( T(V_{out}/R) \).

The current which recharges \( C \) at the positive peak of each cycle is \( i_s \) which flows for a time \( \tau \). As the diode is assumed perfect (no forward drop), \( i_s \) is given by \( (V_{in} - V_{out})/R_s \) so that the charge received by \( C \) each cycle is \( (V_{in} - V_{out})\tau/R_s \). In the equilibrium state where each cycle is identical with the next, the net charge received by \( C \) must be zero, so that \( (V_{in} - V_{out})\tau/R_s = TV_{out}/R \), i.e.

\[ V_{out} = \frac{R}{R + (T/\tau)R_s} V_{in} \]

A simple check on this result is given by making \( \tau = T \), which represents a steady (d.c.) input of \( V_{in} \), resulting in an output of \( R/(R + R_s)V_{in} \). If, on the other hand, \( \tau = 0 \), \( V_{out} \) is zero, since \( C \) never receives charge. An alternative approach to the calculation is to equate the average power supplied by the source with that dissipated in the resistive elements in the circuit.

Now, the average power supplied by the source is

\[ \tau \frac{V_{in}(V_{in} - V_{out})}{T} \]

The average power dissipation in \( R_s \) is

\[ \frac{\tau (V_{in} - V_{out})^2}{T R_s} \]
and that in \( R \) is \( V_{\text{out}}^2/R \). Hence
\[
\frac{\tau \left( \bar{V}_{\text{in}} V_{\text{in}} - V_{\text{out}} \right)}{R} = \frac{\tau (V_{\text{in}}^2 - V_{\text{out}})^2}{R} + \frac{V_{\text{out}}^2}{R}
\]
\[
\therefore \quad \frac{\tau}{TR} (\bar{V}_{\text{in}} - V_{\text{out}}) V_{\text{out}} = \frac{V_{\text{out}}^2}{R}
\]
\[
\therefore \quad V_{\text{out}} = \frac{R}{R + (T/\tau)R} \bar{V}_{\text{in}}
\]
as before.

The trap to be avoided in this type of calculation is the assumption that the average power supplied by the source is the same thing as the average power dissipated in the source resistance \( R_s \), and then to equate this to the power dissipated in \( R \). This would lead to
\[
\frac{\tau}{T} (\bar{V}_{\text{in}} - V_{\text{out}})^2/R_s = V_{\text{out}}^2/R
\]
which is incorrect.

In the case where \( V_{\text{in}} \) is a sine wave \( \bar{V}_{\text{in}} \sin \omega t \), the above procedures are still correct but the analysis is much more complicated, since the time \( \tau \) for which source current flows depends on \( V_{\text{out}} \) and \( V_{\text{out}} \) in turn depends on \( \tau \) and \( R_s \); moreover, the input current and voltage are not constant throughout the charging time \( \tau \). Average charge received by \( C \), or, if the alternative approach is used, the average power supplied by the source and that absorbed by \( R_s \), must be obtained by integration. This is given in Appendix 1, the results being interpreted graphically in Fig. 1.9. The graphs enable the output voltage and the conduction time \( \tau \) to be deduced from the values of \( R_s/R \), assuming negligible diode incremental resistance or forward voltage drop. It will be seen that a particular value of \( R_s \), e.g. \( R/20 \) produces much more loss in \( V_{\text{out}} \) than a simple estimate would indicate: the drop in \( V_{\text{out}} \) from the case where \( R_s \) is zero is from \( \bar{V}_{\text{in}} \) to about 0.7 \( \bar{V}_{\text{in}} \), not, as might be anticipated, 0.95 \( \bar{V}_{\text{in}} \). The reason is that the current which flows through \( R_s \) for a short time each cycle is much larger than the load current, since it has to replenish the charge on \( C \) (lost to the load) in this short time \( \tau \). The voltage drop in \( R_s \) is therefore many times larger than \( I_L R_s \).

For the same reason the power dissipated in \( R_s \) is much higher than \( I_L^2 R_s \); its value is calculated in Appendix 1. Failure to appreciate this can result in unreliable power supplies in which \( R_s \) has been inserted to limit \( I_s \) to a safe level, but where the rating for \( R_s \) has been made much too low, so that this component eventually fails.

To take an example, the rectifier circuit for a radio or television receiver is often of this form, where \( \bar{V}_{\text{in}} \) is 240 \( \sqrt{2} \), \( R_s = 70 \) \( \Omega \) and \( R = 1 \) k\( \Omega \) (effective). Here \( R_s/R = 0.07 \), giving \( \tau/T = 0.25 \) and \( V_{\text{out}}/V_{\text{in}} = 0.7 \), i.e. \( V_{\text{out}} = 240 \) V (d.c.). The required power rating for \( R_s \) is not \( 70 \times t^2_L = 70 \times (0.24)_s^2 = 4 \) W, but must be \( P_{Rs} = 30.8 \) W.

\[\text{Choice of } C\]

It has been assumed throughout the above analysis that \( C \) is so large that \( V_{\text{out}} \) has no a.c. component; this can be literally true only if \( C \) is infinite. However, the results are correct for practical purposes provided \( C \) is large enough to produce an alternating component of \( V_{\text{out}} \) which is negligible in the particular application. This means that the voltage by which \( C \) discharges through \( R \) in one cycle must be a small proportion of \( V_{\text{out}} \). The discharge voltage is obtained from \( i = C \text{d}V/\text{d}t \) and is \( V_{\text{out}}T/RC \), since, for small discharge, the current is constant and equal to \( V_{\text{out}}/R \). Hence, \( V_{\text{out}}T/CR \ll V_{\text{out}} \), giving \( CR \gg T \). The time constant of the load circuit must therefore be much greater than one period of the input if output ripple is to be small, and the analysis correct. The relationship between ripple and time constant is such that a time constant 1 per cent of the period produces ripple which is 1 per cent of the d.c. output.

The capacitor waveform is shown in Fig. 1.10.

\[\text{d.c. Restorer}\]

When the positions of diode and capacitor are interchanged, as shown in Fig. 1.5, the circuit operation is basically unchanged, but
the output is now the difference between the capacitor voltage (i.e. $V_{out}$ for the rectifier circuit) and $V_{in}$. The resulting waveform (Fig. 1.11) has ideally an alternate component which is identical to $V_{in}$ and a d.c. component of $V_{in}$, resulting in the waveform just reaching zero potential at its positive peaks. These peaks are then said to be 'd.c. restored' to earth. Naturally $D_1$ may be inverted and it may be returned to any potential, so that either peak may be made to sit at any desired steady potential. Any resistance between the potential to which $D_1$ is returned and the signal 'earth' must be added to $R_s$ when considering circuit performance.

![Waveforms for d.c. restorer (CR $\gg$ T)](image)

This circuit can again be analysed by the methods used for Fig. 1.4, and circuit performance may in fact be derived directly from that analysis. However, the usual requirement for d.c. restoration is that it should be as near the equivalent of peak rectification as possible, giving the restoration level equal to the diode-return potential. It will therefore normally be designed with $R_s \ll R$, $CR \gg T$ and $V_{in} \gg$ diode forward drop.

Practical problems are that if $C$ is too small, the ripple waveform across $C$ will noticeably affect the waveform $V_{out}$, as shown in Fig. 1.12; on the other hand, if $C$ is very large many cycles of $V_{in}$ will pass before restoration level at the output is reached. $R_s$ should be as low as possible, but, if very low, peak charging current requirement for $C$ will be high and the source may be unable to supply this. As the source temporarily fails, the effective value of $R_s$ becomes high and the restoration level wrong.

Finally it should be noted that any configuration equivalent to that of Fig. 1.5 produces d.c. restoration whether or not intended by the designer.

**Zener Diode Applications**

The Zener diode has the property of sustaining a voltage drop which is virtually constant over a wide range of currents applied in the direction which corresponds to reverse current in a normal diode. Its most obvious use is therefore to produce a stable voltage for supplying other circuit elements when the main supply is subject to wide variations; these variations may be very slow fluctuations or low-frequency or high-frequency ripple, the Zener diode acting as a low impedance over a wide band of frequencies.

![Simple Zener diode stabilizer](image)

The circuit of Fig. 1.13 shows a typical application in which a Zener diode is used to obtain a stable voltage source from a power supply $V_1$ which is itself subject to large variations. In order to illustrate the significance of the various quoted parameters, the circuit performance is assessed below assuming that

(i) $V_Z = 6.8 \pm 10$ per cent for $I_Z = 5$ mA, i.e. $(V_Z(I_Z = 5$ mA) $ = 6.8 \pm 10$ per cent).

(ii) Zener incremental resistance for $I_Z = 5$ mA is $30 \Omega$ maximum, i.e.

$$\frac{dV_Z}{dI_Z} (I_Z = 5$ mA) $ = 30 \Omega$ max.

(iii) Temperature coefficient of $V_Z$ at 5 mA is $\pm 0.02$ per cent per degree $C$, i.e.

$$\frac{dV_Z}{dT} (I_Z = 5$ mA) $ = \pm 0.02$ per cent.
(iv) $V_1$ is $20 \pm 5$ V.
(v) $R_1$ is $2.7$ kΩ $\pm 5$ per cent.

Since the above values for the Zener diode apply for a nominal Zener current of 5 mA, $R_1$ has been given a value which gives approximately this Zener current under nominal conditions. Thus the nominal output $V_Z$ will be $6.8$ V. The effect on the output of the above specification figures is detailed below under the relevant number.

(i) The manufacturer’s tolerance of $\pm 10$ per cent at 5 mA gives an output variation of $\pm 10$ per cent, i.e. $\pm 0.68$ V. The effect when $I_Z$ is not 5 mA is dealt with in (ii).

(ii) The value of $R_Z$ of 30 $\Omega$ at 5 mA implies that any departure of $I_Z$ from 5 mA by an amount $\delta I_Z$ will produce an output voltage change of $R_Z \delta I_Z$; since $R_Z$ is positive, the direction of output change is the same as that of $\delta I_Z$. Since the value of $\delta I_Z$ depends on factors (iv) and (v), and slightly on factor (iii) also, the effect of $R_Z$ cannot at this stage be calculated.

(iii) Temperature coefficient produces a direct effect on the output of the same proportion, namely $\pm 0.02$ per cent per degree C, i.e. $\pm 1.36$ mV/degC.

(iv) Variations in $V_1$ cause changes in $I_Z$; this is the only reason why $V_Z$ is affected by $V_1$. Since $V_1 = 20$ V gives $I_Z \approx 5$ mA with $V_Z \approx 6.8$ V (it is generally unnecessary to be more precise except in highly critical cases), then a change of $\pm 5$ V in $V_1$ will produce a change $\delta I_Z = 5/R_1 = \pm 5/2.7$ mA = $\pm 1.85$ mA.

This gives a change in $V_Z$ of $\pm 1.85 \times 10^{-3} R_Z = \pm 1.85 \times 30$ mV = $\pm 55.5$ mV.

(v) Variation in $R_1$ again changes $I_Z$, and because of $R_Z$ this changes $V_Z$. A tolerance of $\pm 5$ per cent changes $I_Z$ by 5 per cent (since $I_Z = (V_1 - V_Z)/R_1$), i.e. $\delta I_Z = \pm 1/4$ mA and $V_Z$ therefore changes $\pm (1/4)30$ mV = $\pm 7.5$ mV.

Summarizing the above results, the value of $V_Z$ is given by

$$V_Z = 6.8 \pm 0.68 \pm 0.055 \pm 0.075 \pm 0.001 \; 0$$

initial  $V_1$  temperature changes  tolerance coefficient

where 0 is the number of degrees Centigrade temperature variation.

There are several points of interest in the above result.

First, the major item causing output uncertainty is initial tolerance; this, however, does not change with use and is therefore of no importance if the only requirement of $V_Z$ is stability rather than its absolute value, which is often true in practice. When the absolute value is important, it is necessary to specify a tighter initial tolerance and this is expensive, since it involves selection by the manufacturer (it is usually much more expensive for the customer to make the selection).

Secondly, all the causes of change are to a small extent self-compensating. For instance, when $V_1$ increases, $I_Z$ also increases, causing a change of $\delta I_Z R_Z$ in $V_Z$. Because $V_Z$ has now increased, the value of $I_Z$ is not so high as was assumed at the high value for $V_1$. It is in fact given by $I_{ZH} = (V_{1H} - V_Z - \delta I_Z R_Z)/R_1$, where $I_{ZH}$ and $V_{1H}$ are the high values of $I_Z$ and $V_1$; the calculation used earlier in the chapter assumed that $I_{ZH} = (V_{1H} - V_Z)/R_1$.

![Fig. 1.14 Equivalent circuit for simple Zener diode stabilizer (Fig. 1.13)](image)

The result is that the limits given for $V_Z$ were slightly pessimistic. How slight this effect is can be shown by calculating the case for the initial tolerance of $V_Z$, namely $\pm 0.68$ V. Taking the positive limit, $I_Z$ is less than the expected value by $0.68/R_1$, i.e. $0.68/2.7 = 0.25$ mA, giving a drop in $V_Z$ of $30 \times 0.25$ mV = 7.5 mV from its apparent value of $6.8 + 0.68 = 7.48$ V. This is completely negligible in this and almost every other case.

Thirdly, it is easy to see that instead of calculating the changes of current caused by each effect and then using $R_Z$ to put this in terms of $V_Z$, the operation can be simplified by regarding the diode as a perfect battery in series with $R_Z$ (Fig. 1.14).

The value of the 'battery' voltage $V_{Z0}$ is the Zener voltage at a given current $I_Z$ minus the product of $R_Z$ and $I_Z$. In the above example $V_{Z0}$ would be $(6.8 \pm 10\% \text{ per cent}) - 5 \times 10^{-3} \times 30 = 6.8 \pm 0.68 - 0.15 = 6.65 \pm 0.68$ V. If $R_Z$ were constant for all $I_Z$ this equivalent circuit would apply for any condition; in fact, $R_Z$ is a function of $I_Z$ and the quoted $R_Z$ is true for only a small region; e.g. at
$I_Z = 6\ mA$ $R_Z$ may be 25 $\Omega$. Since any calculations where accuracy is important would normally be used only for cases where $I_Z$ has small variations, this equivalent circuit is a useful design aid.

Assuming in Fig. 1.14 that circuit values give $I_Z$ in the region corresponding to the value of $V_{Z0}$ and $R_Z$, the effect of variations is readily calculated; in particular, it is clear that any change $\delta V_1$ in $V_1$ will cause an output change of $\delta V_Z = \delta V_1 R_Z / (R_Z + R_1)$ =

$\frac{5 \times 30 \times 10^{-3}}{2.73}$ = 0.055 $V$, as obtained earlier by direct calculation.

If $R_Z$ is truly resistive (rather than reactive), then the above calculation applies to variations in $V_1$ at any frequency. $V_1$ can have a mean value in our example of +20 $V$ and a ripple content which causes $V_1$ to oscillate by $\pm 5\ V$ about this value. The output $V_Z$ will have a steady value of about +6.8 $V$ and its ripple content will be only $\pm 0.055\ V$. An actual diode will have parallel capacitance which causes this ripple content to be further reduced at frequencies above a few hundred kilocycles per second. At very much higher frequencies the series inductance of diode internal connections causes output ripple in the above circuit to rise again.

From the equivalent circuit of Fig. 1.14 it is evident that for the most constant $V_Z$ the value of $R_1$ should be as high as possible and that of $R_Z$ as low as possible. Changing $R_1$ in our example to, say, 27 k$\Omega$ would give no improvement, however, because $I_Z$ then falls to about 0.5 mA and $R_Z$ would be found to rise by a factor of at least 10. Performance would therefore be unchanged.

The only practical improvement is to raise the mean value of $V_1$ so that $R_1$ may be increased while maintaining the same value of $I_Z$ and, hence, $R_Z$. The available supply voltage will naturally be limited, but the aim should be to use as high a value as possible. The use of a 'constant-current device', to be dealt with later, is sometimes appropriate and involves the use of a transistor (see Chapter 6).

An additional complication which often affects the calculation is the presence of a load current $I_L$ assumed negligible in the above example. If $I_L$ is large, e.g. 4 mA, the value of $R_1$ must be changed. To maintain $I_Z$ at 5 mA, $R_1$ must pass 9 mA and will have a value of about (20 - 6.8)/9 k$\Omega$, namely 1.5 k$\Omega$ (see Fig. 1.15).

By causing $R_1$ to be thus reduced, the presence of $I_L$ clearly reduces the stability of the circuit. Changes in $V_1$ are now reduced by a factor of about 1500/30, i.e. 50/1, instead of 2700/30, or 90/1.

The effect of load current changes are accounted for by noting that the apparent source resistance of the Zener circuit is 30/1500, i.e. about 30 $\Omega$. (It is futile in these calculations to consider small effects such as the influence of 1500 $\Omega$ when placed in parallel with 30 $\Omega$, since the figure of 30 $\Omega$ given by the manufacturer is merely typical and is subject to considerable variation.) Thus a change in load current of $\pm 10\$ per cent, or 0.4 mA will produce an output change of $\pm (0.4 \times 30)$ mV, i.e. $\pm 12\ mV$.

![Fig. 1.15 Practical example with load $R_L$](image)

The design of the simple stabilizing circuit merely requires care in adding all the effects which cause output variation. Improvements to the circuit can be made by using two or even more stages in cascade, i.e. $V_1$ is itself the Zener voltage of another diode.

![Fig. 1.16 (a) Improved stabilizer, (b) equivalent circuit of improved stabilizer](image)

A more subtle improvement is shown in Fig. 1.16 (a), where the 'earthy' side of the load is returned to a potential divider $R_2R_3$. The idea is that when $V_1$ rises, causing the positive output terminal to rise, the negative terminal also rises. If both terminals rise the same amount, $V_{out}$ remains constant, giving perfect stabilization against changes in $V_1$. This condition is achieved if $R_3/R_2 = R_2/R$, as is obvious from Fig. 1.16 (b).
be a sharp-edged square wave, then \( V \) must be much larger than \( V_Z \); the actual rate of rise achieved is

\[
\frac{R_L}{R_s + R_L} \frac{d}{dt} (V \sin \omega t)
\]

at any time when clipping is not taking place.

Temperature affects the mean level and the peak-to-peak value of the output according to the temperature coefficients of \( V_Z \) and \( V_F \). If the positive clipping level is to be constant, then \( V_Z \) should be a low coefficient diode (e.g. 5-6 V); if it is more important that the peak-to-peak be constant, then \( V_Z \) should have a positive coefficient of magnitude equivalent to 2-2.5 mV/degC in order to cancel the \( V_F \) coefficients, e.g. a Zener diode of 8-2 V. In this case the output waveform moves positive by 2-2.5 mV/degC but the amplitude remains relatively constant.

When the output is to be symmetrically disposed about zero level, two Zener diodes can be used ‘back-to-back’ in series—either the two anodes or the two cathodes may be joined (see Fig. 1.18). The output peak value is now \( (V_Z + V_F) \), where \( V_F \) is the drop of the forward-biased diode. Asymmetry is caused mainly by inequality between the two values of \( V_Z \) and to a lesser extent from inequality between the two values of \( V_F \).

Temperature effects are slightly less in this circuit because, if the Zener coefficient is chosen as before to represent +2-2.5 mV/degC voltage change, each clipping level is stable, and no mean level drift occurs.

It must be emphasized that in either circuit such compensation is by no means exact; all that can be said is that an attempt to match coefficients helps, since linear quantities are involved.
Other uses of the Zener diode

As will be seen in later chapters, Zener diodes can often replace resistors, and sometimes capacitors, as coupling devices. By regarding the Zener diode as a battery and series resistor, such circuits are readily designed by the methods described earlier.

2—The transistor: d.c. characteristics

No attempt is made here to describe the physics of transistor operation. Instead its characteristics as a circuit element are discussed and orders of magnitude of parameters are also given, since these must be known in order to understand how the approximations are made when deriving usable formulae.

D.C. OPERATING CONDITIONS

Figure 2.1 shows both $p-n-p$ and $n-p-n$ transistors, and indicates how in some respects the transistor can be represented as two diodes,

![Diode Representation](image)

Fig. 2.1 (a) $p-n-p$ transistor, (b) $n-p-n$ transistor

one representing the emitter–base junction and the other representing the collector–base junction.

It must be emphasized that this diode analogy is correct only in so far as it describes the behaviour of the two junctions separately, but nevertheless the idea is helpful as an aid to visualizing the directions of forward currents and voltages, and also leakage currents.

The Emittter Circuit

From the above representation, and bearing in mind that it is correct only for junctions energized separately, it is readily seen that
if the base is returned to a more positive potential than the emitter on an n-p-n transistor, current will flow and a potential difference will appear across the junction as shown in Fig. 2.2. The magnitude of the drop, named $V_{be}$ in Fig. 2.2, clearly depends upon the current flowing into the emitter-base diode, i.e. it depends on $V$ and $R$; the drop also depends on the forward characteristic of the emitter-base diode.

If a plot of the forward characteristic were available, it would therefore be possible to calculate the current $I_e$ and the voltage $V_{be}$ in Fig. 2.2, either by trial and error (until $(V - V_{be})/R$ gave a current which produced that value of $V_{be}$) or more scientifically by drawing a load line on the diode characteristic.

Such a calculation would, however, be subject to considerable error because of variations between the actual transistor used and the ‘typical’ one for which the curve applies, and also because changes in temperature produce changes in $V_{be}$ at a given current (as for a diode, from $-2$ to $-2.5$ mV/degC). There is therefore no point in going to great trouble to try to establish the exact values of $I_e$ and $V_{be}$. The important thing is to find what values of $V$ and $R$ are required to guarantee that $I_e$ has the intended value within a certain tolerance.

The practical approach to be adopted in working out the current in such examples is as follows.

The temperature range over which the circuit must operate is e.g. 0-50°C. If the transistor is silicon, its $V_{be}$ at the current it is intended to operate will lie between 0.5 and 0.9 V at room temperature (25°C), will be larger by another $(25 \times 2.5)$ mV ≈ 60 mV at 0°C, and smaller by $(25 \times 2.5)$ mV ≈ 60 mV at 50°C. Therefore, the limits of $V_{be}$ are 0.44 and 0.96 V for a temperature range of 0-50°C. It is required that the emitter current be constant to, e.g. ± 10 per cent. If the simple arrangement of Fig. 2.1 is to be used, $V$ must be at least so large that the two limits of $(V - V_{be})$ produce less than ± 10 per cent change in $I_e$. This choice of $V$ leaves no permissible tolerance on $R$ and on $V$ itself, and since precision resistors and stabilized lines are expensive, a practical solution would be to make $R$ a ± 5 per cent type (including temperature effects) and make the $(V - V_{be})$ tolerance less than ± 5 per cent. To achieve this $V$ must be at least $10(0.96 - 0.44)$, i.e. 5.2 V, and it is convenient to use here, say, 10 V which is already available. $R$ is now given by the nominal voltage across it, i.e. $10 - [0.96 + 0.44]/2$ divided by the intended value of $I_e$, i.e. 1 mA, for example. Therefore $R$ is 9.3 kΩ, or to the nearest standard value 9.1 kΩ.

Note that the only transistor information required for the above (other than the desirable operating current, which will be dealt with in Chapter 7) is the value of $V_{be}$, its change with temperature and its variation from one unit to another. The figures used above are typical for a silicon transistor, i.e. 0.5-0.9 V at 25°C for any operating current which is likely to be reasonable for the device. The corresponding figures for germanium are 0.15-0.5 V. Temperature drift of about 2.5 mV/degC applies to both types. When dealing with power transistors carrying more than 1 A it is advisable to check on the maximum values given above.

The emitter circuit of a p-n-p transistor is designed in exactly the same manner, the only difference being the polarity; values and drifts of $V_{be}$ remain the same.

No difficulty need be experienced in remembering the correct emitter circuit polarities for the two types, since the arrow on the circuit symbol points in the direction of conventional current flow (as in the diode symbol) and the external voltage source has to be connected so that current will be supplied by this source in the direction of the arrow.

To establish $I_e$ within the required tolerance, one therefore needs to know that tolerance, the temperature range, and whether the transistor is germanium or silicon. Knowing these facts, $V$ is made sufficiently large and constant, and $R$ is made sufficiently accurate. Apart from the use of temperature-compensating elements to counteract $V_{be}$ temperature changes (to be dealt with later in this chapter) this is all that can be done to be sure of operating at the intended value of $I_e$ — excepting the use of d.c. negative feedback as described on page 45.
The Collector Circuit

If a voltage is applied between the collector and base of an \( n-p-n \) transistor so that the collector is positive, then the collector–base diode is reverse-biased. Provided no emitter current is flowing, because this would cause the two-diode analogy to fail, then the collector current will be merely the leakage of the diode (Fig. 2.3).

This current is known as the collector–base leakage current and is designated \( I_{\text{CEO}} \).

As in a normal diode, \( I_{\text{CEO}} \) is the sum of two components, one of which is invariant with temperature; the other doubles itself every 9–10°C. The value of applied voltage has little effect on \( I_{\text{CEO}} \) until the ‘breakdown’ voltage is reached, when \( I_{\text{CEO}} \) rises rapidly.

Typical values for small silicon types range from about 10 nA at 25°C to 1 µA at 50°C; since \( I_{\text{CEO}} \) depends directly on junction area, power transistors often have values of from 2 or 3 mA (25°C) to 20 mA (50°C).

Germanium values are much higher, e.g. small signal types 2 µA (25°C)–100 µA (50°C). Again, power types may be many orders of magnitude higher than the above.

It is evident that \( I_{\text{CEO}} \) is subject to much more uncertainty than \( V_{\text{BE}} \) and that although the ratio of maximum to minimum \( I_{\text{CEO}} \) for silicon is similar to that for germanium, the absolute values for germanium are much larger.

Because of the exponential way in which \( I_{\text{CEO}} \) varies with temperature, it is almost impossible to predict with any confidence what value \( I_{\text{CEO}} \) will have at, for example, 65°C when its value is known only at 20°C. If its value at 20°C were, for instance, 10 µA ± 10 per cent, the 65°C figure assuming a law where \( I_{\text{CEO}} \) doubles every 9 or 10 degC rise, would have a lower limit of \( 9 \times 2^{4.5} \) i.e. 200µA and an upper limit of \( 11 \times 2^{4.5} \), i.e. 352 µA.

![Fig. 2.3 Collector leakage](image)

**Effect of Collector Load**

The addition of a load \( R_L \) in series with the collector supply causes the applied collector voltage to be reduced by \( I_{\text{CEO}}R_L \) and this voltage drop will be present whenever the collector supply is connected, adding to any other drop caused by transistor action.

As mentioned earlier, increasing the collector supply voltage has little effect on \( I_{\text{CEO}} \) until breakdown of the collector–base diode occurs. This effect is again analogous to the same action in a normal semiconductor diode, so that if \( R_L \) is present and \( V_1 \) is very large, the collector–base voltage rises to the breakdown value \( BV_{\text{CEO}} \) and the collector current is given by \( (V_1 - BV_{\text{CEO}})/R_L \). The collector current can be very large compared with normal values of \( I_{\text{CEO}} \), but damage will not occur provided the power rating of the junction is not exceeded, i.e. provided \( BV_{\text{CEO}}(V_1 - BV_{\text{CEO}})/R < P_{\text{max}} \). Although this mode of operation is not generally useful owing to uncertainty in the actual value of \( BV_{\text{CEO}} \) (often much higher than the guaranteed minimum), such action often occurs transiently under overload conditions, or immediately after switching power supplies on and off. In these cases the power must be calculated and if necessary reduced to a safe level.

Typical breakdown voltages range from 10 to 80 V; less common but obtainable are ratings up to 500 V.

**Comparison between Base–emitter and Base–collector Diodes**

Although, as indicated above, the normal mode of operation is to forward-bias the emitter diode and reverse-bias the collector diode, each can also be used in the opposite connection.

Usually, but not always, the breakdown voltage of the reversed emitter–base diode is less than that of the collector–base diode, especially in diffused transistors (often 1 V only), and the reverse leakage of this diode before breakdown, which is usually less than \( I_{\text{CEO}} \), is called \( I_{\text{CBO}} \).

In other respects the two diodes are similar, and indeed the
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Collector and emitter loads can be interchanged and provided the changed ratings are not exceeded, no damage will be done. However, performance as a transistor will be poor, except for a 'symmetrical' type, as will be seen from the following section.

Transistor Action

If the emitter-base diode is biased forward with a current \( I_e \) and the collector-base diode is simultaneously reverse-biased, the two-diode analogy fails because of transistor action. The resulting currents are shown in Fig. 2.4, which indicates that the collector current has increased from \( I_{cbo} \) to \( (I_{cbo} + \alpha I_e) \), where \( \alpha \) is a transistor parameter whose value is close to but less than unity (generally within 5 per cent). Most of the emitter current therefore flows out of the collector, and since \( \alpha \) is found not to vary appreciably with collector-base voltage (provided this is at least a few hundred millivolts) the total collector current is independent of \( V_1 \).

The addition of \( R_L \) into the collector circuit, as in Fig. 2.5, does not therefore change the currents, provided that \((\alpha I_e + I_{cbo})R_L \) is less than \( V_1 \) by a few hundred millivolts, i.e. provided the collector junction is still reverse-biased.

In designing a practical circuit it will always be necessary to know the collector potential and so the designer must be aware of parameter variations which cause drift in this voltage.

Naturally, variations in \( \alpha \) and in \( I_e \) (discussed earlier) result directly in variations in collector current \( I_c \) and, hence, in \( V_{cb} \). Variations in \( I_{cbo} \) again directly affect \( V_{cb} \). The rate of change of \( \alpha \) with temperature is ill-defined but usually lies between 0 and \( +1/25 \) per cent per degree \( C \); its value also varies from one unit to another of the same type by a spread of approximately \( \pm 1 \) per cent.

The variations of \( \alpha \) have usually little direct effect on the collector circuit in comparison with the effect of \( V_{cb} \) (on \( I_e \)).

In the practical design previously discussed \( I_e \) was established to within \( \pm 10 \) per cent by using \( V = 10 \) V, \( R = 10 \) k\( \Omega \), giving \( I_e = 1 \) mA. Suppose now that \( V_1 = 10 \) V and \( R_L = 5.6 \) k\( \Omega \), and that it is required to calculate the value and expected variation in \( V_{cb} \). The designer proceeds as follows.

\( I_e \) is known to be nominally 1 mA so that \( V_{cb} \) is 10 V less the drop in \( R_L \), which is nominally 5.6 V, giving \( V_{cb} = 4.4 \) V.

Note the assumptions made that \( \alpha = 1 \) and \( I_{cbo} = 0 \). The correct allowance for these quantities is not made at this stage; since if the very simple nominal calculation gave \( V_{cb} = 0 \) V, which could easily be the case if, e.g. \( R_L = 10 \) k\( \Omega \) or \( V_1 = 5.6 \) V, the circuit would clearly not operate and a new value of \( R_L \) or \( V_1 \) would have to be allocated. The argument that the correct allowances for \( \alpha \) and \( I_{cbo} \) might influence the answer and then give a reasonable value for \( V_{cb} \) is invalid, because if these side-effects give a marked change in \( I_e \), the design is bad (spurious variable effects should not predominate); if the effects have only slight influence, a slight change in \( R_L \) or \( V_1 \) would again lead to circuit failure. This example where the failure criterion is \( V_{cb} \leq 0 \) is naturally only a particular case; for many applications it could be that \( V_{cb} \geq V^* \) is a failure condition, where \( V^* \) is a voltage limit determined by signal levels or by a following circuit. Returning to the design procedure, it has been established that nominally \( V_{cb} = 4.4 \) V.

The actual value of \( V_{cb} \) is affected by \( I_e \), which causes \( \pm 10 \) per cent variation in that part of \( I_e \) which does not include \( I_{cbo} \). This is a change
of about 0-1 mA, which produces 0-56 V change across \( R_L \) and therefore in \( V_{eb} \). Change in \( z \) from 0 to 50°C will be about 2 per cent at worst and changes from one transistor to another will be another 2 per cent. Since the minimum \( \alpha \) for any transistor of the type to be used is, e.g. 0-95 at 0°C, the maximum could be 0-99, so that a nominal 3 per cent must be taken from the assumed value of 1 mA and a spread due to \( \alpha \) of ±2 per cent taken as tolerance. Hence, the drop in \( R_L \) (assumed 5-6 V) should be

\[
[5.6 - (3/100)5.6] = 5.43 \pm 0.56 \pm (3/100)5.43 = 5.43 \pm 0.75 \text{ V}
\]

giving \( V_{eb} = 4.57 \pm 0.75 \text{ V} \). The \( I_{eb0} \) contribution is highest at 50°C, when \( I_{eb0} \) for this transistor is 100 \( \mu \)A, giving an additional drop in \( R_L \) of 5-6 mV. The tolerance on \( V_{eb} \) is therefore 4.57 ± 0.75 – 0.0056 V.

The above calculation is not strictly accurate, since 10 per cent change in \( I_e \) does not represent 0-1 mA change in \( aI_e \) but more nearly 0-097 mA. This 'error' is deliberately presented to emphasize the futility of making exact calculations when the quantities being dealt with experience wide variations.

The above procedure, although apparently tedious, takes little time in practice and leads to a better understanding of which are the worst contributors in a particular case to variations in \( I_e \) or \( V_{eb} \).

It is clear, for example, that the value of \( I_e \) is uncertain by the proportion that \( I_{eb0} \) represents in \( I_e \), quite apart from any other cause. If, therefore, \( I_{eb0, max} = I_e/10 \), then \( I_e \) has at least 10 per cent uncertainty from \( I_{eb0} \) alone, because the \( I_{eb0} \) of some transistors of the specified type may be almost zero, whereas others will be \( I_{eb0, max} \). Similarly, if the expected \( V_{be} \) variation is equal to \( V_e/10 \), the uncertainty in \( I_e \) is again about 10 per cent from this cause alone.

These points, if borne in mind at the beginning of a design, will enable at least a reasonably good first attempt to be made. Corrections can then be made after calculating errors. If, instead, equations such as

\[
V_{eb} = V_1 - (V_e - V_{be})/R_e + I_{eb0}R_L.
\]

are solved and prove that \( V_{eb} \) is in error, the necessary corrective measures are not obvious.

Using the methods described above, the designer can now calculate the emitter and collector currents, and collector voltage for the circuit of Fig. 2.4. By using the successive approximation method presented he can equally well calculate the values of \( R_e \) and \( R_L \) necessary for a specified \( I_e \) and \( V_L \) to be obtained.

**Effect of Base Circuit Resistance**

The inclusion of a base resistor \( R_b \) as shown in Fig. 2.5 can have a considerable effect on the values of \( I_e \), \( I_c \), and \( V_{eb} \). The circuit equations, as shown in Appendix 2, now appear very complicated and are of little use in practical design. The reason for all these changes in operating conditions is, however, very simple: the base is no longer at zero potential, because the difference current between \( I_e \) and \( I_c \), which must flow out of the base, causes a voltage drop in \( R_b \). The magnitude of the base current is clearly \( I_b = (I_e - I_c) \), i.e. \( I_e - (\alpha I_e + I_{eb0}) \), or \((1 - \alpha)I_e - I_{eb0}\), and therefore the base voltage differs from zero by \( V_b = R_b[(1 - \alpha)I_e - I_{eb0}] \). The complication caused by this is evident, because the calculation of \( I_b \) is immediately changed to \((V_e - V_{be} - V_b)/R_b \), which itself is dependent on \( I_e \). Note that \( I_b \) can have either polarity and may be zero.

The way in which the designer minimizes the problems of the calculation is very simple. He merely assumes that \( V_b \) is so small that it may be disregarded, and then, knowing \( I_b \) from the simply calculated \( I_e \), chooses \( R_b \) so that \( V_b \) really is negligible. An objection which may be raised here is that perhaps it would be advantageous to allow \( V_b \) to be large so that large values of \( R_b \) could be used (even though it puts the designer to more trouble to work out his sums). This is rarely valid in serious design, however, because of the nature of \( I_b \). Referring to the equation \( I_b = (1 - \alpha)I_e - I_{eb0} \), it will be noted that the first part of the expression has a factor \((1 - \alpha)\) and that the second term is \( I_{eb0} \). Now, \( \alpha \) varies in transistors of any type by a few per cent and since \( \alpha \) is close to unity, \((1 - \alpha)\) varies widely for these changes in \( \alpha \). A typical spread in \((1 - \alpha)\) is 3 or 4 to 1. The \( I_{eb0} \) component varies by about 100 to 1 in most transistors, and is often comparable in maximum value to the first term (e.g. in germanium transistors and in silicon transistors run at low current). It is clear therefore that the value of \( I_b \) is uncertain by a factor of at least 3 or 4 to 1 and, hence, so is \( V_b \). If \( V_b \) is allowed to have a maximum value which is enough to affect \( I_e \) significantly, the spread in \( I_e \), and therefore in \( I_c \) and \( V_{eb} \) will be very large. Since it is rare that large spreads (of more than about 20 per cent) can be tolerated, the whole idea of allowing \( V_b \) to become significant must be discarded in good design practice.

The procedure then is to ignore \( R_b \) initially, calculate \( I_e \), \( I_c \), and \( V_{eb} \) as if \( R_b \) were zero, then assess \( I_b \). If \( R_b \) is already fixed (i.e. it is
part of a previous drive circuit), \( I_e \) is now recalculated knowing that \( V_b \) is not zero but is really \( I_\alpha R_b \). This gives a new value of \( I_e \); if this is different from the original value by more than the tolerable uncertainty, the design is unsatisfactory. If acceptable, the new, slightly different, \( I_e \) is now used to recalculate \( I_b \) and \( V_b \). This again affects \( I_e \) but considerably less even than before. \( V_b \) can yet again be recalculated, but convergence will have been obtained in most circuits by this stage.

If in the above design \( R_b \) is not known but is to be specified by the designer, a reasonable value of \( V_b \) is decided (a certain percentage of \( V_e \)), \( I_b \) is calculated, and \( R_b \) chosen to give not more than that value of \( V_b \) for the highest limit of \( I_b \). As above, the value of \( V_b \) changes \( I_e \): hence, \( I_b, V_b, I_e \), and again \( I_b \), but the sequence converges rapidly.

This iterative process is similar to that used for most practical circuit design. Although the description is long the actual process is not, and the designer soon becomes aware of which circuit values need respecifying and in which direction.

Taking the values previously used in illustrating the calculation of \( V_{eb} \), the permissible value of \( R_b \) will now be worked out using the (arbitrary) condition that the presence of \( R_b \) should not cause \( I_e \) to change by more than 5 per cent.

The first step is to calculate \( I_e \) approximately (in view of the corrections which will follow as a result of \( R_b \) there is no point in exact calculation). This is easily done and yields, as before, \( I_e \approx 1 \) mA.

Now, \( I_b = (1 - \alpha) I_e - I_{bo} \) and \( \alpha \) was assumed to vary from 0.95 to 0.99, giving a variation in \((1 - \alpha)\) of 1/20–1/100. The value of \( I_{bo} \) can be anywhere in the range 0–1 \( \mu \)A. With nominal \( I_e \) (1 mA) the limits of \( I_b \) are therefore from \((1/20 \text{ mA} - 0)\) to \((1/100 \text{ mA} - 1 \mu A)\), i.e. from \(+50\) to \( 9 \mu A \). The effect of \( I_{bo} \) is therefore less than that of \((1 - \alpha)\) in this example and leads to a maximum \( I_b \) of \(50 \mu A \). If \( I_b \) is to be upset by only \( \pm 5 \) per cent, then \( V_b \) must be only \(1/20 \ V_e \), i.e. 0.5 V. Therefore \( R_b \) should not exceed 0.5/50 M\( \Omega \), or 10 k\( \Omega \). A value of 4-7 k\( \Omega \) is therefore satisfactory. The value of \( I_e \) will now be different and the limits of \( I_b \) can be used to calculate the limits of \( I_e \). These will clearly be within the 5 per cent limits set (from the effects of \( R_b \) only), so that the design is successful.

**Incompatible design requirements**

It may occur (and very often does, in the first attempt to meet a given specification) that the permissible maximum \( R_b \) obtained from the indicated design procedure is too small for other circuit requirements. In such a case the designer must resist the temptation to play down some of the possible variations which led to the low value of \( R_b \). The possible solution may be to change the transistor for a higher \( \alpha \) type, since the closer \( \alpha \) is to unity the smaller is \((1 - \alpha)\), so that variations in it produce smaller changes in \( I_b \). Similarly, a reduction in \( I_e \), if allowable from output considerations, also reduces \( I_b \).

Failure of these simple remedies means that too much is being asked of one transistor and either the specification must be changed or an additional transistor used as described in Chapter 7.

**Temperature Drift**

So many complicated equations are presented for the determination of the effects of temperature on operating conditions that the student will be pleased to know that the procedure outlined in the previous sections already includes all necessary figures.

The claim that a circuit is stabilized against temperature drift merely implies that the designer has been able to produce a circuit meeting the required specification while taking into account (as in the above examples) the parameter spreads with temperature as well as spreads from one unit to another.

It may be difficult to imagine a circuit capable of operating correctly with any transistor of a specified type and yet failing because of temperature change. This can, however, come about for the following reasons. First, if the operating temperature rises by, for example, 40 degC, the value of \( I_{bo} \) is likely to rise by a factor of 16, which would usually represent a much greater current change than the possible values of \( I_{bo} \) for all transistors of that type at normal room temperature. Secondly, most transistor parameters, although differing widely from one transistor to another of the same type, remain stable for any particular transistor at constant temperature. For example, a high-\( \alpha \) transistor will maintain its high value indefinitely; \( V_{be} \) does not normally change during the life of a transistor; \( I_{bo} \) is not so predictable but generally falls slightly as the transistor ages (rising \( I_{bo} \) with operating time indicates an unreliable transistor owing to contamination, often caused by imperfect sealing from the atmosphere). This means that if factory pre-adjustment of one of the circuit values which determine operating conditions is allowed, enabling for example \( V_{eb} \) to be set to a particular voltage by selection of \( R_b \), the circuit will remain stable thereafter provided the
temperature conditions are also stable. This procedure is often used for 'entertainment' circuits, where the use of the smallest number of components is more important than great temperature stability.

In most industrial and military designs, however, wide temperature variations are encountered and it is rarely practicable to construct a constant-temperature enclosure to house the electronic circuits. It should nevertheless be borne in mind that small temperature-controlled ovens are often used to enclose certain critical components such as quartz crystals, reference Zener diodes, and sometimes transistors.

It has already been established that temperature drift is caused by three transistor parameters. The list below explains how each of these affects the value of \( I_e \) and, hence, \( V_{be} \).

(a) \( V_{be} \). For a given value of \( I_e \) the corresponding value of \( V_{be} \) varies with temperature at a rate which usually lies between \(-2\) and \(-2.5\) mV/\( \text{degC} \). Note that this is a linear relationship and that although the spread quoted is wide, this covers all junction types and includes both germanium and silicon transistors. Any one type of transistor (i.e. alloy, planar, diffused, etc.) has a much narrower variation of the order of \( \pm 0.1 \) mV/\( \text{degC} \). The value of \( I_e \) also affects the \( V_{be} \) coefficient to about the same extent over the useful emitter current range of the transistor.

This \( V_{be} \) variation, as has already been shown, has a direct effect on the value of \( I_e \) in Figs. 2.4 and 2.5. The magnitude of the change in \( I_e \) is simply calculated, as has been seen, and its direction is such that \( I_e \) increases when temperature rises.

(b) \( I_{ebo} \). \( I_{ebo} \) rises with temperature exponentially, doubling itself every 9–10 degC rise. Since, strictly, only part of its low-temperature value obeys this law (the other part remaining constant), the value at high temperature cannot be predicted from a low-temperature figure. By assuming that all of the low-temperature \( I_{ebo} \) obeys the law, a pessimistic estimate for high temperatures can be made; in practice this is not far wrong for germanium but is often greatly pessimistic for silicon. It is usually necessary for design therefore to know absolute maximum \( I_{ebo} \) for the transistor at the maximum junction temperature to be used in the circuit (i.e. the maximum ambient temperature plus the rise due to dissipation within the transistor).

Its effect on the circuit has been seen to be twofold. First, \( I_{ebo} \) adds directly to the collector current. Secondly, the same \( I_{ebo} \) is flowing into the base and therefore causes a base voltage drop of \( R_b I_{ebo} \) in such a direction as to increase \( I_e \) as the temperature rises.

The latter effect usually predominates and its magnitude is simple to calculate, since it is equivalent to an increase in \( V_e \) of \( R_b I_{ebo} \).

(c) \( \alpha \). As stated previously, the variation in \( \alpha \) is approximately 0 to \(+1/25\) per cent per degree C. Although for a particular transistor the law is more or less consistent over a wide temperature range, different transistors of one type often have \( \alpha \) temperature coefficients at the two extreme limits.

Like \( I_{ebo} \), \( \alpha \) variation has two effects on operating conditions. The first, direct, effect is caused by the relationship \( I_e = \alpha I_o + I_{ebo} \). The maximum magnitude of this effect is therefore approximately \(+1/25\) per cent change in \( I_e \) per degree C, generally negligible in comparison with other drifts. The second effect is usually much more significant and occurs by virtue of the dependence of \( I_b \) and, hence, \( V_b \) (when \( R_b \) is present) upon \( (1 - \alpha) \), which in turn depends critically on \( \alpha \). For most transistors the \( \alpha \) temperature coefficient can therefore more conveniently be described in terms of \( \beta = (1 - \alpha) \), which normally has a maximum temperature coefficient of \(+2\) per cent per degree C. The quantity \( \beta \) is often called \( \beta \) and since \( \alpha \) is close to unity, \( 1/(1 - \alpha) \) is approximately \( \beta \).

As in the case of \( I_{ebo} \), the effect of \( \beta \) temperature coefficient on the operating conditions of the circuit of Fig. 2.5 is simply calculated by working out the change in \( I_b \) and, hence, \( V_b \) when \( \beta \) changes by the expected maximum amount. This should be calculated using the value of \( \beta \) which is the minimum for the transistor type at the lowest operating temperature to be considered, and then assuming \(+2\) per cent per degree C rise, since lowest initial \( \beta \) gives the worst case.

Note that, as with \( I_{ebo} \), \( \beta \) variations change \( V_b \) in such a way that \( I_e \) rises as temperature increases.

Summary of temperature effects

Three parameters drift with temperature: \( V_{be} \), which varies linearly at a rate of \(-2\) to \(-2.5\) mV/\( \text{degC} \); \( I_{ebo} \), which doubles some or all of its low-temperature value for every 9 or 10 degC rise; and \( \alpha \), which drifts in such a way as to cause \( \beta \) to vary 0–2 per cent per degree C.

The main effects on the circuit are caused by the direct influence
of $V_{be}$ on $I_e$ and by the changing base current caused by both $I_{ebo}$ and $\alpha$, which cause $V_b$ to differ from its intended value, and thus cause $I_e$ to change.

Smaller effects are caused directly by the change in $I_e$, even at constant $I_e$, brought about when $\alpha$ and $I_{ebo}$ vary.

All these effects cause the collector current to increase as temperature rises.

**Design for stability**

As indicated, the procedure described earlier for ensuring that circuit conditions are as required includes temperature-stability criteria. The changes in procedure for exceptionally wide temperature ranges (such as $-55^\circ$ to $+100^\circ$C) in some military applications are changes in parameter magnitudes only. For $I_{ebo}$ the $100^\circ$C $I_{ebo,max}$ figure would be used (or more if transistor dissipation raises the junction temperature appreciably); for $\beta$ a temperature rise of $155^\circ$C would be used and the worst case obtained by using the $-55^\circ$C minimum $\beta$ for the transistor; for $V_{be}$ would be assumed a drift of $-(2.5 \times 155)$ mV. Otherwise the procedure is unchanged.

**Design example**

As a practical example, the circuit of Fig. 2.6 will be examined and its temperature drift from 0 to $50^\circ$C predicted, assuming knowledge of the relevant transistor parameters.

It will be assumed that $T_1$ is a small signal (300 mW) silicon transistor having a minimum value for $\beta$ of 20.

![Fig. 2.6 Practical example of bias circuit](image)

First, low-temperature operating conditions are found by assuming $R_b$ to be zero and $V_{be}$ its maximum value, e.g. 0.9 V.

Then, 

$$I_e = (20 - 0.9)/4.7 \text{ mA}$$

$$= 4.06 \text{ mA}$$

since 

$$\beta_{min} = 20$$

$$\alpha_{min} = 0.95$$

and 

$$I_e = \alpha I_e = 0.95 \times 4.06$$

$$= 3.86 \text{ mA}$$

Hence, 

$$V_{eb} = 15 - 3.86/(2.2)$$

$$= 6.5 \text{ V}$$

Using the principle of calculating changes from nominal rather than recalculating every value for each extreme, the effect of $R_b = 3.9 \text{ k}\Omega$ can now be accounted for.

$I_b$ is known to be $I_e/\beta$, i.e. 0.2 mA, giving 

$$V_b = -0.2 \times 3.86$$

$$= -0.77 \text{ V}$$

The value for $I_e$ should therefore be 

$$I_e = (20 - 0.9 - 0.77)/4.7$$

$$= 3.9 \text{ mA}$$

$$I_e = 0.95 \times 3.9$$

$$= 3.7 \text{ mA}$$

and 

$$V_{eb} = 15 - 3.7/(2.2)$$

$$= 6.86 \text{ V}$$

Note, then, $I_b = I_e/\beta$ is still about 0.2 mA, so no further correction is required.

So far, temperature effects have been ignored, and since the minimum value of $\beta$ and the maximum value of $V_{be}$ have been used, the above values of $I_b$, $I_e$, and $V_b$ represent the low-temperature limit. The changes which occur when the ambient changes by 50 degC are as follows.

(a) $V_{be}$. $V_{be}$ falls by 125 mV. Since $V_b$ is assumed constant (for the moment base voltage changes are to be ignored), the change in $I_e$ must be $125/R_e \text{ mA}$, i.e. $125/4.7 \mu\text{A}$, or 26.6 $\mu\text{A}$. This causes a change of $0.95 \times 26.6 \mu\text{A}$ in $I_e$, i.e. 25 $\mu\text{A}$, and hence, a change in $V_{be}$ of $25 \times R_e \mu\text{V} = 25 \times 2.2 \mu\text{V}$, or 55 mV. The direction of the $V_{be}$ change increases $I_e$ and $I_e$ and therefore reduces $V_e$. 

D
(b) $I_{CBO}$. In the absence of data a maximum value of 10 $\mu$A at 50°C for almost any small silicon transistor is a slightly pessimistic figure and this will be assumed here.

There are two effects, as described earlier. The first is that $I_C$ increases by 10 $\mu$A because $I_{CBO}$ adds directly to the collector current. This gives a $V_{CE}$ change of $10 \times 2.2$ mV, i.e. 22 mV, and this is again a reduction in $V_{CE}$.

The second effect is to change $V_T$ by $10 \times 3.9$ mV, i.e. 39 mV. This movement of $V_T$ will itself cause $I_E$ to change by $39/R_E$ mA, i.e. $39/4.7$ mA, or $8$ $\mu$A, giving a change in $a_T$ of $0.95 \times 8$, i.e. $7.6$ $\mu$A. This causes $V_{CE}$ to change by $7.6 \times 2.2$ mV, i.e. 17 mV, and again $V_{CE}$ is reduced.

Hence, the total effect of $I_{CBO}$ is a $V_{CE}$ reduction of 39 mV.

(c) $\beta$. The minimum value for $\beta$, namely 20, was used to derive the 0°C conditions. Change in $\beta$ totals $2 \times 50 = 100$ per cent and $\beta$ therefore changes from 20 at 0°C to 40 at 50°C.

The direct $\alpha$ change therefore alters $I_B$ by 2-5 per cent, giving a change of $3.7 \times 2.5/100 = 92.5$ $\mu$A, reducing $V_{CE}$ by $92.5 \times 2.2$ mV = 0.2 V.

When $\beta$ changes from 20 to 40, $I_E$ changes by $I_e[(1/20) - (1/40)]$, i.e. by $0.025 I_e$, or $3.9 \times 0.025 = 97.5$ $\mu$A, giving a $V_T$ change of $97.5 \times 3.9$ mV = 380 mV. $I_E$ therefore changes by $380/R_E = 380/4.7 = 81$ $\mu$A, $I_C$ by $0.95 \times 81 = 77$ $\mu$A, and $V_{CE}$ by $77 \times 2.2$ mV = 0.17 V.

Hence, the total effect of $\beta$ is a $V_{CE}$ reduction of $0.2 + 0.17 = 0.37$ V.

The total drift of $V_{CE}$ from all transistor changes with temperature is therefore $0.055 + 0.039 + 0.37 = 0.46$ V for silicon.

This method of calculating drift is not exact, since each parameter drift affects the contribution caused by another; for instance, when $\beta$ changes from 20 to 40 the effect of a change in $I_E$ due to $V_T$ change is different in the collector circuit, since $I_C$ drifts will be multiplied by 0.975 instead of 0.95.

In practice, however, the method is quite satisfactory, because these interacting effects are important only when one of them represents a large percentage change from the nominal state. If this proves to be true the design will require modification, and, using this method of calculation, it will be immediately obvious which remedy to take.

**Temperature compensation**

In view of the severe effects of drift in transistor circuits, attempts are often made to obtain stability by adding another temperature-sensitive element in such a way as to oppose drift caused by the transistor.

Since, as described in this chapter, transistor drifts are predictable only in their maximum values, it is generally impossible to provide an equal and opposite drift unless the compensating device can be adjusted at two or more different temperatures until its law is correct. This is normally out of the question, representing as it does a prolonged setting-up procedure, but may occasionally be accepted if the alternative is a more expensive system (e.g. chopper amplifier, Chapter 9).

The exception is $V_{BE}$ temperature variation which, first, has a limited range of values (from −2 to −2.5 mV/degC) and, secondly, is linear with regard to temperature. This means that if it can be arranged to ‘back off’ $V_{BE}$ changes, and the backing-off device and the transistor have the opposite extreme values, then at least the drift has been reduced to ±0.5 mV/degC. With a little more care, by using for the compensating device a similar junction type (e.g. alloy, diffused, planar, etc.) and running the device and transistor at similar currents, the spread is greatly reduced and is typically ±0.1 mV/degC, an improvement of about 10:1 on the uncompensated transistor. The compensating element may be a diode or, for better matching, an extra transistor preferably mounted within the same can as the amplifier transistor.

The actual arrangements for compensation may be the circuit of Fig. 2.7, or any configuration in which the added device does not seriously affect circuit operation and in which its drift and the transistor’s act in opposite directions and with equal significance.

**Other Bias Arrangements**

Although the bias circuit discussed is the most usual arrangement, there are a few other possibilities. The emitter may be held at a fixed potential (e.g. ‘earth’) and a base current supplied as shown in Fig. 2.8. In this circuit the base current is accurately known provided $V_{ES} \gg V_{BE}$ but the resulting collector current depends greatly on $I_{CBO}$ and $\beta$ (see Appendix 2, standard bias circuit where $R_E = 0$). The practical implication is that in order to be certain that the transistor
collector voltage will not reach zero even with a high-β transistor, $R_L$ must be much smaller than the value it would be given in the standard bias circuit. The output voltage swing has to be restricted to a small fraction of $V_T$ to avoid possible cut-off or saturation, and so the circuit is useful only in simple applications where temperature drift is unimportant and signal levels are low, e.g. domestic radio equipment.

An even worse version of Fig. 2.8 is obtained by omitting the base resistor, and capacitor coupling the signal to the base. The defects of this method are even greater than those of the last, since the collector current is now given by $β_{CEO}$ (often called $I_{CEO}$), which is an amplified version of the extremely variable $I_{CEO}$.

A much better attempt is shown in Fig. 2.9, which uses d.c. negative feedback to define the collector voltage. Note that in the intended operating condition $T_1$ base will be just positive and, if the base current is assumed negligible, then the current in $R_b$, approximately $V_n/R_b$, must equal that in $R_b$, namely $V_c/R_b$. Therefore $V_c = V_nR_b/R_b$. This suggests that $R_b$ has no influence on $V_c$ and is substantially true provided $T_1$ passes current and that $I_b$ remains $< V_n/R_b$. To design this bias circuit, just assume that conditions are as desired and use Ohm's law to deduce circuit values. For example, suppose that $V_n = 10$, then $V_c$ is to be approximately $+5$ V, and that $T_1$ collector current is to be $3$ mA and its $β$ at least 30.

$T_1$ base current is at most $3/30 = 0.1$ mA. Let $V_n/R_b = 1$ mA, giving $R_b = 10$ kΩ. $V_o = +5$, so that $5/R_b = 1$ mA, giving $R_b = 5$ kΩ. The current in $R_L$ is the sum of $1$ mA from $R_b$ and $3$ mA from $T_1$, so that $R_L = (V_p - V_o)/4 = 1.2$ kΩ.

These figures are all approximate and no allowance for $I_b$ was made in calculating $R_b$.

To understand fully the operation of this bias circuit, note the effect of variations in $V_p$, $V_n$, $R_b$, $R_b$, $R_c$, $V_{bo}$, $β$, and $I_{CEO}$. An increase in $V_p$ increases $(V_p - V_o)/R_L$ thus increasing $I_c$ and $I_b$, but until $I_b$ becomes significant $V_c$ remains constant. An increase in $V_n$ increases $V_n/R_b$, so that $R_{bo}$ drops more voltage, thus increasing $V_c$ in direct proportion to $V_n$. Decrease in $V_p$ similarly has no effect on $V_c$ until $(V_p - V_o)/R_L$ is equal to the current in $R_b$, implying $I_c = 0$. The transistor is now cut off, its base voltage is no longer just below zero, and the stabilizing loop fails. Changing $R_b$ or $R_b$ is similar to changing $V_n$ and a proportionate change in $V_c$ results.

The temperature variations of $V_{be}$, $β$, and $I_{CEO}$ are of special interest.
in that the method for assessing their effect is applicable to all d.c. negative feedback systems.

\[ V_{be}, \text{which was ignored in the previous calculations, affects the} \]
\[ \text{conditions by determining the potential at the junction of } R_{b1} \text{ and} \]
\[ R_{b2} \text{ and the correct equation for } V_c (\text{still ignoring } I_b) \text{ is} \]
\[ (V_u + V_{be})/R_{b1} = (V_c - V_{be})/R_{b2} \]

When \( V_{be} \) varies by \( \delta V_{be} \), it changes \( R_{b1} \) current by \( \delta V_{be}/R_{b1} \) and so changes \( V_c \) by \( (\delta V_{be}/R_{b1}) + \delta V_{be}(R_{b2} + R_{b1})/R_{b1} \). Therefore \( V_c \) increases by \( (R_{b2} + R_{b1})/R_{b1} \times (2 \text{ to } 5) \text{ mV}/\text{degC} \).

\( \beta \) and \( I_{be} \) change \( I_b \) and the resulting change in \( V_c \) is approximately \( \beta I_b R_{b2} \). This is a most useful result for the assessment of current drifts arising at the feedback input terminal of a negative d.c. loop. The argument is very simple: \( I_b \) will clearly affect the base potential but the collector will move much more, so that in calculating voltage drops in \( R_{b1} \) and \( R_{b2} \) the base variation is negligible. Current \( V_u/R_{b1} \) can then be taken as unchanged and any \( I_b \) change flows into \( R_{b2} \), thus varying the drop in \( R_{b2} \). Since the base is fixed, the collector moves by this amount, and if \( I_b \) is inwards and then increases by \( \delta I_b \), \( V_c \) will become more positive by \( \delta I_b R_{b2} \) V.

Sometimes the circuit of Fig. 2.9 is used with \( R_{b1} \) omitted. Although this is advantageous in removing the need for \( V_u \), the current in \( R_{b2} \) is now \( I_b \), so that \( V_c \) depends greatly on \( \beta \) and \( I_{be,0} \), which in Fig. 2.9 could be made only a small fraction of \( V_u/R_{b1} \). Collector voltage is therefore badly defined, but not so badly as if \( R_{b2} \) were returned direct to \( V_p \).

Finally, it should be pointed out that the commonly used bias circuit of Fig. 2.10(a), which requires only a single supply, is basically the same as that of Fig. 2.5. This is easily proved by Thévenin's theorem and its equivalent circuit is given in Fig. 2.10(b). For simplicity of description, most circuits in Part I use the form of Fig. 2.5.

![Fig. 2.10](image)

To summarize, designing a biasing circuit for a single transistor consists in ensuring that the operating currents and voltage remain within limits set by various external circuit requirements. This is achieved by making an initial nominal choice of values and assessing by calculation the effect of each varying parameter on the operating conditions. In the light of these effects, values are modified until the operating conditions are within the required tolerance limits.

Temperature stabilizing is not a separate problem and the calculation of 'stability factor', although indicating how bad stability is, does not point to the particular cause of drift. Instead the recommended procedure is to include temperature effects when assessing the effect of parameter variations.

Temperature-compensation techniques are of doubtful value except to make an already good or marginal design better. Reliance on these methods for correction of large drifts is unsound because the compensating device and the transistor may not always be at the same temperature; even if they are, a small imperfection in matching can give large drifts. In particular, \( I_{be,0} \) cannot be satisfactorily compensated for because of its great variability; the same applies to \( \beta \), but to a very much smaller extent. \( V_{be} \) can be balanced out provided an improvement of no better than 10:1 is anticipated; special pairs of transistors ('dual' transistors) and housed in one encapsulation do, however, give excellent balance of better than 100:1. The inherent \( V_{be} \) and \( \beta \) matching which exists between transistors on the same chip is exploited by the integrated-circuit designer in circuit configurations which would be unsound using discrete devices.
3—The transistor as a switch

One of the assumptions made in the last chapter on the biasing of a transistor was that the collector−emitter voltage would never be allowed to reach zero. This chapter deals with the use of values deliberately chosen to make $V_{ce}$ as near zero as possible.

Figure 3.1 shows a simple circuit which in terms of normal bias arrangements is unsatisfactory because the only current directly determined by the circuit appears to be $I_b$, which must be $V_p/R_b$ provided $V \gg V_{be}$. Hence, $I_c$ must be $\beta I_b = \beta V_p/R_b$ and $V_L = \beta V_p R_L/R_b$,

which is greatly dependent upon $\beta$. But suppose that, even with the lowest possible value of $\beta$, $\beta R_L/R_b > 1$; then $V_L$ is apparently $> V$. This is clearly impossible, since the collector voltage would then become negative with respect to earth and no negative supply exists which could cause this to happen.

In fact, as $V_{ce}$ approaches zero $\beta$ falls until, with $V_{ce}$ almost zero, $\beta = 0$. Since $\beta$ is the ratio of a change in $I_e$ to the change in $I_b$ causing it, this implies that further increase of $I_b$ has no further effect on $I_e$ and $I_e$ remains equal to $V_p/R_L$.

In this condition the transistor is said to be 'saturated' or 'bottomed', and to ensure this state the designer merely makes $I_b > I_c/\beta_L$, where $\beta_L$ is the large signal current gain* from 0 to the desired $I_c$. When the same supply rail is used for $R_b$ and $R_L$ and its voltage is much greater than $V_{be}$, then for saturation $R_b < R_L/\beta_L$.

A similar situation is shown in Fig. 3.2, where the collector is fixed and two supplies are used. To ensure saturation the same criterion must be obeyed, $I_b > I_c/\beta_L$ or in this case $V_p/R_b > V_n/(R_n/\beta_L)$.

In the saturation condition the transistor has some especially useful properties. Consider the circuit of Fig. 3.2 to be changed by reversing the polarity of $+V_n$; the circuit is now identical in form to Fig. 3.1 except that the emitter and collector are interchanged. It has already been pointed out in Chapter 2 that a transistor operates quite normally in this condition provided ratings are observed. The only significant change is that $\beta$ becomes much lower (e.g. 3 or 5) in this direction except for a symmetrical transistor, where $\beta$ is similar in both connections.

Hence, provided that $V_p/R_b > V_n/(R_n/\beta_L)$, where $\beta_L$ is the $\beta_L$ in reversed connection, the $V_{ce}$ will remain zero in the Fig. 3.2 circuit even when $-V_n$ is changed to $+V_n$. This means that the transistor acts as a short-circuit to currents passing through $R_n$ in either direction.

This concept is most important in understanding transistor switching circuits (especially d.c. choppers). Although it is possible to design the circuit to saturate only with $-V_n$ negative (by $V_p/R_b > V_n/R_n/\beta_L$, but $< V_n/(R_n/\beta_L)$), it is equally possible to design for bidirectional saturation by making $V_p/R_b > V_n/(R_n/\beta_L)$.

**SATURATION POTENTIALS**

The collector−emitter voltage at saturation, known as $V_{cesat}$, depends on several factors: the type of transistor construction, the

* $\beta_L$ is the ratio of collector current to base current at a given $V_n$ and $\beta$ is the ratio of a small change in collector current to the change in base current which causes it, at a specified value of $I_b$ and $V_n$. 

---

*Fig. 3.2 Alternative saturation circuit*
value of load current $I_L (=I_e$ or $I_O)$ and its direction, the base
current, and, to a slight extent, the junction temperature.

In general, alloy transistors, germanium and silicon, $p-n-p$ and
$n-p-n$ have low values of $V_{ce(sat)}$ Planar epitaxial transistors have
values almost as low. Alloy diffused, 'straight' planar, and diffused
types have much higher values.

The load current naturally influences $V_{ce(sat)}$, because, when it
increases beyond $\beta_L I_B$ or $\beta_d I_L$, the transistor begins to leave
saturation and so $V_{ce}$ rises. If this is offset by arranging an increase in $I_B$
when $I_L$ rises, $V_{ce}$ may still rise or alternatively may change sign, passing
through zero at some value of $I_L$. This is puzzling at first sight but careful examination of Figs. 3.1 and 3.2 reveals
how this comes about. In Fig. 3.1 it is clear that $V_{ce}$ is positive, i.e.
collector positive to emitter. An increase in $I_c$ accompanied by an
increase in $I_B$ would maintain saturation (assuming $\beta_L$ constant), but
$I_e$ still has to flow through some bulk semiconductor not under the
influence of transistor action and the voltage drop of this part of
$V_{ce}$ must increase. In Fig. 3.2, if the special case is considered where
$I_e = 0$, $V_{ce}$ will clearly be negative, i.e. emitter positive to collector
since no supply exists which could cause the open-circuit emitter to
fall below earth potential. On reconnecting $I_B$, the emitter potential
can be negative or positive depending on $I_B$ and the bulk resistance
through which $I_e$ flows. As $I_e$ rises from zero, $V_{ce}$ therefore changes
sign, becoming positive, and then continues to increase in this
direction.

The effect of increasing the base current depends on its initial value.
If it is assumed to be zero and is increased, $V_{ce}$ will approach zero
as $I_B$ approaches $I_L/(\beta_L$ or $\beta_d)$. As $I_B$ further increases, $V_{ce}$ becomes
even closer to zero, and then either increases again with the same
sign as with low $I_B$, or passes through zero, changes sign and then
increases. The reasoning for similar behaviour for changing $I_L$
is again applicable: the circuit of Fig. 3.1 always has $V_{ce}$ positive, and
that of Fig. 3.2 will change sign.

Temperature rise has two main effects: first, $\beta$ increases with the
same results as a change in $I_B$ and, secondly, the bulk resistance
changes, causing a direct change in $V_{ce(sat)}$. If the circuit is designed
to be well in saturation at all temperatures of operation, a 50 degC rise
typically causes $V_{ce(sat)}$ to change 100 per cent, which is usually
acceptable, since the circuit is normally designed to make $V_{ce(sat)}$ itself
negligible.

**Significance of the Above Results**

Where low $V_{ce}$ is of great importance and base current of $I_L/\beta_L$
and $b_t$ cannot be assumed >2) is available, the circuit of Fig. 3.2
is better than that of Fig. 3.1 whether the emitter load is returned to
$+$ or $- V_n$.

When, however, a slightly higher figure for $V_{ce}$ can be tolerated,
Fig. 3.1 requires much less $I_B$; this is particularly important when $I_L$
is really large, such that $I_L/\beta_L = I_L/2$ would represent a large drain
from the base supply circuit.

In practice this generally means that Fig. 3.2 is used when $I_L < 5$
mA and Fig. 3.1 for higher currents: it is fortunate that higher $V_{ce}$
can usually be tolerated in high-current circuits.

When $I_L < 100 \mu$A, then, by using Fig. 3.2 and $I_B = 1/4$ to 1 mA,
$V_{ce(sat)}$ is as low as 1 to 4 mV for most alloy transistors, germanium
or silicon. Planar epitaxial values are about 5-20 mV. In the Fig. 3.1
circuit under optimum $I_B$ conditions, the best result is usually 10-20
mV for alloy and 50-100 mV for planar epitaxial.

At values of $I_L$ above 100 $\mu$A $V_{ce(sat)}$ increases and at 1 mA
is usually 10 mV, corresponding to the best case above of 1 mV.

For much higher currents e.g. 10 A $I_L$ with 1 A $I_B$, $V_{ce(sat)}$, for
Fig. 3.1 is usually from 0.3 to 1 V for alloy types. The connection of
Fig. 3.2 is then generally unsatisfactory if $V_n$ is reversed, since 5 A $I_B$
may be required.

**CUT-OFF CHARACTERISTICS**

In order to reduce load current to zero in Fig. 3.1, the base–emitter
junction must be reverse-biased or short-circuited, or at least the
base must be more negative than the turn-on potential for the
transistor. In Fig. 3.2 it is insufficient to drop $V_B$ to earth potential—it
must be more negative than $-V_n$. The only safe criterion is that for a
$p-n-p$ transistor the base must be more positive than the emitter and
collector for turn-off. Similarly, for $n-p-n$ the base must be more
negative than emitter and collector.

When 'cut-off', currents still flow from the transistor; $I_{ebo}$ flows
out of the collector and $I_{eb}$ out of the emitter. Generally, but not
necessarily, $I_{ebo}$ is less than $I_{eb}$, but if no $I_{eb}$ value is quoted by the
manufacturer, it can only be assumed to be less than or equal to
$I_{ebo}$.

The effect of these currents is shown in Figs. 3.3 and 3.4, which
correspond to Figs. 3.1 and 3.2, respectively. In the first case $I_{ebo}$ flows into $R_L$, giving a voltage drop $I_{ebo}R_L$ across the load. In the second, $I_{ebo}$ causes a voltage drop $I_{ebo}R_L$ across the load.

Temperature affects $I_{ebo}$ as described in Chapter 2, and $I_{ebo}$ follows the same law, so that any calculation must be made at the highest temperature of interest. In the majority of practical applications this effect rules out the use of germanium types for low-level circuits whenever $R_L$ exceeds about 1 kΩ.

**SWITCHING FROM 'ON' TO 'OFF'**

When a transistor connected as shown in Fig. 3.1 or Fig. 3.2 is alternately in saturation and cut-off, the base conditions have to change from a circuit giving a known $I_b$ to one giving a known $V_b$. A simple series resistor driven by a voltage source is usually satisfactory, but care must be taken that the potential $V_1$ reached on the positive swing is much greater in magnitude than $V_{be}$ so that $I_b$ is known to be $\approx V_1/R_b$ (Figs. 3.5, 3.6).

**THE TRANSISTOR AS A SWITCH**

A capacitor-coupled drive circuit often causes difficulty and can only be used safely under special conditions. The snag is (see Fig. 3.7) that on the first positive swing, $T_1$ conducts, charging $C$ as shown; on the next negative swing $T_1$ cuts off leaving $C$ charged. On the second positive swing $C$ charges further and still does not discharge on the next negative swing. Eventually $C$ becomes charged to $(V_1 - V_{be})$ and $T_1$ base never turns on again; in other words, $C$ and the transistor have 'd.c. restored' the input waveform which appears on the base going negative only (see Chapter 1). One solution is to allow $C$ to discharge through a resistor connected to a positive potential, while $T_1$ is off, so that $T_1$ will conduct on the next positive swing to replenish the lost charge (Fig. 3.8). This can be satisfactory if the input base waveform is regular in mark space, since $I_b$ which flows to replenish $C$ can then be calculated; on the whole however the situation is best avoided. If the earthed emitter circuit is being used, a simpler and better remedy exists. A diode between base and emitter enables $C$ to discharge and recharge every cycle. This can clearly be done only in the earthed emitter circuit, since in the earthed collector version $V_b$ must fall to at least $-V_n$.

Transformer coupling may be used as shown in Fig. 3.9 to avoid
transient effects

When a transistor is saturated and its base-emitter voltage is then reversed, there is a time delay before the current decreases to leakage level. This is quite distinct from the time taken for the collector-emitter voltage to reach its ultimate 'off' condition, which is determined by the stray capacitance and the value of the load.

The effect is similar to the turn-off characteristic of a single junction (see Chapter 1), and again the delay time is proportional to the degree of saturation before turn-off and inversely proportional to the current which flows 'backwards' in the base circuit when turn-off is initiated. The degree of saturation refers not only to the emitter-collector current flowing when saturated but also to how much base current is used which is in excess of the minimum to achieve saturation. The reverse base current which flows when cutting off is calculated by assuming that initially the base potential remains unchanged when bias is reversed. Knowing this base potential and the source e.m.f. and resistance, the reverse current is easy to calculate.

As in the case of diodes, Q figures of stored charge in the base circuit are often quoted for transistors and have the same meaning (see Chapter 1). Some manufacturers quote instead delay or hole-storage times under specified conditions. Knowing that these times increase in proportion to 'on' base current and decrease in proportion to 'off' base current, the designer can readily convert the information to suit his circuit conditions.

Transistor inter-electrode capacitances also affect switching performance, and because of these and the other effects just described, transistors with f₀ of 1 MHz are useful as heavily saturating switches only up to a repetition rate of a few kilohertz.

**Transformer Power Switch**

Transistors are used in the saturated switching mode at least as much as in any other connection. The most obvious use is the replacement of a mechanical switch or relay where moving parts have disadvantages. Where there is a fire hazard, where the equipment must operate under severe vibration, where a circuit must be switched by remote control, where high operating speed is required, the transistor can out-perform all but the most expensive mechanical arrangements.

The simplest application is perhaps the direct use of the circuit of
Fig. 3.1 where $R_L$ is a load to be connected or disconnected from voltage $V$, $T_1$ taking the place of a relay. A relay has the advantage of alternately giving a good short- and a good open-circuit compared with a series $V_{oct}$ and a leakage $I_{ebo}$, but has shorter life owing to contact wear and cannot generally be operated so fast.

Design is straightforward. Find the load current ($V/R_L$) and select a suitable transistor type (i.e. alloy or planar epitaxial, germanium or silicon, $p-n-p$ or $n-p-n$, according to price, tolerable 'off' leakage, and supply polarity) having useful large-signal current gain $\beta$ at $I_e = V/R_L$. Supply $I_b > V/\beta R_L$ to turn on, allowing an extra factor of 2 if convenient; ensure $V_{be}$ at least reaches zero (preferably reverses) to turn off.

![Fig. 3.10 Power switch](image)

For example, one terminal of a 30 $\Omega$ load is permanently connected to $-30$ V d.c.; the other terminal is to be connected to and disconnected from zero potential by changing the voltage at the remote end of a twin cable attached to the switch. The current along the cable must not exceed 100 mA and its total resistance is 20 $\Omega$.

This requires the circuit arrangement of Fig. 3.10, which is similar to 3.1 except that $R_b$ is returned to a separate supply.

The transistor can be a $p-n-p$ alloy type rated at least 1 A and 30 V, and this itself almost dictates the use of germanium, since silicon $p-n-p$ types of this rating are very expensive. Fortunately, no leakage limit is quoted in the circuit specification. A suitable type is the NKT403, which has a guaranteed $\beta$ of at least 30 at 1 A, so that for the saturated condition $I_b$ must exceed 1/30 A and could nominally be 1/15 A, i.e. 66 mA. The use of such a generous extra $I_b$ saves the designer the tedious task of adding tolerances, because so long as $V_s$ which supplies $I_b$ is $> V_{be}$, saturation is certain even with wide component variations.

Now, $V_{be}$ at saturation is quoted as 0-75 V, so a supply of 7-5 V is adequate for the base circuit. (Almost as good would be 0-75 V, and even better, 10 V.) Base current is therefore $(7.5 - 0.75)/(R_b + 20)$ and this must equal approximately 66 mA. Therefore

$$R_b = \frac{6750}{66} = 20 \approx 82 \Omega$$

Connection of the 7.5 V supply in the direction shown in Fig. 3.10 therefore connects the load if $R_b \leq 82 \Omega$. Because $V_{oct}$ is 0.75 at 1 A, the load actually receives 29.25 V.

If the remote 7.5 V supply is reversed, then the transistor cuts off. To check this, first assume it to be true, then confirm that $V_s$ is positive when leakage is taken into account. The base current is $(I_{ebo} + I_{ebo})$ and this causes the base to fall relative to emitter by $(R_b + 20)(I_{ebo} + I_{ebo})$, so that the actual base potential $V_b$ will be $7.5 - (R_b + 20)(I_{ebo} + I_{ebo})$. It must be confirmed that, at the maximum operating temperature, this is still positive or zero. At 80°C, for example, $I_{ebo(max)} = 10$ mA, $I_{ebo(max)} = 10$ mA, giving $V_b = 7.5 - 102(20) \times 10^{-8} = +5.5$ V, which is satisfactory. Note, however, that in the cut-off direction the 7.5 V supply were reduced to $+2$ V, then with 'bad' transistors at 80°C the circuit would only just turn off the load current.

Transistor power in the 'off' state is $(V_s - I_{ebo}R_L)I_{ebo} \approx 300$ mW at 80°C and in the 'on' state is $I_eV_{oct} + I_bV_{be}$, i.e. 0.75(66 + 1000) $\approx 800$ mW. Mean power depends on mark/space, but if the 'on' state lasts for more than a few milliseconds the transistor rating would be taken as 800 mW.

It is important to remember that, although these static transistor dissipation levels are low compared with the load power of 30 W, a slow transition between the two states can cause high transistor dissipation. Clearly the worst case is when about 15 V appear across the load, giving a current of 0.5 A, and a transistor dissipation of 7.5 W—as in the load. Care must therefore be taken either that the transition is over in one or two milliseconds or that a heat sink is used to enable 7.5 W to be dissipated. It is not sufficient that the transition is short compared with the on and off times: it must be short compared with the transistor thermal time constant.

STANDARD CIRCUITS USING TRANSISTOR SATURATION

Following the above principles of transistor switching, the design of several standard switching circuits becomes easy, provided the
function of the circuit is clearly understood. As in the case of most switching circuits, the waveforms must be assessed by using the capacitor charging laws set out in Chapter 1. It is then possible to calculate values to ensure correct operation.

In the following examples p-n-p versions of the standard circuits have been used for illustration; n-p-n devices are also suitable and it is useful for the reader to become familiar with both forms of all the circuits he may encounter.

**Free-running Multivibrator**

The standard multivibrator of Fig. 3.11 is intended to operate with $T_1$ and $T_2$ alternately cut-off or saturated. Assume then that $T_1$ is saturated, so that $V_{ce}$ is zero; if $T_2$ is to be cut-off, $T_2$ base must be positive and $T_2$ collector at, or approaching, $-V_s$.

The action from now on is that $R_4$ causes $T_2$ base to approach $-V_s$ at a rate determined by $C_2R_3$. When it reaches $-V_{eb}$, $T_2$ begins to conduct and will eventually saturate provided $R_4 < \beta_2R_3$. As $T_2$ saturates, $V_{ce}$ rises from $-V_s$ to earth and therefore causes $V_{eb}$ to rise an equal amount, that is by $V_s$, cutting off $T_1$. This state remains as $R_4$ pulls $T_1$ base towards $-V_s$ at a rate determined by $R_1C_1$. In the meantime $T_1$ collector falls to $-V_s$ at a rate given by $C_2R_3$. As $V_{eb}$ reaches $-V_{eb}$, $T_1$ begins to conduct and soon saturates, provided $R_1 < \beta_2R_2$.

The whole action is then repeated, giving the waveforms shown in Fig. 3.12. The time taken for $T_1$ base to reach $-V_{eb}$ when starting at $(-V_{eb} + V_s)$ above earth, and aiming at $-V_s$ with a time constant $C_1R_1$, is given by

$$t_1 = C_1R_1 \log \frac{2V_s}{V_s - V_{eb}}$$

This approximates to $t_1 = C_1R_1 \log 2$ provided $V_s \gg V_{eb}$. This period is the time for which $T_1$ is cut off and $T_2$ is saturated;

the other "half"-cycle has a period of $t_2 = C_2R_3\log 2$ provided $V_s \gg V_{eb}$.

Note that for correct action $R_1 < \beta_1R_1$ and $R_4 < \beta_2R_3$, which is the normal static condition for saturation; as suggested previously, a further factor of 2 avoids all tolerance problems ($R_1 < \beta_1R_1/2$, $R_4 < \beta_2R_3/2$). Note also the need for $V_s \gg V_{eb}$ and $V_s \gg V_{eb}$; although the circuit would operate, the timing would depend on $V_{eb}$ and $V_s$ if this condition were not obeyed.

The above equations enable simple design to be carried out. If external loads are negligible and if the circuit values are symmetrical ($R_1 = R_4$, $R_2 = R_3$, $C_1 = C_2$), all will be well. There are one or two tricky points which can arise when these conditions do not apply.

**External load**

If a load $R_L$ is added from $T_1$ collector to earth, it is clear that when $T_1$ cuts off it falls only by a fraction of $V_s$ given by $V_sR_L/(R_2 + R_L)$. When $T_1$ turns on again its change of collector voltage is $V_sR_L/(R_2 + R_L)$, and this is also the positive swing on $T_2$ base. The timing equation is now

$$t = C_2R_4 \log \frac{R_2 + 2R_L}{R_2 + R_L}$$

and so depends on the value of $R_L$.

If, on the other hand, $R_L$ appears directly in parallel with $R_2$ then either $T_1$ no longer bottoms, giving either no oscillation or a frequency and amplitude which depend critically on $\beta_1$, or $T_1$ still bottoms, giving unchanged timing and amplitude.

Where possible, any external load should therefore be connected in parallel with $R_2$ or $R_3$ (not to 'earth') and $R_1$ and $R_4$ should be designed to maintain saturation with the additional collector current. If this cannot be done and the load must go to earth, an emitter-follower may be placed between the collector and the load (see Chapter 4) so that the influence of the load current is reduced by the $\beta$ of the transistor.

**Asymmetry**

To appreciate the difficulties of asymmetrical operation, it is necessary to examine the collector waveforms more closely. In Fig. 3.12 it can be seen that when $T_1$ cuts off, $T_1$ collector falls with a time constant $R_2C_2$ (since the right-hand side of $C_2$ is fixed at just below earth by $T_2$ base), reaching its ultimate level of $-V_s$ after a time of about $4R_2C_2$ (within a few per cent). In the normal description
of its action it is assumed that this state has been reached before $T_1$ turns on again.

Suppose that $C_1$ is only 1/100th of $C_2$ and that $R_1 = R_4, R_2 = R_3$; then a ratio of 100:1 would be expected in the on and off times of $T_1$. With the ratio given $T_1$ would be cut off for a time of about $C_1R_1 \log 2$ and $T_1$ collector would be falling during this interval, reaching $-V_s$ in a time $4C_2R_2$. Now, $C_2 = 100C_1$ and $R_1 \approx \beta R_2/2$, e.g. 15$R_2$. This gives a cut-off time of 0.7 $C_1R_1$ and a collector time of 27 $C_1R_1$ to reach $-V_s$.

It is clear that $T_1$ collector does not have time to fall far before $T_1$ turns on and $T_1$ collector rises to earth again. This means that $T_2$

![Fig. 3.12 Waveforms for free-running multivibrator (Fig. 3.11)]

base rises only slightly and soon recharges, so that the 'T2-off' interval is much shorter than anticipated.

The circuit therefore has a limit to the mark/space ratio which may be obtained by increasing $C_2/C_1$, given roughly by 0.7 $C_1R_1 = 4C_2R_2 = 8C_3R_3/\beta$, i.e. $C_2/C_1 = \beta/11.5$. This may be exceeded by a factor of about 2 before serious drop of output from $T_1$ accompanied by inaccurate timing occurs, corresponding to a practical limit of 10:1 if $\beta \approx 50$.

Another method to change mark/space is to vary $R_1$ and $R_3$ in the same proportions while leaving $C_1$ equal to $C_2$. This avoids the above situation, since the fall time of $T_1$ collector is reduced in the same proportion as the cut-off time for $T_1$. Another difficulty now appears, however, because when $T_2$ saturates, its load $R_2$ has $R_1$ in parallel and it is $T_2$ collector which supplies the current $V_2/R_1$ which raises $T_1$ base to $+V_s$. (This is naturally true also when the circuit is symmetrical, but then $R_1$ is $(\beta/2)R_3$ and may be ignored.) The consequence of this may be seen by assuming $R_1 \approx R_3$ in an attempt to obtain a mark/space of $\beta/2$ to 1. Because the collector load of $T_2$ is halved at initial saturation, $R_4$ must be half its normal value, i.e. $R_3/\beta/4$. This halves the cut-off period for $T_2$ and the resulting mark/space is $\beta$ to 4, not $\beta$ to 2.

The best method for high mark/space ratio is therefore to use a combination of these two methods, making $C_2/C_1 \approx \beta/11.5$, $R_4/R_1 \approx 4/1$, $R_3/R_2 \approx 4/1$ giving a mark/space of about 20:1 if $\beta = 50$.

**Temperature effects**

Since the timing equation used earlier involves $V_{eb}$, change of temperature clearly affects the timing unless $V_s \gg V_{eb}$.

Other effects not mentioned in the equation are $V_{cossat}$ and leakage current. The former is always negligible if the precautions for good bottoming dealt with earlier in this chapter are taken. $I_{ebo}$ is a more serious problem, since when $T_1$ is cut off and $T_1$ base is falling as $C_1$ charges, $(I_{ebo} + I_{eb})$ adds to the charging current of $R_1$. Cut-off time is reduced, the law being complicated since $(I_{ebo} + I_{eb})$ charges $C_1$ linearly and $R_1$ charges $C_1$ exponentially.

For correct design the current in $R_1$ at the end of the cut-off period of $T_1$ (namely $V_s/R_1$) must therefore be much larger than $(I_{ebo} + I_{eb})$ at the maximum operating temperature.

A secondary effect of $I_{ebo}$ is that $T_1$ collector swings from $V_{ebmax}$ to $(-V_s + I_{eb}R_2)$, not to $-V_s$ as assumed. This again affects timing and also reduces the output swing so that a second condition is that $I_{ebo}R_3 \ll V_s$. Since $R_1 \approx \beta R_3/2$ it is sufficient to satisfy the previous condition, which is $\beta/2$ times more stringent.

Temperature effects therefore dictate $V_s/R_1$ and $V_s/R_4 \gg (I_{ebo} + I_{eb})_{max}$.

**Voltage ratings for $T_1$ and $T_2$**

From the circuit action it is evident that at saturation, dissipation in $T_{1,2}$ will be $(V_{cossat})_2 V_s/R_2 + V_{eb} + V_s/R_1$, which will usually be negligible.

The base waveform goes positive by $V_s$, so that $V_{eb}$ reverse must be rated at $V_s$. $V_{eb}$ rating is also $V_s$, and the dissipation at cut off is $I_{ebo}(V_s - V_{ebo}R_2)$, which is very small.

The above shows how thorough knowledge of the operation of the transistor as a switch enables the standard multivibrator to be designed and the effects of parameter variation easily predicted. Such circuits can be designed to operate correctly with almost any transistor so long as the minimum $\beta$ and the voltage ratings are known.
For example, a standard multivibrator is to drive a load of 1 kΩ, connected between T2 collector and a 10 V negative supply. The frequency is 1 kHz and the mark/space is 1:1.

The simplest approach is to make \( R_3 = R_4 = 1 \) kΩ, where \( R_3 \) is the load. Select a transistor type in common use and find its minimum \( \beta \) at 10 mA, e.g. 20. Then, \( R_1 = R_4 = 20 \) kΩ/2 = 10 kΩ. Now, each half-period is given by \( t_1 = t_0 = 0.7 C_1 R_3 = 0.7 C_2 R_4 \) and this is to be 0.5 msec, giving \( C_1 = C_2 = 0.07 \) μF.

Charging current near the end of a half-cycle is \( V_0/R_1 = 1 \) mA, so at the maximum temperature (\( I_{eb} + I_{ce} \)) must be less than \( N/100 \) mA, where \( N \) is the maximum percentage error which can be tolerated from this cause. For some applications (e.g. 1 per cent drift at a temperature of 60°C) this will dictate the use of silicon transistors; this must be rated at 10 V reverse \( V_{eb} \), which is met only by alloy types.

Timing errors are naturally caused by resistor tolerances, only \( R_3 \) and \( R_4 \) being significant. Changes in the -10 V line would have no effect whatever on timing if \( V_{eb} \) were negligible, but would cause a proportionate increase in collector swing.

**Use of Planar Transistors**

One of the most frequently made design errors in this circuit is failure to observe the reverse \( V_{eb} \) rating. The result may be the catastrophic failure of both transistors if the coupling capacitors are large enough to store sufficient energy for transistor destruction. Values in excess of 1 μF are almost certain to achieve this. When this occurs the reason is soon appreciated and a more careful design is then calculated.

Much less obvious is the case where, although base-emitter breakdown occurs, it is non-destructive. The effect is that the capacitors discharge rapidly on reaching the reverse breakdown voltage and then discharge from that point in a normal manner. A superficial examination of circuit waveforms does not show up the peculiar mode of operation, but the timing cycle is seen to be shorter than predicted. This is naturally due to the smaller voltage through which the capacitor has to recharge (with a 10 V supply and 5 V breakdown, for example, the multivibrator period is halved).

The inexperienced designer assumes that the short timing is merely the result of yet another miscalculation of the type he keeps making and changes the capacitor in the prototype until the correct timing is obtained. Apart from being bad practice this adjustment to the calculated value can still give erratic timing due to variations in reverse breakdown voltage; moreover the transistors may fail after a few hours of operation even if the capacitors do not carry sufficient charge to give instant destruction.

Several remedies are possible and are discussed at length in the companion volume, *Circuit Consultant's Casebook* (Business Books, 1970). Where the accuracy of output swing is unimportant a simple cure is to insert diodes in each emitter lead; another is, of course, to use a safe supply voltage.

**One-shot Multivibrator**

This is one of the most useful circuits ever devised and seems to find application in every pulse system. It is also known as a 'flip-flop' (though many use this term to describe the 'bistable' or 'two-state' device), as a 'monostable multivibrator' and as a 'delay multivibrator'.

The circuit is basically a standard multivibrator in which one coupling is direct; it can take two forms, according to whether this coupling is by common emitter connection or from collector to base. Since the main intention of these examples is to familiarize the student with designing saturating circuits, the second form will be considered (Fig. 3.13).

**Circuit function**

In Fig. 3.13 the values are designed so that in the quiescent condition \( T_2 \) is saturated and, by virtue of the coupling to \( T_1 \) base, \( T_1 \) is cut off.

When an input pulse is applied which is sufficient to turn \( T_1 \) on, the resulting rise in potential of \( T_1 \) collector is coupled to \( T_2 \) base, taking it to \( +V_n \), and \( T_2 \) cuts off. \( R_2 \) is now connected by \( R_4 \) to the negative line and the values of these resistors are designed to be low enough to cause \( T_1 \) to saturate.

The input pulse can now disappear and \( T_1 \) will remain in saturation until \( T_2 \) conducts again, which occurs when \( C \) has recharged through \( R_4 \) to \( -V_{eb} \) (a similar action to the free-running multivibrator). When this takes place, \( T_2 \) finally saturates, causing \( T_1 \) to turn off. After a further time of about \( 4CR_b \), \( T_1 \) collector reaches \( -V_n \) and the original quiescent conditions are regained.

The collector waveform of \( T_2 \) is a negative pulse (see Fig. 3.14) of width determined by \( V_n \), \( C \), \( R_5 \) and \( V_{eb} \), and amplitude determined by \( V_p \), \( V_n \), \( R_1 \), \( R_2 \), and \( R_4 \). Neither of these depends on the magnitude
or width of the input trigger pulse unless this is either so short that $T_1$ has no time to saturate through $R_4$, or so long that it continues to hold $T_1$ on after $C$ has recharged. The minimum input signal to trigger is simply that which can drag $T_1$ base negative in spite of $R_3$, $R_2$, and $V_p$.

*Uses of the one-shot circuit*

The collector waveform of $T_2$ has the property that its rising edge as the circuit reverts to its quiescent state occurs a certain time later than the input trigger negative-going edge. This time depends on circuit values, so that a variable delay unit can easily be made.

If the input consists of a series of pulses with minimum interval greater than the time for $C$ to recharge, then the output consists of a set of pulses of uniform width and height. The average value of these pulses obtained by a smoothing circuit is directly proportional to the mean input frequency. The one-shot circuit is therefore an alternative to the diode or transistor pump used as a frequency discriminator.

![Fig. 3.13 One-shot multivibrator](image)

When a pulse signal is intended to indicate its moment of arrival by operating a gate which requires an input of longer duration than the signal itself, the signal can usually be made to trigger a one-shot and the output can then operate the gate. The one-shot is then acting as a pulse-stretching circuit.

*Design*

The two basic design requirements are that in the quiescent state $T_2$ is saturated and $T_1$ cut off, and in the triggered condition $T_1$ is saturated by the current in $R_2$ and $R_4$ even if the input has been removed.

For reasons which will be dealt with later, it is usually convenient to run both transistors at similar values of current when saturated (to within a factor of 2). The minimum current is dictated by the useful $\beta$ of the transistors and the demand of any external collector load. The maximum current depends on transistor and supply power ratings and the power available for triggering.

$R_3$ and $R_4$ are now decided and $R_5$ is given by $\beta R_4/2$ to ensure saturation of $T_2$. The value of $R_1$ is now determined by the $I_{DSS}$ of $T_1$ and the $V_{CE(sat)}$ of $T_2$, which both try to turn on $T_1$ in the quiescent state; provided $V_P > 2$ V (i.e. $\gg V_{CE(sat)}$), then $R_1$ current will simply be $\gg I_{DSS}$, e.g. 0.5 mA for germanium or 50 $\mu$A for silicon at 50°C.

The triggered condition may now be examined, the required condition being that the current in $(R_2 + R_4)$ with $T_2$ cut off must saturate $T_1$. If $R_1$ were not present then $(R_2 + R_4)$ would be $\leq \beta R_4/2$ by the usual criterion for good bottoming; because of $R_1$ this must be modified to pass an extra 50 $\mu$A (or whatever the current in $R_1$ is).

There remains the choice of C, which obeys the same charging laws as in the previous free-running multivibrator, giving an over-and-back time $\tau$ of $0.7 CR_3$.

As an example assume that $R_4$ is to be the load and is 2-2 k$\Omega$, supply lines are +5 and −10 V, and silicon transistors having $\beta_{DSS}$ of 25 at 5 mA are to be used. The over-and-back time $\tau$ is to be 1 msec.

Then, $R_5 \leq \beta R_4/2 \leq 27.5$ k$\Omega$, e.g. 22 k$\Omega$

$R_1 \leq (V_p/50)$ M$\Omega$, e.g. 82 k$\Omega$

$R_3 = R_4 = 2.2$ k$\Omega$

The current in $T_1$ when saturated is 10/2-2 ≈ 4.5 mA, so that the base current for bottoming should be $\geq (2/\beta)4.5 = 0.36$ mA.
\[ \frac{V_n}{274} \cdot \frac{R_2}{R_2 + R_4} = 37 \mu \text{s} \]

The rise time is therefore 3.7 per cent of the interval \( \tau \), and this will hold for any value of \( C \) until \( 3.7\tau/100 \) is comparable with transistor turn-on time.

Limitations in use

This standard one-shot circuit is simple to design but has certain limitations. If \( \tau \) is to be variable, then \( C \) may be changed, but where times greater than a few tens of microseconds are required the large value of \( C \) makes its continuous variation impractical and it must be switched. \( R_3 \) may be varied but its maximum value is restricted to about \( \beta_L R_4 \) and its minimum to about \( R_3 \), since it presents an additional load on \( T_3 \) collector (see asymmetrical operation of the free-running multivibrator). This is why approximately equal values were chosen for \( R_3 \) and \( R_4 \).

A less obvious restriction applies when the input is a train of pulses with a minimum interval comparable with \( \tau \). The circuit will operate correctly on the first pulse, but if the second occurs before \( T_3 \) collector has reached \(-V_n\), then the next value of \( \tau \) will be smaller. If this continues and the input pulse spacing is constant, each output pulse after the first will tend to an equilibrium width which is less than \( \tau \). When the output from \( T_2 \) is smoothed and the resulting d.c. used as an indication of input frequency, the law of \( V_{out} \) against \( f_{in} \) will become non-linear when \( f_{in} \) is high enough to cause the effect in question. The reason that the effect is often overlooked is that the time constant \( R_3 C \) is much less than \( R_3 C \) and the designer feels that this is a good reason to forget it. Unfortunately the collector circuit aiming and final potentials are the same \((-V_a)\) whereas the base circuit aiming potential is \(-V_e\) and its final level \(-V_{eb}\). The collector circuit therefore recovers only after about \( 4R_3 C \) compared with the base circuit time of \( 0.7 R_3 C \).

Any attempt to reduce the recovery time by reduction of \( R_3 \) requires lower \( R_3 \), thus reducing the output swing from \( T_2 \). The basic problem is to recharge \( C \) as rapidly as possible, and one simple method is the addition of an emitter-follower \( T_3 \) between \( T_1 \) collector and \( C \) (Fig. 3.15), its emitter load \( R_6 \) being less than \( R_3 \) by a factor of about \( \beta_3/5 \). This speeds up circuit recovery by a factor of 5 but adds only 20 per cent extra load to \( T_1 \) collector circuit.

An alternative method which is not quite so effective is to catch...
Triggering methods and their properties are discussed more fully in *Circuit Consultant's Casebook* (Business Books, 1970).

**The Standard Bistable**

This circuit differs from the previous multivibrator in that both collector-base feedback paths are directly coupled (Fig. 3.17).

**Circuit function**

The circuit is designed so that if $T_1$ is cut off the current in $(R_3 + R_5)$ minus the current in $R_4$ is sufficient to saturate $T_2$. When $T_2$ is saturated, the current in $R_1$ exceeds $I_{b(t)}$ plus the current in $R_3$, so that $T_1$ is maintained in the cut-off condition, resulting in a stable state.

In the opposite condition, values are similarly chosen so that another stable state exists with $T_1$ saturated and $T_2$ cut off. This usually, but not inevitably, leads to a symmetrical design where $R_1 = R_4$, $R_2 = R_5$, and $R_3 = R_6$.

**Use of the bistable**

The most common use for this circuit is in binary counters in computer systems. A method of input trigger routing (described later) is added which makes the circuit change state alternately when...
an input pulse train is applied. The output from either collector is then differentiated, giving a pulse train at half the input frequency; by using this to trigger a second bistable the process of frequency division continues. If \( n \) such stages are used, the final binary changes state after a total of \( 2^{n-1} \) input pulses have been applied, so that the system may be used as a binary counter. There are many variations on these lines and the interested reader is referred to the many volumes devoted to computer systems and circuits.

The bistable is also useful in converting a momentary signal into a permanent state for use as an alarm or interlock signal.

In most modern equipment, discrete circuit bistables are seen only rarely owing to the advantages given by standard microcircuits. However, knowledge of the design procedure is important in fully understanding the counting process and in enabling the student to appreciate the complex operation of the circuits within a J-K or D-type bistable.

**Design**

Design follows the same lines as the previous circuits in this chapter. Assuming the symmetrical form, determine the required loads \( R_3, R_6 \). Design \( R_1, R_4 \) as before, so that \( V_p/R_1,4 = V_p/R_4 \gg I_{eb0,2} \); then finally design \( R_2, R_4 \) to pass more than \( V_p/R_4 + 2V_n/\beta R_3 \) to ensure saturation.

For example, assume \( V_p = 10, V_n = 15, R_3 = R_6 = 4 \cdot 7 \kOmega, \) and that silicon transistors with \( \beta_L \) of 30 min at 4 mA are to be used. Then

\[
R_1 = R_4 < V_p/I_{eb0(max.)} < 200 \kOmega, \text{ e.g. } 18 \kOmega
\]

**Current for saturation**

\[
2V_n/\beta \cdot 7 + V_p/18 = 0.20 + 0.55 \approx 0.75 \text{ mA}
\]

**Therefore**

\[
R_3, R_6 < V_n/0.75
\]

**Temperature effects**

When \( T_1 \) is cut-off \( I_{eb0} \) tends to lower \( T_1 \) base potential and cause \( T_1 \) conduction, and the circuit fails unless the current \( V_p/R_1 \) exceeds \( I_{eb} \) at the maximum temperature.

The output voltage swing is from \( V_{ccmin} \) (almost zero) to

\[
\frac{V_nR_2}{R_2 + R_6} + \frac{I_{eb}R_6R_2}{R_3 + R_6}
\]

and so varies with \( I_{eb0} \).

In most applications the first temperature effect is the more critical. If silicon transistors are used it is often possible to obtain satisfactory high-temperature operation even if \( V_p \) is zero, by ensuring that \( I_{eb0}R_1/R_3 \) is less than about 50 mV so that the transistors cannot turn on owing to \( I_{eb0} \). A similar criterion is often used for germanium types, but then \( R_1 \) would be very small (because \( I_{eb0} \) is large and the transistor turn-on voltage small) and would rob \( T_1 \) of much of its turning-on base current in the opposite state. This usually limits the reliable operating temperature to well below 50°C and the use of \( V_p \), even if only 1 V, improves this situation considerably.

**Triggering circuits**

There are many methods for triggering the bistable, and only two will be considered here. If the circuit is to be used for alarm indication where an input signal pulse changes the state of the binary which is later reset by a different signal, the input may simply be coupled by a diode to either base, and the other signal to the other base (Fig. 3.18).

The circuit is 'set' so that the transistor connected to the signal is cut off and when the signal appears the circuit changes state, provided the input signal can supply more current than \( V_p/R_1 \) and enough voltage to bring the base below earth. This voltage is the forward diode drop plus \( (V_pR_3)/(R_1 + R_6) \) which is the amount by which the base is positive.

The re-setting signal can be coupled in the same manner or can be manually operated as shown, where \( R_9 \) must pass more current than \( V_p/R_4 \), i.e. \( R_9 < (V_n/V_p)R_4 \).
A much more difficult situation arises when the bistable is to be used as a counter. Each successive input pulse must change the state of the circuit so that a ‘steering’ or ‘routing’ circuit must be added to direct the input pulses to each base alternately.

The usual arrangement is shown in Fig. 3.19, which has superficially a simple action. Supposing T1 is on, then T1 base is just below earth so that D1 is near conduction; T2 collector–base is at \( + V_p R_2/(R_4 + R_3) \) so that D2 is cut off by a few volts. If a positive input pulse of a few volts is applied, this is immediately coupled to T1 base and T1 therefore cuts off. No pulse was applied to T2 base, so T2 is turned on by T1 turning off. When a second input pulse is applied, the situation is completely reversed and again the state of the circuit changes. The bistable therefore operates as a counter or frequency divider as intended.

The above explanation is by no means sufficient, because T2 base cannot fall any lower than D2 allows, so that if the input remains positive both transistors will cut off. When the input finally falls, there is no reason why the transistor which was previously cut-off should now turn on, since all circuit conditions are symmetrical. If any unintentional asymmetry exists, the tendency will be for one particular state to be preferred and the circuit may never change state. If, on the other hand, the signal rapidly cuts off T1 and is removed before T2 base has descended appreciably, then there is no reason why the changeover which has been initiated should continue; the more likely event is that T1 will resume conduction.

It is only the addition of capacitors C1 and C2 which enables the desired action to take place. If T1 is saturated and T2 cut-off, the upper connection of C2 is at \( V_p/[R_2/(R_4 + R_3)] \) and its lower connection at \( V_{ce(sat)} \).

The upper connection of C1 is at \( -V_{ce} \) and its lower connection at about \( -V_p/[R_3/(R_4 + R_3)] \). The charges on C1 and C2 are therefore as shown in Fig. 3.20, and it is clear that C1 carries a greater voltage than C2. (In the practical example, \( V_p = 10 \), \( V_n = 15 \); \( R_1, R_2 = 18 \, \text{k} \Omega \); \( R_3, R_5 = 12 \, \text{k} \Omega \); \( R_8, R_9 = 4.7 \, \text{k} \Omega \), giving \( V_{ce} = 10.8 \, \text{V}, \, V_{ce} = 4 \, \text{V} \).

![Fig. 3.19 Binary counter](image)

![Fig. 3.20 Practical bistable](image)

![Fig. 3.21 (a) Base voltages in Fig. 3.20, (b) equivalent circuit of (a)](image)

When the input pulse arrives, T1 is cut off, and if the input remains positive, T2 base descends only until D2 turns on; both transistors remain with bases positive and are consequently both cut off. If the input now disappears rapidly before capacitors C1 and C2 have appreciably altered their charge, the circuit now appears as shown in Fig. 3.21 (a), where the capacitors are represented by batteries and the transistors are omitted, as they are still cut off at this time.

By Thévenin’s theorem the circuit can be represented by Fig. 3.21 (b) from the point of view of the two bases. Both base circuits have the same source resistance, but the e.m.f. for T1 base circuit is not...
|E₁| = I₁R₁ - V₀ and for T₂, |E₂| = I₂R₂ - V₀. Since V₀ > V₀, I₁ < I₂ and therefore |E₂| > |E₁| (assuming a workable design where V₀ is not so large that saturation will never occur). In the example, V₀ = 11 V and V₀ = 4 V, giving I₁ = 14/22.7 ≈ 0.6 mA and |E₁| ≈ 1 V; I₂ = 21/22.7 ≈ 0.9 mA and |E₂| ≈ 6 V. The source resistance to both bases is R₁/R₆ = R₂/R₆ = 4.7/18 = 3.7 kΩ.

As the input signal falls, the initial base current for T₁ will be 1/3.7 mA and for T₂ 6/3.7 mA. Provided β(T₁) is not > β(T₂) by a factor of 6, it will therefore be T₂ which conducts first, and as it does so its collector rises and reduces T₁ base current.

It is now clear that only C₁ and C₂ cause correct changeover action, and although they have also the effect of improving collector-base coupling during transition, their main purpose is the storing of a charge in accordance with the state of the circuit. For this reason they are often called 'memory' capacitors, as they remember the previous state.

The correct values are obtained from a study of the above action. The charge on C₁ and C₂ must remain from the moment of input triggering to the end of the trigger pulse. On the other hand, the charge must change when the binary settles in a new state before the next input trigger pulse begins. The time constant associated with C₁ is C₁R₂/R₆ when T₁ has just turned on, but is C₁R₂/(R₁ + R₆) when T₁ is off. Similarly, C₂ time constant is C₂R₆/(R₂ + R₆).

In the circuit given, the input trigger pulse length is determined by the differentiating circuit CR, where R includes circuit-loading with T₁ and T₂ cut off.

The criteria for correct memory are therefore CR < C₁R₂/(R₁ + R₆) < T, where T is the input signal period.

Note that owing to C₁ and C₂ the collector waveforms have slow falling edges, the fall time constant being equal to the memory time constant C₁R₂/(R₁ + R₆).

Other Switching Circuits
There are many other circuits in which the transistor is used as a switch, and also several versions of the above standard circuits. The reader should experience no difficulty in their design provided that he makes certain of the precise mode of operation before calculating circuit values.

The use of a transistor as a low-level chopper switch requires a different design approach and is dealt with in Chapter 9.

4—Transistor T-equivalent circuit

For most design purposes the well-known T-equivalent circuit shown in Fig. 4.1 gives an adequate representation of the low-frequency behaviour of a transistor.

The use of this circuit enables any normal circuit to be analysed, by simple if tedious application of Kirchhoff's laws, provided the signal currents involved are only a small fraction of the transistor's operating currents. For large current changes the circuit is still correct but each parameter in it has to be considered variable and dependent upon the current flowing in it; using the circuit is then hardly practicable.

Some of the results obtainable by the use of this circuit are given in Appendix 3, and it is shown that by making certain assumptions about the relative values of the equivalent circuit components these results can be greatly simplified. These simplified expressions are those used by practical designers and the complete form is resorted to only in unusual conditions, a typical example being the circuit described in Chapter 12.

In the present chapter it is assumed that circuit values allow the
simplified forms to be used, although pitfalls will be pointed out where the necessary conditions may be violated inadvertently.

**TYPICAL VALUES IN THE T-EQUIVALENT CIRCUIT**

The simplest way to become familiar with practical values is to examine the characteristic curves for a range of general-purpose transistors of various power ratings and junction type.

Collector resistance $r_c$ is the slope of the graph relating $V_{cb}$ with $I_c$ for constant $I_e$. Its value is usually in the megohm region for transistors of less than 1 W rating (e.g., 2N930) and is as low as 10 kΩ in a power transistor rated at tens of watts (e.g., 2N3055).

The current generator coefficient $a$ is approximately equal to $\alpha$, which is given by the ratio $I_e/I_0$ at constant $V_{ce}$. In practice this is difficult to read from the curves with sufficient accuracy, and since $a$ generally appears in the equations in the form $(1 - a)$, i.e. approximately $1/\beta$, it is more useful to read $\beta$ directly from the slope of the graph $I_e/I_0$ ($I_e/I_0$ is nearly the same) at constant $V_{ce}$. $\beta$ usually lies between 15 and 300, falling at very low and at very high currents.

Note that the slope of the $V_{ce}/I_e$ graph for constant $I_b$ is $r_e/\beta$.

The values of $r_e$ and $r_b$ cannot readily be obtained directly from the curves, but examination of the equivalent circuit analyses in their simplified form shows that $r_e$ and $r_b$ invariably occur as either $(r_e + r_b(1 - a))$ or $(r_b + r_e(1 - a))$. Since $(1 - a) \approx 1/\beta$, these expressions are $(r_e + r_b/\beta)$ and $\beta(r_e + r_b/\beta)$.

These can be obtained directly, because $(r_e + r_b/\beta)$ is the slope of the curve $I_e/V_{eb}$ for constant $V_{eb}$ and $\beta(r_e + r_b/\beta) = (r_b + \beta r_e)$ is the slope of the curve $I_e/V_{eb}$ for constant $V_{ce}$. $(r_e + r_b/\beta)$ is typically 50 Ω at $I = 1$ mA for any transistor and is roughly inversely proportional to $I_e$.

The quantity $(r_e + r_b/\beta)$ has the same significance as $1/g_m$ has in valve circuitry: it enables the emitter or collector current to be calculated in terms of the input voltage which caused it. Thus, if the input voltage appearing between base and emitter is $v_{eb}$, the resulting emitter current is $v_{eb}/(r_e + r_b/\beta)$ and the collector current is $\alpha v_{eb}/r_e + r_b/\beta$, which is roughly the same, since $\alpha \approx 1$. In view of this relationship, this quantity $(r_e + r_b/\beta)$ is often called $1/g_m$, where $g_m$ is mutual conductance, and this terminology will be used here.

Should the values of $r_e$ and $r_b$ require to be known separately, $r_e$ can be calculated as $(12.5/I_e)$ Ω, where $I_e$ is in milliamps, and $r_b$ can be deduced knowing $1/g_m$, $r_e$, and $\beta$.

In the next section the derivation of the circuit equations for typical applications is given, and the assumptions leading to more useful simplified equations are stated.

**CIRCUIT ANALYSIS USING T-EQUIVALENT CIRCUIT**

The first configuration yields the voltage gain, input, and output impedance of the emitter follower and the grounded emitter amplifier. Since the analysis applies for small signals only (because large-signal currents change the values of $r_e$, $r_b$, etc. during one cycle), the external components $Z_a$, $Z_b$, and $Z_L$ represent any external circuits which behave like impedances of these values to small signals at the input frequency.

![Fig. 4.2 Basic emitter follower/grounded emitter base](image)

In small-signal analysis, supply lines are assumed to have negligible impedance to 'earth' (supply common). This is not always true, especially at high frequencies, in which case the actual supply impedance $Z_p$ must be inserted.

The circuit of Fig. 4.2 is called an emitter follower when the output is taken from the emitter; the presence of $Z_L$ is shown by the analysis to make little difference to performance as an emitter follower. When the output is taken from the collector, the circuit is a grounded emitter amplifier; the values of $Z_L$ and $Z_B$ are of great significance in its performance.

By calculating $v_{eb}/v_e$, $v_{eb}/v_{in}$, $Z_{in} = v_{eb}/i_{in}$ and $Z_{out} = v_{out}/i_{in}$, the performance of both circuits is established, since $Z_B$ or $Z_L$ can be zero if required.

The procedure is to replace the transistor by its T-equivalent circuit and analyse the resulting network, using Ohm's law and Kirchhoff's laws. In carrying out this analysis it will be found helpful to regard
all signal voltages as measured from earth (common); an additional point which ensures the correct relative polarities is that when \( i_e \) is shown to be flowing into the emitter (as shown in Fig. 4.3), the current generator \( a_i \) must be in the direction indicated, i.e., tending to cause collector current to flow outwards into the load \( Z_L \). By using these methods the phasing of the various signals is consistent and if, for instance, \( v_L = -k v_b \) this means that an inversion takes place from \( v_b \) to \( v_L \), if \( k \) is a real constant.

![Fig. 4.3 Equivalent circuit of basic emitter follower/grounded emitter base (Fig. 4.2)](image)

Note that the parameters \( r_e, r_b \), etc. are correct for small signals only and cannot be used to calculate bias conditions; these bias conditions in fact determine the values of the network parameters.

**Network Currents**

Starting with the given \( i_e \) and \( i_B \), the remaining currents are readily obtained by Kirchhoff’s first law (the sum of currents at a junction is zero). Thus, the current leaving the junction of \( r_e \) and \( r_B \) is \( (i_e + i_b) \) and this same current must emerge from the collector. The only tricky point is the polarity of the current in \( r_e \) which the beginner often reverses; an easy way to think about this is to imagine the collector load to be open-circuit when it is clear that the current generator \( a_i \) circulates its current through \( r_e \), so that in Fig. 4.3 \( a_i \) flows downwards in \( r_e \).

The detailed calculations are worked out in Appendix 3 and more practical simplified calculations are also given which are correct for most designs.

**Significance of the Results of Analysis**

For the simplified results, which rely on assumptions usually valid in practical circuits, it soon becomes clear by inspection that a simple picture of transistor circuit performance will yield correct answers. To recall the important simplified equations (Appendix 3):

**Emitter follower**

- **Input impedance**
  \[ Z_{in} = \beta [Z_e + 1/g_m] \]  
  ...(A.25)

- **Gain**
  \[ \frac{v_o}{v_b} = \frac{Z_e}{Z_e + 1/g_m + Z_e/\beta} \]  
  ...(A.28)

- **Output impedance**
  \[ Z_{out} = Z_e/\beta + 1/g_m \]  
  ...(A.33)

**Earthen emitter amplifier**

- **Input impedance**
  \[ Z_{in} = \beta [Z_e + 1/g_m] \]  
  ...(A.25)

- **Gain**
  \[ \frac{v_o}{v_b} = \frac{Z_L}{Z_e + 1/g_m + Z_e/\beta} \]  
  ...(A.35)

- **Output impedance**
  \[ Z_{out} \approx r_e(Z_e \text{ large}) \]  
  \[ \approx r_e/\beta(Z_e \text{ zero}) \]  
  ...(A.37)

Note that the input impedance to both circuits is equal to the sum of two components in series: \( 1/g_m \), due to transistor internal resistances, and \( \beta Z_e \), which is the emitter load impedance increased by a factor \( \beta \). This is expressed in Fig. 4.4.

![Fig. 4.4 Transistor input impedance as seen by source (enables \( v_b \) to be calculated)](image)

Examination of equation (A.28) shows that the gain would be unity except for the terms \( 1/g_m \) and \( Z_e/\beta \) in the denominator. The emitter circuit therefore behaves like a generator of source e.m.f. \( v_b \).
and internal impedance $\frac{1}{g_m} + Z_e/\beta$, as indicated in Fig. 4.5, giving

$$v_e = v_b \left( \frac{Z_e}{Z_e + (1/g_m) + (Z_e/\beta)} \right)$$

An alternative way of looking at this circuit is to calculate from Fig. 4.4 the voltage $v_b$ actually reaching the base, namely

$$v_b = v_b \left( \beta/g_m + \beta Z_e \right)$$

As far as the transistor is concerned, this is a zero resistance source connected directly to the base: the transistor functions according to

$$v_b = v_b \left( \frac{\beta/g_m + \beta Z_e}{Z_e + \beta/g_m + \beta Z_e} \right)$$

Fig. 4.5 Transistor emitter output as seen by $Z_e$ (enables $v_e$ to be calculated and, hence, $i_e$)

Fig. 4.6 Alternative forms of Fig. 4.5 give identical results

the voltages which are applied to it and it cannot know what lies beyond. Hence, the emitter circuit can be regarded as shown in Fig. 4.6. The emitter voltage using this circuit would be

$$v_e = v_b \left( \frac{\beta/g_m + \beta Z_e}{Z_e + \beta/g_m + \beta Z_e} \times \frac{Z_e}{Z_e + 1/g_m} \right) = v_b \left( \frac{Z_e}{Z_e + 1/g_m + Z_e/\beta} \right)$$

as before.

Having used the emitter equivalent circuit of Fig. 4.5 or 4.6, the emitter voltage can therefore be calculated, and so the emitter current is known by dividing by $Z_e$. Alternatively, the same answer is obtainable by dividing $v_e$ by $[Z_e + 1/g_m + Z_e/\beta]$ or by dividing $v_b$ by $[Z_e + 1/g_m]$; one of these two must be used if $Z_e = 0$ to avoid an indeterminate result.

By using one of the equivalent circuit forms given above, the input impedance, emitter voltage, and emitter current can therefore be calculated. By assuming that the collector signal current is approximately equal to emitter signal current (which is true for the conditions which lead to the simplified equations), the signal collector voltage is known by multiplying this current by $Z_L$.

This is illustrated by the dismembered form of equivalent circuit shown in Fig. 4.7, 4.7 (a) showing the input circuit and 4.7 (b) the output circuit. Using this representation, all linear circuits may be analysed very simply, although the full T-equivalent circuit has to be resorted to for exceptional conditions which make the approximations invalid.

Fig. 4.7 (a) Input, and (b) output equivalent circuits

Variations in Circuit Performance

It will be seen that many of the analytical results are very dependent on transistor parameters, particularly $\beta$ and $g_m$. These parameters are by no means constant between various specimens of one type of transistor or even for one particular transistor when the ambient temperature or operating current changes. The practice of selecting transistors from a batch to a tight specification giving, for example, a spread in $\beta$ of 1.5 to 1 instead of the normal figure of perhaps 3 or 4 to 1, is unsatisfactory in at least two respects. It leads to higher transistor cost because of the work involved in selection and the possibly poor yield of acceptable transistors, and it leads to a great variety of transistors with only slightly different specifications.

The designer must therefore ensure correct circuit performance in spite of these variations and this aspect of design is at least as important as, and is indeed complementary to, his concept of the configuration to be used. One part of this process has been covered in Chapter 2, where the importance of the bias arrangements was stressed, in order to stabilize the operating currents and voltages.
This involves arrangements which reduce base voltage variations and also ensure by the use of large emitter resistors returned to a high voltage that any base variations have the least possible effect on emitter current (and therefore collector voltage). This second precaution can be regarded as keeping low the transistor stage gain from base to collector for very slow-moving signals, which is indeed achieved (see Fig. 4.7) by high $R_e$.

This provides a clue to a method for stabilizing the gain at higher frequencies. Instead of making $Z_e$ zero at the signal frequency (by capacitor to earth) which would give the highest obtainable gain, a series resistor can be included as in Fig. 4.8 (a) and (b). The gain equation $\frac{v_c}{v_b}$ is $RL[1/g_m + R_e']$ (if $X_e$ is negligible at the signal frequency), and if $R_e' \gg (1/g_m)$, variations in $1/g_m$ have no effect on gain. Knowing the possible variations in $1/g_m$, $R_e'$ can be designed to reduce their effect by any desired extent. Unfortunately, the gain itself is reduced by the same amount, so by varying $R_e'$ the gain could vary from, e.g., $100 \pm 30$ per cent to $30 \pm 10$ per cent or $10 \pm 3$ per cent, etc.

This method of overcoming the effects of variables which are outside the designer's control by adding fixed components to swamping the variables is used in many designs.

In the present example it can be regarded as a form of negative feedback, a subject discussed in Chapter 9. The ideal is to make the circuit performance depend only on static components, the accuracy of which are well specified. In an actual design transistor parameters will inevitably affect performance but, in a correct design, only to an extent which can be allowed by the circuit performance specification. 

![Fig. 4.8 Addition of $R_e'$ to stabilize gain](image)

The following chapters give procedures for the design of a variety of circuits using the above philosophy.

**Emitter Coupled Pair**

This circuit (Fig. 4.9) is a very useful amplifier having two input and two output terminals. Its bias arrangements present more problems than the simple earthed emitter amplifier and will be dealt with later in the chapter; only its small a.c. signal behaviour will be discussed here.

![Fig. 4.9 Emitter-coupled pair](image)

Since the signal currents are assumed to be small compared with transistor bias currents, operation is almost linear; therefore the outputs produced by the two inputs $v_{b1}$ and $v_{b2}$ can be assessed separately and the results added, by the principle of superposition.

If $v_{b1}$ is present and $v_{b2}$ short-circuited, the common emitter voltage $v_E$ can be calculated from Fig. 4.10, showing $v_{b1}$ connected to $E$ through an apparent source resistance $(R_{s1}/\beta_1 + 1/g_{m1})$, $E$ being loaded by $R_E$ in parallel with the emitter impedance of $T_2$, namely $[R_{s2}/\beta_2 + 1/g_{m2}]$. For simplicity assume $T_1$ and $T_2$ are identical.
and that \( R_{S1} = R_{S2} \), and further, that \( R_E \) is very large in comparison with \( (R_S/\beta + 1/g_m) \). In this case the source resistance between \( v_{S1} \) and \( v_E \) and the loading on \( v_E \) are equal, giving \( v_E = \frac{1}{2} v_{S1} \). Hence,

\[ i_{S1} = (v_{S1} - v_E)/(R_S/\beta + 1/g_m) = \frac{1}{2} v_{S1}/(R_S/\beta + 1/g_m), \]

and

\[ v_{C1}/v_{S1} = -\frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

(The reason for the negative sign is obvious from inspection of the direction of \( i_{S1} \) in Fig. 4.10.)

In this special case, where \( R_E \) is very large, \( T_2 \) and \( T_3 \), \( R_{S1} \) and \( R_{S2} \) are identical, and only \( v_{S1} \) is connected, the voltage gain from \( v_{S1} \) to \( T_1 \) collector is \( 1/2 \) that obtained from a single transistor stage having an earthed emitter.

Continuing the analysis, \( T_2 \) emitter current is also

\[ \frac{1}{2} v_{S1}/(R_S/\beta + 1/g_m) \]

so that

\[ v_{C2}/v_{S1} = \frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

the voltage gain from \( v_{S1} \) to \( T_3 \) collector is therefore equal but opposite in sign to the gain to \( T_1 \) collector.

The effect of a finite value of \( R_E \) is to change \( v_E \) from \( v_{S1}/2 \) to

\[ v_{S1}/2[R_S/\beta + 1/g_m]/R_E + \frac{1}{2} v_{S1}/[R_S/\beta + 1/g_m] \]

which is slightly smaller. The value of \( T_1 \) emitter current, i.e. \( (v_{S1} - v_E)/(R_S/\beta + 1/g_m) \), is therefore increased and \( T_2 \) emitter current i.e. \( v_E/(R_S/\beta + 1/g_m) \), is decreased. The gain to \( T_1 \) collector is therefore higher than to \( T_2 \) collector.

\[ v_{C1}/v_{S1} \text{ being } -\frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

and

\[ v_{C2}/v_{S1} \text{ being } \frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

Returning to the simple case (\( R_E \rightarrow \infty \)), the effect of source \( v_{S2} \) will be equal and opposite to that of \( v_{S1} \), so that

\[ v_{C1}/v_{S2} = \frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

and

\[ v_{C2}/v_{S2} = -\frac{1}{2} R_L/(R_S/\beta + 1/g_m) \]

With both inputs present the collector voltages are, by superposition,

\[ v_{C1} = -v_{S1}/4 R_L/[R_S/\beta + 1/g_m] + v_{S2}/4 R_L/[R_S/\beta + 1/g_m] \]

\[ = -\frac{(v_S1 - v_S2)/4 R_L}{R_S/\beta + 1/g_m} \]

and

\[ v_{C2} = -(v_{S2} - v_{S1})/4 R_L \]

The significance of these results is that if \( v_{S1} = v_{S2} \), then \( v_{C1} = v_{C2} = 0 \); and if \( v_{S1} = -v_{S2} \), then

\[ v_{C1} = -v_{S1}/4 R_L/R_S/\beta + 1/g_m + v_{C2} = + v_{S1}/4 R_L/R_S/\beta + 1/g_m \]

This amplifier therefore produces equal and opposite outputs dependent only on the difference between the two input signals and not on the sum. Thus, if

\[ \frac{R_L}{R_S/\beta + 1/g_m} = 30 \]

and two sine-wave signals were applied, identical in frequency and phase, \( v_{S1} \) being 100 mV peak, \( v_{S2} \) being 95 mV peak, then \( v_{C1} \) would be \( (100 - 95)30 = 150 \) mV peak and in antiphase with \( v_{S1} \), and \( v_{C2} \) would be 150 mV peak and in phase with \( v_{S1} \).

This result is easy to understand by considering the current flow caused by \( v_{S1} \) and \( v_{S2} \). If these are identical signals and \( R_E \) is infinite (see Fig. 4.10), \( v_E \) will also move identically with them. (If it did not, then the currents flowing out of the \( v_E \) junction into \( T_1 \) \( T_2 \) would change so that the current in \( R_E \) must change, which is impossible; therefore \( v_E \) must have moved in such a way as to cause no current change in \( T_1 \) or \( T_2 \), i.e. \( v_E \) must have followed \( v_{S1}, v_{S2} \) identically.) No signal current flows in \( T_1 \) or \( T_2 \), and no collector voltage change occurs.

When \( v_{S1} \) and \( v_{S2} \) differ, then a current flows along the \( (R_S/\beta + 1/g_m1) \) and \( (R_S/\beta + 2/g_m2) \) paths, causing a current change which is one way for \( T_1 \) and the other for \( T_2 \), giving equal and opposite polarity collector voltages.

A finite \( R_E \) clearly changes the first result, since if \( v_{S1} = v_{S2} \), the current in \( T_1 \) and \( T_2 \) does not now remain constant but the total current changes by \( v_{S1}/R_E \). If the \( T_1 \) and \( T_2 \) are identical and are
sharing the current equally the result is a change of current in both 
$T_1$ and $T_2$ of $\frac{1}{2}v_{S1}/R_E$. Each produces an output voltage of $-\frac{1}{2}v_{S1}R_L/R_E$
and each output is in antiphase with $v_{S1}$ and $v_{S2}$.

The presence of a finite $R_E$ therefore results in an appreciable output even when $v_{S1} = v_{S2}$.

When an amplifier is to be used for measuring the small difference
between two large signals, the criterion for good performance is
the ratio between 'push-pull' gain $v_{out}/(v_{S1} - v_{S2})$ and 'push-push' gain $v_{out}/(v_{S1} + v_{S2})$, often called 'rejection ratio'. In the emitter-coupled pair, this is proportional to $R_E$, as shown above, and is typically between 20 and 200.

The use of a constant-current device (Chapter 6) instead of $R_e$
 improves this figure to between 2000 and 20 000.

**Biasing the emitter-coupled pair**

In designing a practical emitter-coupled pair amplifier, the bias
conditions present a special problem. If the basic circuit is used as it stands, both bases being returned through resistors to a fixed potential (e.g. 'earth') and the common emitter connection taken to a suitable supply, then although the current in $R_B$ is well defined, the ratio in which it divides between $T_1$ and $T_2$ is unknown. With normal transistor spreads for $V_{be}$ against $I_e$, one transistor may be cut off while all the current from $R_B$ passes into the other. This is no exaggeration, as a glance at production spreads of $I_e$ for constant $V_{be}$ soon confirms.

The corresponding valve circuit, the cathode-coupled pair, behaves in the same way if two pentodes are used. This is the main reason why triodes are preferred, since by using equal anode loads any tendency to cut off causes the appropriate anode voltage to rise, thus increasing the current. With pentodes, as with transistors, a change in anode or collector voltage has little effect on the current and the unbalance remains.

If the amplifier is to operate down to zero frequency, nothing can be done about balance except to add a zero-setting potentiometer (see Chapter 9): any feedback loop designed to balance the circuit automatically also reduces the zero-frequency gain.

For the amplification of alternating signals only, which is a very common application, the zero-frequency gain should be reduced as shown in Figs. 4.11 and 4.12. In Fig. 4.11 each transistor is separately biased in a normal manner and $C_e$ must be a low reactance

compared with the emitter circuit impedance at the lowest angular
operating frequency $\omega_L$, i.e.

$$\frac{1}{\omega_L C_e} \ll \frac{R_{e1}}{|(1/g_{m1} + R_{S1})/R_{e1} + (1/g_{m2} + R_{S2})/R_{e2}|}$$

In calculating rejection ratio, $R_e$ must be taken as $R_{e1}/R_{e2}$. At frequencies much below $\omega_L$ the rejection ratio will be degraded, since the push-pull gain falls greatly, and the push-push gain falls only by a factor of 2 if $R_{e1} = R_{e2}$.

![Fig. 4.11 Bias circuit for emitter-coupled pair](image1)

![Fig. 4.12 Alternative bias circuit for emitter-coupled pair](image2)

In Fig. 4.12 $R_{e1}$ and $R_{e2}$ are arranged to drop a voltage greatly in excess of $V_{be1}$ and $V_{be2}$. For a low-gain application $C_e$ may be omitted, but since $R_{e1}$ and $R_{e2}$ reduce the push-pull gain much more than the push-push gain, rejection ratio deteriorates. If $C_e$ is added, then a similar criterion applies, as stated for Fig. 4.11. The equation is slightly altered and becomes

$$\frac{1}{\omega_L C_e} \ll (R_{e1} + R_{e2})/\left|(1/g_{m1} + R_{S1}/R_{e1} + 1/g_{m2} + R_{S2}/R_{e2})\right|$$
The emitter-coupled pair may be used for differential amplification, its rejection ratio being as high as 20,000 (86 dB) per stage if the emitter feed is a constant-current source. Special precautions must be taken in fixing the operating point, otherwise severe unbalance occurs.

5—Linear sweep circuit

A linear voltage sweep can be produced across a capacitor which is changing its charge. The charging equation for a capacitor is $i = C dV/dt$, and for the special case where $dV/dt$ is constant, i.e. the voltage sweep waveform is linear, this reduces to $i = CV/t$.

For a constant rate of voltage per second, i.e. a linear sweep, it is therefore necessary to charge the capacitor with a constant current. There are several circuit configurations such as the Miller integrator, constant-current source, bootstrap sweep generator, which produce a linear voltage sweep, and although they are at first sight quite different, they all work on this principle of constant-current charging of a capacitor.

In this chapter the bootstrap version is described in its simplest form and a typical design procedure is given with a numerical example. It becomes clear that the design of such circuits is very simple in principle and that the main task of the designer is to minimize unwanted side-effects which tend to change the current which is to be held constant.

THE BOOTSTRAP SWEEP CIRCUIT

Fig. 5.1 illustrates how use of the bootstrap principle can produce a constant charging current for a capacitor $C$.

Assume initially $S_1$ rests in the position shown, that $C'$ is very large and that the box marked 'X1' is a unity-gain amplifier having high input impedance and low output impedance.

Initially, $B$ is therefore at zero potential, $A$ is at $VR_a/(R_1 + R_3)$, and the current in $R_1$ and $R_2$ is $V/(R_1 + R_2)$.

On opening $S_1$, the current $V/(R_1 + R_2)$ which had been flowing through $S_1$ now charges $C$ at an initial rate given by $dV/dt = i/C$. 

G
that is \( \frac{V}{C(R_1 + R_2)} \) V/sec. None of this current passes to the amplifier, since its input impedance is assumed to be very high, and if the amplifier output were not connected through \( C' \) to \( A \), capacitor \( C \) would charge exponentially towards +V, because as \( V_e \) rises the charging current decreases.

With the connection to \( A \) made as shown, however, point \( A \) rises exactly in step with \( B \), provided \( C' \) is very large. This means that the potential difference \( V_{AB} \) is the same as in the original state with \( S_1 \) closed, namely \( VR_a/(R_1 + R_2) \), so that the current in \( R_2 \) remains \( V/(R_1 + R_2) \) throughout the sweep and the charging rate for \( C \) is constant, being \( V/C(R_1 + R_2) \) V/sec.

The output voltage waveform \( V_e \) is therefore a linear ramp until either \( S_1 \) is closed, giving a fast return to zero output, or the amplifier ceases to operate, which will occur in practice when \( V_e \) exceeds some fraction of the supply rail voltage of the amplifier.

Before considering practical design, it is important to note one or two points which can become confusing later if not completely understood at this stage.

First, it is quite possible for point \( A \) to rise to a voltage higher than +V, even though no higher supply rail is used for the amplifier. The source of the energy which produces this effect is the capacitor \( C' \) which has been pre-charged to a voltage \( VR_a/(R_1 + R_2) \).

Secondly, it is clear, especially after appreciating the first point, that \( R_1 \) is redundant as soon as the sweep process begins. Its only function is to help determine the initial charging current which the bootstrap loop then maintains. When the sweep begins, \( R_1 \) constitutes a load on the amplifier through \( C' \), making the design of the amplifier more difficult and increasing the required value of \( C' \). The circuit is therefore improved if \( R_1 \) is replaced by a diode (Fig. 5.2) which cuts off when \( A \) rises.

Thirdly, the linearity can be spoilt in three main ways. (1) The amplifier may modify the charging current received by \( C \) if its input impedance is too low. (2) \( C' \) may be inadequate, the effect being that the proportion of voltage lost across \( C' \) (by discharge through \( R_1//R_2 \)) during the sweep, compared with the initial voltage on \( R_2 \) \( VR_a/(R_1 + R_2) \) will produce that same proportion of non-linearity.

(3) The amplifier gain may not be held closely to unity throughout the sweep, the amount of departure from unity producing the same proportion of non-linearity.

Finally, the output can with advantage be taken from point \( D \) since the waveform is similar to that across \( C \), and the load will not then affect significantly the charging current for \( C \).

Bearing the above points in mind will make the design of the practical circuit of Fig. 5.3 easy to understand.

Assume for example that a sweep output is required, after opening \( S_1 \), having a rate of 5 V/msec, into a 10 kΩ load from + and - 10 V rails. Typical values will be calculated and the expected linearity assessed using a single emitter-follower \( T_1 \) as shown.

Since the quiescent output level is just below zero volts and the maximum possible output will be +10 V (\( T_1 \) will then be bottomed)
the maximum load current is 1 mA, reached at the end of the sweep.

**Effect and Choice of \( R_B \)**

Resistor \( R_B \) can be omitted completely; this means that while \( C \) charges for the first few hundred millivolts \( T_1 \) remains cut off and \( V_{out} \) remains at zero. When \( V_C \) has reached the turn-on voltage for \( T_1 \), conduction begins and \( V_{out} \) follows the sweeping waveform on \( C \). There are two snags in this action: there is a delay between operation of \( S \) and commencement of \( V_{out} \); and the transitional period as \( T_1 \) is beginning to conduct will imply a high output impedance from \( T_1 \), so that the bootstrap loop will have noticeably less than unity gain, giving non-linearity near the start. In many applications these may be unimportant defects and \( R_B \) and the \(-10 \text{ V} \) line can be omitted.

Assuming these effects are to be avoided, \( R_B \) should provide a standing current of about the same order of magnitude as the maximum load current, so that when the output rises, the change of current in \( T_1 \) (due to the load current) is not large. It is unfortunate that the currents in \( R_B \) and \( R_L \) both increase as the output rises causing parameter variations in \( T_1 \) which change its gain. If the negative supply were many times the value given, or if a constant-current device were used, this particular effect could be overcome by making the standing current for \( T_1 \) many times the maximum load current. This would then, however, cause further difficulties owing to the resulting rise in base current leading to a vicious circle to be solved only by the use of more transistors.

\[ R_B \] will therefore be given the (non-critical) value of 10 kΩ, giving an initial current into \( R_B/R_L \) of 1 mA and a final current of 3 mA when \( V_{out} \) is at \(+10 \text{ V} \).

**Choice of \( T_1 \)**

A tentative specification for the transistor \( T_1 \) must now be assumed before the design can begin. The designer may have a particular type in mind, or may be prepared to search for one which is ideal for the purpose, depending mainly on how critical is the specification for the performance of the circuit.

For the purposes of this example, a non-linearity of 1 per cent has been taken as the tolerable limit, and the experienced designer will immediately realize that a high-gain transistor will be required. For the moment, a minimum \( \beta \) of 50 will be specified over the range of operating conditions which apply throughout the sweep.

\( T_1 \) may be silicon or germanium, the important differences in this circuit being the higher \( V_{cb} \) for silicon, so that the output swings from about \(-0.7 \text{ V} \) to \(+9.3 \text{ V} \), rather than \(-0.2 \text{ to } +9.8 \text{ V} \), and the leakage of germanium which robs \( C \) of some of the current in \( R \). Since \( I_{dso} \) varies greatly with temperature, the use of germanium for \( T_1 \) will cause drift in the rate of sweep with temperature, the magnitude of which depends on the value of \( I_{dso} \) compared with the charging current through \( R \), which is not yet decided.

The voltage rating for \( T_1 \) is 10 V for \( V_{dd} \); the power rating cannot be decided until \( R \) is chosen.

**Choice of \( R \)**

This component is the only one in the circuit which causes difficulty, and this is because almost any value would seem to be satisfactory. How is the designer to know whether a charging current of 1 mA or 50 mA would be preferable?

In such cases the possible values should be taken to opposite extremes in order to arrive at more reasonable limits. Here, a charging current of 1 μA (giving \( R = 10 \text{ MΩ} \)) can be taken as one extreme and 1 A (\( R = 10 \text{ Ω} \)) as the other. The consequences of these clearly wrong values can be evaluated and from the result it will be obvious which values are really possible.

If \( R = 10 \text{ MΩ} \), the charging current is not in fact 1 μA, but is 1 μA minus the base current of \( T_1 \). From the (reasonable) assumption that \( \beta \) is always at least 50, this base current will be at most 1/50 mA.
at the beginning of the sweep and 3/50 mA at the end, assuming a sweep to +10 does take place. It is clear that no sweep occurs, because the charging current is actually negative and C will charge negatively with a current starting at 1 μA minus 1/50 mA. R cannot therefore have this high value and in fact must be low enough to ensure that the current it passes greatly exceeds 1/50 mA.

On the other hand, if \( R = 10 \, \Omega \), charging current is about 1 A, as intended, but it must not be forgotten that the transistor emitter current, which varies from 1 to 3 mA due to \( R_E \) and \( R_L \), includes the current in \( R \) as soon as \( D_1 \) cuts off, i.e., after a few hundred millivolts of sweep. Base current is therefore 1/50 (1 mA + 1 A) at the beginning and 1/50 (3 mA + 1 A) at the end. Excepting for the first few hundred millivolts, when it is 1/50 (1 mA). This small value of \( R \) therefore produces a large enough current to swamp the base currents caused by \( R_E \) and \( R_L \), but there is a sudden step in current when \( D_1 \) cuts off when the charging current changes from 1 A to

\[
\left(1 - \frac{1}{50}\right) \text{A. To avoid this effect } R_E \text{ could be reduced to give a comparable current to that in } R \text{ but then the original difficulty returns because } R_E \text{ current varies during the sweep. Another snag in using a low value for } R \text{ is, of course, the power rating required of the transistor (about 5 W) and the drain on the power supply.}
\]

The compromise is easy to make: R must be small enough to avoid base current variations of 1/50–3/50 mA during the sweep causing more than the allowed non-linearity (note that base current caused by \( R \) itself, namely \( V/50R \), is constant through the sweep and causes no non-linearity); \( R \) must be large enough to avoid unnecessarily high dissipation in the transistor.

In the example under consideration, a value of 10 kΩ for \( R \) gives a nominal charging current of 1 mA. Base current is at most 1/50 mA initially, changes to 2/50 mA when \( D_1 \) cuts off, and ends at 4/50 mA at +10 V output. Charging current therefore varies from \( \left(1 - \frac{1}{50}\right) \) mA to \( \left(1 - \frac{1}{50}\right) \) mA, a variation of 6 per cent. If the first phase is ignored, variation is 4 per cent.

A value for \( R \) of 2.2 kΩ gives nominal charging current of 10/2.2 = 4.5 mA. Base current begins at 1/50 mA, changes to \( (1/50 + 4.5/50) = 5.5/50 \) mA when \( D_1 \) cuts off, and ends at \( (3/50 + 4.5/50) = 7.5/50 \) mA at full output. Total variation in charging current is therefore 6.5/50 parts in 4.5, i.e. 3 per cent, or ignoring the first phase, 2/50 in 4.5, or 0.9 per cent.

This circuit in its simple form always suffers from this initial non-linearity until \( D_1 \) cuts off, as is shown by the above calculations. Since this phase occurs only for a few hundred millivolts, however, it scarcely affects the overall linearity figure and the second figure for each case above is the relevant one. The value of 2.2 kΩ appears to be satisfactory, and although it would be tempting to make \( R \) smaller still, to meet the linearity requirement easily, discretion must be used: the letter of the specification may well be met but the 'kink' appearance of the initial rise may be important to the user (though not specified) and dissipation will be raised without real justification.

**Choice of \( D_1 \) and C**

The charging rate for C depends slightly on the forward drop of \( D_1 \), and so the type of diode should be specified before calculating C. The dependence of the charging rate on \( D_1 \) is so small that for practical purposes it is only necessary to know whether \( D_1 \) is to be silicon or germanium, giving a typical voltage drop of 0.5 V or 0.1 V, respectively. The reverse leakage of \( D_1 \) has little effect on performance and merely adds slightly to the current which tends to discharge C' during the sweep (which mainly consists of the current in \( R \)).

Assuming a germanium type is to be used (almost any signal diode is satisfactory, such as OA10, HG1005), a nominal allowance of 0.1 V drop can be made in calculating the charging current for C, giving \( i = 9 - 2 = 4.5 \) mA. To obtain the required sweep rate of 5 V/m sec, C must therefore be

\[
\frac{4.5 \times 10^{-3} \times 10^{-3}}{5} F
\]

or 0.9 μF.

**Choice of C'**

C' must now be determined, and since it has been assumed that linearity is important, it will be allowed to degrade linearity by only \( \frac{1}{5} \) th per cent. The effect of C' in practice is rather different from that of changing base current in \( T_1 \); both cause the same kind of non-linearity because the charging current for C is made to fall during the sweep; but whereas a 'worst-case' design for \( T_1 \), i.e. assuming its
gain to be minimum, usually is pessimistic, the calculation for \( C' \) will be correct and the designer has no right to expect practical performance for a given value of \( C' \) to be better than predicted. The current for \( C' \) is, as above, 4.5 mA. The voltage across \( C' \) during the sweep must fall by no more than 1/8 per cent of the voltage across \( R \), i.e. \((1/800) 9.85 \) V. The total sweep time is about 2 msec (10 V at 5 V/msec), so that, using \( i = C' \frac{dV}{dt} \),

\[
4.5 \times 10^{-3} = C' \frac{9.85}{800} \times 10^3 \frac{1}{2}
\]

giving \( C' = 730 \) \( \mu \)F. In view of the wide tolerances of electrolytic capacitors \( C' \) must be given a nominal value of 1000 \( \mu \)F.

**Tolerance and Temperature Effects**

The design is now complete except for calculation of tolerances and temperature effects. Because of the simplicity of the charging equation for \( C \), errors in the +10 V line, and in \( R \) and \( C \) all produce the same percentage errors in sweep rate. Temperature effects on these same parameters give sweep rate errors in the same way, and the only effects not so far included are the change in the forward drop of \( D_1 \), the \( \beta \) of \( T_1 \) and the \( V_{ob} \) of \( T_1 \). The first changes the current in \( R \) calculated from \((10 - V_P)/R\), knowing that \( V_P \) changes at \(-2.5\) mV/degC at worst. The second causes linearity to improve and sweep rate to increase with temperature because base current is reduced. The third gives a d.c. output change of \(+2.5\) mV/degC (positive because the base voltage is unchanged and the emitter-base voltage decreases as temperature rises).

Summarizing, the suggested design has a tolerance on rate of sweep of 7 parts in 250 plus the effects, in proportion, of \(+10\) V variation and \( R \) and \( C \) tolerance. Linearity is better than 3/8 per cent. Provided transistor \( \beta \) over the range 5 to 7 mA exceeds 50 at the lowest operating temperature, the only detrimental effect of temperature change is a d.c. output voltage change of at worst \(+2.5\) mV/degC.

The sweep rate tolerance (+8 per cent even with perfect positive line, \( C \) and \( R \)) is too great for many applications but this can be improved in this simple circuit only by a higher-\( \beta \) transistor or a decrease in charging current, leading to worse linearity.

A much-improved circuit results if \( T_1 \) is replaced by the complementary emitter follower circuit described in Chapter 10. Circuit design is the same as above but \( \beta \) becomes \( \beta_1 \beta_2 \), thus greatly reducing the effect of amplifier current on the charging current for \( C \).

**Practical Problems**

In making practical use of the circuit just designed, it would normally be necessary to use an electronic switch for \( S \), such as a transistor driven between saturation and cut-off. Depending upon the rate at which this switch is operated, the output is then a continuous sawtooth waveform or a succession of sawteeth each separated by a waiting period. Both waveforms are often required for time-base generation for oscilloscopes, and the operation of electronic switch \( S \) to its closed condition, which terminates the sweep, is derived partly from trigger circuits operated by the signal being examined and partly from the waveform \( V_{out} \) itself.

Design of such a loop becomes complex when many different time-base frequencies have to be generated, especially when the required speeds approach the limit for available transistors.

A full appreciation of the problems would require detailed discussion of all the elements comprising the loop, but some insight can be obtained by noting the following points.

The design procedure above was concerned only with the generation of the ascending linear sweep and \( T_1 \) was taken to be \( n-p-n \) in order to illustrate the way in which the cathode of \( D_1 \) is driven more positive than any supply rail. If \( T_1 \) is \( p-n-p \), then when \( V_{out} \) reaches +10 V, it is evident that a more positive line has to be available merely to supply load current, thus masking the above phenomenon.

When the closure of \( S_1 \) is considered, at the end of the sweep, all appears to be well provided that it is realized that some small series resistance in \( S_1 \) and \( C \) is inevitable. \( C \) discharges rapidly to earth (infinitely rapidly and with infinite current if no resistance is assumed) and \( V_{out} \) follows.

In practice \( R_L \) will always have in parallel with it some stray capacitance \( C_S \) and when \( C \) descends to zero in a very short time, \( V_{out} \) can follow at the same rate only if current is available from some source which is sufficient to charge \( C_S \) at the high rate (\( dV/dt \)) required. In Fig. 5.3 the only sources which can charge \( C_S \) towards zero are the paths through \( R_E \) and \( R_L \), since any transistor emitter current, other than the leakage, will flow in the opposite direction and can only charge \( C_S \) positively.

It is clear that if the sweep rates involved are such that \( C \) becomes
as small as a few hundred picofarads, then typical load strays which are rarely less than 5 pF will cause the flyback time (i.e. \( V_{\text{out}} \) returning to zero) to be an appreciable proportion of the sweep. Reducing \( R_{\text{E}} \) enables faster discharge of \( C_{\text{S}} \) to take place but degrades the linearity of the sweep as shown in the initial design.

The root of the difficulty is that \( T_1 \) is unable to pass current in the right direction to discharge \( C \) rapidly and a \( p-n-p \) type would be more appropriate (see Fig. 5.4). Here the situation is reversed: \( R_{\text{E}} \) must supply the charging current for \( C \) and the load current. Maximum total current is at +10 V when \( R \) takes 4-5 mA (as always) and \( R_L \) takes 1 mA. The voltage across \( R_{\text{E}} \) at this time is least, being

\[
(V_2 - 10) \text{ or } 10 \text{ V if } V_2 = +20 \text{ V. } R_{\text{E}} \text{ must therefore be at most } 10/5.5 = 1.8 \text{ kΩ. This leaves no margin for tolerance and also leaves } T_1 \text{ just cut off when } V_{\text{out}} \text{ reaches } +10 \text{ V. A safe value is } 1.2 \text{ kΩ.}
\]

When \( S_1 \) closes and \( C \) discharges, \( C_{\text{S}} \) is now charged by as much current as \( T_1 \) emitter can supply. Since \( T_1 \) base has dropped by 10 V, the current which could flow if \( T_1 \) emitter did not follow would be virtually unlimited; \( C_{\text{S}} \) therefore discharges to zero virtually as quickly as \( C \).

**General Application of the Above Results**

The calculations required in the design of the above circuits are typical of many non-linear circuits in that the main consideration is the provision of adequate currents to obtain the required voltage swings. Several spurious effects combine in an attempt to frustrate the designer by adding unwanted voltage drops, stray capacitance, and temperature drifts.

Knowing the semiconductor properties outlined in the first and second chapters, only Ohm’s law and the charging equation \( i = C \frac{dV}{dt} \) are normally required to complete these designs satisfactorily, provided the function of the circuit is clearly understood.
6—Constant-current circuits

The term ‘constant-current source’ is usually applied to a circuit which supplies direct current, the magnitude of which is independent of the load into which it flows. This property can in practice hold only over a certain range of load conditions since there will always be a limit to the magnitude of the load voltage before the constant-current source limits.

Sometimes such a source is required to deliver current which is also constant with respect to changes of supply voltage and temperature, and although the same name is given to such a circuit and the configuration is similar, detailed design is more complicated.

BASIC CIRCUIT

The simplest circuit to produce constant current is a voltage source in series with a resistor. The current in the load is then given by \((V_1 - V_L)/R_1\) (see Fig. 6.1), so that, provided \(V_L\) is always small compared with \(V_1\), load changes have little influence on the current. If the load is a simple resistor \(R_L\), the condition for constant current is \(R_L \ll R_1\); if the load is a capacitor, the current remains constant until the capacitor voltage becomes comparable with \(V_1\). The departure from constant current is in fact \(100V_L/V_1\) per cent.

In many applications \(V_L\) is required to be several volts in magnitude and the current is to be held to within a few per cent. This can only be achieved in the simple circuit of Fig. 6.1 if \(V_1\) is a few hundred volts, which is often inconvenient.

Figure 6.2 shows how the use of a transistor solves this problem even with low supply voltages. In this circuit \(T_1\) base is held at a potential \(V_1R_3/(R_2 + R_3)\) and \(T_1\) emitter will be a few hundred millivolts more positive. The current in \(R_1\) is therefore approximately \((V_1/R_1)[1 - R_3/(R_2 + R_3)]\), i.e. \(V_1R_3/[R_1(R_2 + R_3)]\). \(T_1\) collector current and, hence, the load current are therefore \(V_1R_3/[R_1(R_2 + R_3)]\), provided \(T_1\) is not saturated, i.e. provided \(V_L\) does not exceed \(V_1R_3/(R_2 + R_3)\).

![Fig. 6.2 Constant-current source](image)

For different loads obeying this condition the collector voltage of \(T_1\) varies, but this causes only a very small change in collector current. In fact, \(T_1\) behaves like a source having resistance \(r_\beta\) if \(R_1\) is large, tending to \(r_\beta/\beta\) when \(R_1\) is zero (see Appendix 3, page 270). Since \(r_\beta\) is typically 1 M\(\Omega\) and \(T_1\) can pass several milliamps, the simple non-transistor circuit would require a supply of a few kilovolts to equal this performance.

Design of Constant-current Device

Circuit values are dictated by the available supply voltage \(V_1\) and the maximum load voltage for which current is to remain constant. When possible, as in cases where \(V_{L\text{max}}\) is much smaller than \(V_1\), but not quite small enough to allow a simple resistor supply to be used, the voltage on \(T_1\) base relative to ‘earth’ should be as small as possible, i.e. just larger than \(V_{L\text{max}}\).
This leads to the highest possible value for $R_1$, giving the highest effective source resistance from $T_1$ into the load; and also makes the actual current less dependent on transistor $V_{eb}$ variations, since these are a smaller proportion of the voltage across $R_1$ which determines the current.

Normal considerations for biasing a transistor must also be observed, in particular the base current of $T_1$ multiplied by $R_2/R_3$ must produce negligible voltage drop compared with the voltage across $R_2$ (or $R_1$). If this is not observed in design, the load current will be less than intended and will vary appreciably for different transistors and with temperature.

Temperature drift, again as in any normal bias circuit, is caused by $V_{eb}$ changes and $\beta$ and $I_{eb}$ changes. $V_{eb}$ drift causes the voltage across $R_1$ to change by $+2.5 \text{ mV/deg}^\circ \text{C}$ even if the base voltage remains constant. $\beta$ and $I_{eb}$ change the base current and move the base by this change multiplied by $R_2/R_3$ and in addition cause the collector current to differ from the emitter current as given by $I_c = aI_e + I_{eb}$.

Supply voltage variations in this circuit cause proportional changes in the voltage across $R_2$ and therefore across $R_1$. (This may be easier to see if the rail named $+V_1$ is regarded as stationary and the ‘earth’ line is taken to vary.) This results in a proportional change in emitter and therefore in load current. If the $V_{eb}$ of $T_1$ is comparable with the drop across $R_1$, the load current will change by a larger percentage than line changes.

**Typical Design**

Assume $V_1$ is 20 V and a constant current of 1 mA $\pm$ 5 per cent is required for a load which may have any value from zero to 5 k$\Omega$. Supply voltage $V_1$ is to be assumed constant.

A simple resistive supply is clearly inadequate since, if the resistor is chosen to give 1 mA into zero ohms, i.e. 20 k$\Omega$, it will deliver only 4/5 mA into a 5 k$\Omega$ load. The necessary criterion for the success of this circuit, that the maximum load voltage (5 V) shall be only 5 per cent of the supply voltage, is not met (the percentage being 25).

The circuit of Fig. 6.2 will therefore be used, and since $V_{be,\text{max}}$, is $+5 \text{ V}$, the voltage across $R_2$ will be made slightly greater, e.g. $+8 \text{ V}$.

The load current is 1 mA, and if a transistor is used having a minimum $\beta$ of 25, maximum base current will be 1/25 mA.

Values for $R_3$ and $R_4$ can now be calculated: the ratio $R_3/(R_2 + R_4) = 8/20$, and if the base current (1/25 mA) is allowed to cause a change of, e.g., $1/2$ per cent of the voltage across $R_2$ (0.06 V) the parallel resistance $R_2$ and $R_4$ is given by $R_2 R_4/(R_2 + R_4) \leq 0.06 \times 25 \text{ k}$\$.

This suggests

$$R_2 \times 8/20 \leq 0.06 \times 25 \text{ k}\$$$

i.e.

$$R_2 \leq 3.75 \text{ k}\$$$

and

$$R_4/R_2 = 8/12 = 2/3$$

Now comes a form of juggling peculiar to the circuit designer, made necessary by the ‘standard value’ system of resistor manufacture. By use of a slide rule it is easy to find standard values for $R_2$ and $R_4$ which are near to the ratio 2/3, but if this nearness is in error by more than a few per cent the voltage across $R_2$ will no longer be 12 V and this would mean that $R_1$ (given by $(V_{R1} - V_{eb})/(1 \text{ mA})$, or nearly $V_{R1}/1 \text{ mA}$) could not be given the convenient standard value of 12 k$\Omega$.

![Fig. 6.3 Typical design of constant-current source](image-url)

A careful study of the table of 5 per cent standard values (and resistors of this tolerance or better would have to be used) together with a slide rule search of 2/3 yields $R_2 = 3.3 \text{ k}$\$, $R_3 = 2.2 \text{ k}$\$. This gives the desired ratio, so that $V_p = +8 \text{ V}$ and $V_{R1} = 12 - V_{be} \approx 12 \text{ V}$; hence, $R_1 = 12 \text{ k}$\$ (see Fig. 6.3).

Having now designed the circuit, the designer must calculate its performance with respect to supply and temperature changes, even if this is not called for in the specification. This practice is always desirable, as it immediately reveals the shortcomings of a bad circuit, or bad choice of values, which can lead to vast performance changes for small supply or ambient temperature variations.

As indicated previously, a change of, e.g., 10 per cent in the +20 line causes a change in output current of 10 per cent, i.e. 0.1 mA in
this design. A temperature rise of 10 degrees reduces $V_{be}$ by 25 mV, thus increasing the current in $R_1$ by $25 \times 10^{-9}/R_1 = 25/12 \, \mu A = 2 \, \mu A$, thus increasing load current by 2 $\mu A$. $\beta$ can rise by 2 per cent per degree C, giving a change of $\beta$ from 25 to 30 for 10 degrees rise. This causes $V_{eb}$ to rise by $(1/25 - 1/30)R_2/R_3$, where $R_2$ and $R_3$ are in kilohms, giving $1/150 \times 1.32 \, V = 8.8 \, mV$. This in turn increases $V_{eb}$ by the same amount, and $I_e$ by $8.8 \times 10^{-9}/R_1 = 0.7 \, \mu A$.

For the sake of example, it will be assumed that $I_{eb0}$ rises by 0.5 $\mu A$. Then $V_{be}$ rises by $0.5 \times 3.3 \times 2.2/5.5 = 0.66 \, mV$, increasing the current in $R_1$ by $0.66/12 = 0.05 \, \mu A$. In addition, the load current is directly increased by 0.5 $\mu A$, giving a total increase of 0.55 $\mu A$ due to $I_{eb0}$. Summarizing temperature drift, the output increases 2 $\mu A$ due to $V_{be}$, 0.7 $\mu A$ due to $\beta$ for 10 degC rise, and typically 0.55 $\mu A$ due to $I_{eb0}$.

**Choice of Transistor**

Unless high-temperature operation is important, most $p-n-p$ small-signal types are satisfactory, provided $\beta_{min}$ is at least 25 at 1 mA and that its $V_{ce}$ rating is at least 8 V with base circuit resistance of 1-3 k$\Omega$.

Suitable types are the 2N3702 and 2N2906.

If the supply line were negative (or the load connected to the supply instead of to earth), the circuit could be inverted and an $n-p-n$ type used; suitable types are the 2N930 and BC108.

**Stabilized Current Source**

Although temperature-stability is adequate, the output of the circuit of Fig. 6.3 is no more stable than the supply, $+V_1$. A simple method to overcome this defect is shown in Fig. 6.4, where $R_2$ is replaced by Zener diode $ZD_1$. To understand the action of this circuit, assume $ZD_1$ is a perfect Zener diode, that is it behaves like a zero resistance battery. Then the voltage across $R_1$ is almost equal to the Zener voltage $V_Z$, provided this is much larger than $V_{be}$, and so the load current will be $V_Z/R_1$, which is independent of $V_1$.

This will not be strictly true with an actual Zener diode having series resistance $R_S$, and in such a case the effect of changing $V_1$ is easily seen if the $V_1$ line is taken to be fixed while the 'earth' connection moves by a fraction of $V_1$. Any change will appear at $T_1$ base reduced by the factor $(R_{3'} + R_S)/R_Z$ and this will then change $I_S$ and $I_L$ as before. The improvement is given by comparing $(R_{3'} + R_S)/R_Z$ with $(R_S + R_S)/R_Z$, giving a stability against supply changes in the new circuit which is better by a ratio of

$$\frac{(R_{3'} + R_S)/R_Z}{(R_S + R_S)/R_Z}$$

over the original.

A typical value of $R_Z$ for a small Zener diode of 12 V nominal $V_Z$,

![Fig. 6.4 Stable current source](image)

run at 5 mA would be 20 $\Omega$ and $R_{3'}$ would be 6/5 = 1.2 k$\Omega$. The improvement ratio is therefore

$$\frac{(1200 + 20) \times 3300}{5500} = \frac{20}{20}$$

i.e. 36-6 to 1. A 10 per cent increase in $V_1$ will now produce about 3 $\mu A$ change in load current instead of 100 $\mu A$.

The circuit of Fig. 6.4 has another virtue: the effective value of $R_Z$ is now a few tens of ohms instead of 3-3 k$\Omega$. This reduces the effects of base current variation because less voltage change results.

These improvements are to some extent offset by the additional temperature drift caused by the Zener diode. Its temperature coefficient is about +0.07 per cent per degree C, so a 10 degC rise gives 0.7 per cent rise in $V_Z$ and, hence, in load current. If necessary, this could be improved by adding one or two forward-biased diodes in series with $ZD_1$ and using a 10 or 11 V Zener diode. However, although the temperature coefficient is improved, the absolute value of load current would be less certain.
SUMMARY
Design of these circuits is simple and operation satisfactory, provided that \((V_i - V_b)\) is at least a few volts, \(R_e\) at least a few kilohms, and base resistors low enough to avoid temperature drift.

This is one of the most commonly used circuits, appearing in, for instance, time-base circuits, stabilizer circuits, and differential amplifier emitter circuits.

Where much greater precision is required the circuits described in Circuit Consultant's Casebook (Chapter 16) (Business Books, 1970) offer possible solutions.

7—Practical design of simple amplifiers

This chapter combines the results of Chapters 2 and 4 to give practical design procedures for the most used amplifier circuits.

EMITTER FOLLOWER

This is the configuration of Fig. 7.1. To begin design, the first thing the designer needs to know is the peak output voltage \(V_{out}\). This determines the peak output current \(I_{out} = \frac{V_{out}}{R_L}\), and \(R_e\) must be designed to ensure that this current can be supplied by the circuit. It also determines the minimum value for \(V_{in}\), which must exceed \(V_{out}\) by at least 1 V to prevent saturation on negative signal peaks.

![Fig. 7.1 Practical emitter follower](image)

When the output reaches \(V_{out}\), current \(I_{out}\) is flowing through \(C_2\) into the load and the transistor emitter current is given by \((V_p - V_{in})/R_e - I_{out}\), i.e., the difference between the current passing through \(R_e\) at that instant and \(I_{out}\). It is essential to the circuit action that this nett emitter current be greater than the useful minimum for the transistor. If \(V_p > V_{in}\) the condition can be restated: \(the\ quiescent\ emitter\ current\ must\ exceed\ the\ peak\ load\ current\). The excess must
be sufficient to allow for all tolerances, so that even in the worst case the transistor cannot approach cut-off.

When $V_p$ is comparable with $V_{cm}$, the above statement is insufficient and can be modified to (i), the current in $R_s$ at the positive peak of $V_{out}$ must exceed peak load current; or (ii), the quiescent emitter current must exceed peak load current where $R_s$ is to be considered to be part of the load.

Fig. 7.2 Insufficient standing current: (a) p–n–p, (b) n–p–n (resistive load)

Failure to meet this condition results in distortion and low output, as shown in Fig. 7.2.

Having determined the operating current, the circuit values and resulting performance may be calculated (Chapters 2 and 4). It may be that the input impedance $R_0/\beta(R_L + 1/\gamma_m)$ is too low for the application and a second emitter follower may be added (Fig. 7.3). This may usually be operated at lower current, since it does not have to supply $I_{Qn1}$, and this results in a larger $R_0$ and higher input impedance.

Provided the extra $V_{eb}$ voltage drop is unimportant, it may be directly coupled as shown. It is common practice to omit $R_{s1}$ so that $R_1$ base current is also $T_1$ emitter current. This is satisfactory only if this current is sufficient to operate $T_1$ and is in the right direction!

If $T_3$ is germanium, especially a power type, it is quite normal at high temperatures for the base current to flow inwards for a p–n–p transistor. This is also quite possible with silicon transistors of very high $\beta$ when operated at rather low emitter current. The omission of $R_{s1}$ would in these cases cause $T_1$ to cut-off and the only safe procedure is to design $R_{s1}$ to pass, when $V_{cm}$ is at its positive peak, at least $I_{ebo}$ plus a reasonable minimum current for $T_1$ emitter.

A similar criterion naturally holds for a pair of n–p–n transistors; a comparable situation arises when complementary types are used in the same cascaded system (Fig. 7.4). In this case the base current of $T_2$ when in its 'normal' direction, i.e. outwards, reduces $T_1$ emitter current. $R_{s1}$ must therefore be designed so that $(V_n - V_{cm})/R_{s1} = I_{ebo}/\beta_2$ is large enough for $T_1$ emitter current. The danger of $T_1$ cut-off increases in this circuit at low temperatures when $\beta_2$ becomes low.

Practical Example (Fig. 7.1 and 7.3)

Assume $V_p = +10$, $V_n = -10$, $R_I = 1$ k$\Omega$, $V_{cm} = 3$ V, and input impedance is to be $\geq 10$ k$\Omega$. Then, peak output current if gain is unity $= 3$ mA. When $V_{cm}$ is at its positive peak, the voltage across $R_s$ is $(7 - V_{eb})$, and at this instant the current in $R_s$ must exceed 3 mA, e.g. 4 mA. Therefore, $R_s \leq 7/4 \times 1 = 1.75$ k$\Omega$, e.g. 1.5 k$\Omega$. This gives a standing current of about 6–5 mA. A suitable transistor is chosen (e.g. 2N2906) with minimum $\beta$ of, say, 50, so that standing base current can be as large as 130 $\mu$A. If maximum temperature is 50°C, $I_{ebo}$ may be 4 $\mu$A, and at this temperature for a high-$\beta$
sample $I_e/\beta$ may be negligible. Maximum variation in $I_b$ is therefore from $+130$ to $-4$ $\mu$A, i.e. $134$ $\mu$A.

About 1 V shift of base voltage from zero could be tolerated without reducing the current in $R_b$ below its critical value, so that $R_{b(max)} = (1/134) \mu \Omega = 7.5$ k$\Omega$.

This clearly gives too low input impedance from $R_b$ alone, so an additional transistor of the same type will be added, as in Fig. 7.3.

$R_{b2}$ is decided as before (though the extra $V_{eb}$ due to $T_1$ drops $R_{b2}$ current a little) and $R_{b2}$ must pass $I_{o2}$ plus enough current to operate $T_1$, e.g. $1/2$ mA total, giving $R_{b2} \approx 20$ k$\Omega$, e.g. $18$ k$\Omega$. $T_1$ base current is therefore at maximum $1/\beta_1$ of $(500 + 170)$ $\mu$A, i.e. about $14$ $\mu$A and at minimum is $-4$ $\mu$A, a variation of $18$ $\mu$A. If $T_1$ base variation of $1$ V is allowed, $R_b \approx (1/18) \mu \Omega$, e.g. $47$ k$\Omega$.

![Fig. 7.5 Insufficient standing current: (a) p-n-p, (b) n-p-n (capacitive load)](image)

Input impedance can be calculated from the formula given earlier, but when working out an initial design it is a good idea to check roughly that the design is not grossly in error. The reasoning in this case would be: the total load on $T_2$ emitter is $R_{b2}/R_L = 1$ k$\Omega$/15 k$\Omega \approx 660$; the load on $T_1$ emitter is therefore $\beta_2(660) = 33$ k$\Omega$, in parallel with $R_{b2} = 18$ k$\Omega$, giving about $11$ k$\Omega$. The input impedance to $T_1$ base is $R_b/(\beta_1 \times 11$ k$\Omega) = 43$ k$\Omega$ which is well above the specified minimum of $10$ k$\Omega$.

The designer should now recalculate all currents, bearing in mind component and supply tolerances to confirm the soundness of the design. $C_1$ and $C_2$ are finally calculated such that $1/\omega C_1 \ll$ input impedance and $1/\omega C_2 \ll R_L$ at the lowest operating frequency.

Special care must be taken when the load is a shunt capacitance to earth, since the value of $I_{out}$ then depends on $dV_{out}/dt$, not simply on $V_{out}$. When the waveform is a pulse, the rate of rise or fall (depending on whether the transistor is $p-n-p$ or $n-p-n$) determines $I_{out}$, which is $C(dV/dt)$. If the standing current is inadequate, the transistor cuts off and $C$ charges from $R_c$ until $V_{out}$ is reached, giving a slow transition. On the opposite edge of the pulse the transistor merely takes extra current and reproduces the input correctly, provided the resulting power dissipation during the transient is not destructive (Fig. 7.5).

**EARTHED Emitter Amplifier**

The bias arrangements here are similar and the criterion for minimum standing current still applies (Fig. 7.6).

![Fig. 7.6 Earthed emitter amplifier](image)

Additional complications arise because $T_1$ must not be allowed to saturate when $V_{out}$ swings to its positive peak, so that the standing collector-base voltage must exceed $V_{out}$ by at least $1$ V.

$R_c'$ is now designed to give the required mid-frequency gain (where $C_e$ is assumed to have zero reactance), by using the simplified formula

$$\frac{V_{out}}{V_{in}} = \frac{R_c/R_L}{R_c' + 1/g_m}$$

Note that when $R_c' \gg 1/g_m$

$$\frac{V_{out}}{V_{in}} \approx \frac{R_c/R_L}{R_c'} \quad (R_e \gg R_c')$$

which is independent of the transistor, and the h.f. cut-off of the amplifier is approximately $f_o$ of the transistor. If $R_c' = 0$, gain has a maximum value of $g_mR_c//R_L$ but its h.f. cut-off frequency is only $f_o/\beta$. 
$C_e$ must be chosen so that its reactance $1/\omega L C$ at the lowest signal frequency is low compared with $R_e' + R_L/(1/g_m)$; when $R_e' = 0$, $(1/\omega L C)$ must be $\ll R_e/(1/g_m)$, not simply $\ll R_c$. At the frequency when $1/\omega L C = R_e' + R_L/(1/g_m)$, the gain will be reduced by a factor of $\sqrt{2}$, i.e. 3 dB.

Finally, $C_1$ and $C_2$ are chosen so that $(1/\omega L C_1) \ll Z_{in}$ and $(1/\omega L C_2) \ll R_c + R_L$.

As in the emitter-follower circuit, it may be that $Z_{in}$ is too low, in which case an emitter follower may be added between $R_b$ and $T_1$ base, observing the precautions previously mentioned (Fig. 7.7).

![Fig. 7.7 Amplifier with input and output buffer stages](image)

It often happens that $R_L$ is variable (sometimes to infinity) and yet $V_{out}/V_{in}$ is to be constant. In this case $R_L$ must be isolated from the collector circuit and an emitter follower may again be used; it can usually be directly coupled to the collector as shown in Fig. 7.7. It is now $T_3$ which requires a standing current capable of supplying the load peak current, and $T_3$ current will normally be decided by $R_c$. The loading of $T_3$ (including $R_L$) on $T_2$ collector is approximately $\beta_2(R_e/R_L + 1/g_m)$ and since the idea is to avoid changes in $T_2$ gain when $R_L$ varies, $R_c$ is chosen so that $R_L \ll \beta_2(R_e/R_L + 1/g_m)$. $T_2$ current may now be designed as if the only load were $R_c$. As in previous circuits, any or all of the transistors may be $n-p-n$ or $p-n-p$.

**Practical Example**

An amplifier is required to have a gain of 10, frequency response of 50 Hz to 50 kHz, $-3$-dB frequencies, load of 1.5 kΩ, $V_{out}$ of 3 V, input impedance of $\geq 10$ kΩ. Removal of the load must cause $< 10$ per cent rise in output. Supply lines $\pm 15$ V.

The load conditions immediately dictate the use of buffering (or an amplifier with overall negative feedback, see Chapter 8); the need for an input emitter follower is not yet known.

Referring to Fig. 7.8, $T_2$ collector potential must be known before $T_3$ emitter current can be determined. If loading on $T_3$ collector (other than $R_c$) is to be negligible, the standing requirement for $T_2$ is that $T_2$ collector potential must stand at least 3 V positive from $-V_e$ (i.e. $> -12$ V) and at least 4 V negative from $T_2$ base (i.e. $-4$ V). A tentative value could therefore be $-8$ V.

![Fig. 7.8 Practical amplifier (see text)](image)
At this point the transistors may be specified as \( \beta_{\text{min}} = 50, I_{\text{ebo}} \leq 4 \, \mu\text{A} \) at maximum junction temperature, collector voltage ratings \( \geq 15 \, \text{V} \), power ratings \( \geq 50 \, \text{mW} \), \( f_0 \geq 400 \, \text{kHz} \). The 2N3703 is suitable, as are many other small-signal transistors. The use of very-high-gain \( n-p-n \) planar silicon types (e.g. 2N930) would have greatly simplified design and led to the omission of \( T_1 \), but it is more instructive to overcome the difficulties of low gain and high \( I_{\text{ebo}} \).

**Using Power Transistors**

The design of linear higher-power amplifiers follows the principles just described, but transistor power dissipation is usually more significant. The instantaneous power dissipated in a transistor is the product of collector voltage and collector current plus the product of base–emitter voltage and base current, the latter product usually being negligible. This causes temperature rise in the junction which is dissipated by conduction to the transistor case and from there to the surrounding air via a heat-sink if fitted.

The thermal time lag between heat at the junction reaching the case depends on the construction of the transistor, but is usually of the order of 20 msec. The designer must therefore ensure that the average dissipation over a time of 20 msec is within the transistor rating. Within this limit, peak dissipation may be several times the rating. For example, a transistor rated at 100 mW can safely dissipate 1 W for 1 msec and then zero for 11 msec; it could not, however, dissipate 1 W for a month and then zero for a year, as destruction would have occurred within the first 20 msec.

Use of a heat-sink can increase greatly the mean power rating, and manufacturers usually quote the junction to case thermal resistance \( \theta_{\text{jc}} \) in degC/W and also the maximum permissible junction temperature \( T_{\text{J(max)}} \). If the case is assumed to be mounted on a perfect (i.e., infinite mass) heat-sink, then for an ambient temperature \( T_{\text{amb}} \), the maximum permissible dissipation \( P_M \) is that which raises the junction temperature to \( T_{\text{J(max)}} \), i.e., \( P_M \theta_{\text{jc}} = T_{\text{J(max)}} - T_{\text{amb}} \).

If the heat-sink is not perfect, it has also a thermal resistance (heat-sink to ambient) of \( \theta_{\text{hs}} \); there is also an imperfect thermal connection between transistor and heat-sink represented by \( \theta_{\text{t}} \). In this case

\[
P_M(\theta_{\text{jc}} + \theta_{\text{hs}} + \theta_{\text{t}}) = T_{\text{J(max)}} - T_{\text{amb}}.
\]
Some data sheets omit $\theta_M$ and give instead a graph of permissible dissipation versus ambient temperature in free air (i.e. without heat-sink) and with infinite heat-sink.

The only special precautions to be observed in these calculations are the duration of any peaks of high power and the use of worst case figures for thermal resistance (i.e. highest values of $\theta_M$).

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8—Negative feedback

Complete understanding of negative feedback systems is difficult, but Bode and Nyquist (p. 292) have succeeded in explaining feedback behaviour in practical terms by interpretation of the results of analysis.

In this chapter only the simpler consequences of the application of feedback will be dealt with in order to give a basic understanding of the problems involved. The methods discussed here for preventing instability in feedback amplifiers are easy to apply and usually adequate, but better overall performance can sometimes be obtained by using the more sophisticated techniques described by Bode and Nyquist.

**BENEFITS OF NEGATIVE FEEDBACK**

The reasons for using feedback are best illustrated by a simple example (Fig. 8.1), where an inverting amplifier of gain $-A$ and

![Fig. 8.1 Simple feedback amplifier](image-url)

infinite input impedance has added to it the input resistor $R_1$ and the feedback resistor $R_2$. It is shown in Appendix 4 that the gain $v_{out}/v_{in}$ is given by $-R_2/R_1$ if $A$ is very large.

This result can be obtained without complete analysis by the following argument. Suppose that an input is present and that the
resulting output is within the linear range of the amplifier. Under these conditions the input voltage of the amplifier itself, \(v_1\), must equal \(-v_{\text{out}}/A\), and if \(A\) is very large the implication is that \(v_1\) is very small.

If \(v_1\) is much smaller than \(v_{\text{in}}\), then the input signal current \(i_{\text{in}}\) is \(v_{\text{in}}/R_1\); since \(Z_{\text{in}}\) for the amplifier is infinite, all of this current must flow into \(R_2\), giving a voltage \(-v_{\text{in}}R_2/R_1\) across \(R_2\). As \(v_1\) is very small, the voltage across \(R_2\) is equal to \(v_{\text{out}}\). Therefore \(v_{\text{out}} = -v_{\text{in}}R_2/R_1\).

This result implies that variations in \(A\) have no effect on the overall gain provided \(A\) is always sufficiently large, so negative feedback enables an amplifier of accurately known gain to be obtained from an amplifier of high but badly defined gain. Since accurate gain is essential in most designs, the use of feedback is very common.

As calculated in Appendix 4, feedback in the form shown has other advantages also: the input impedance is known to be \(R_1\); the output impedance is reduced; the bandwidth is increased. Feedback does not improve signal/noise performance, nor does it reduce zero drift in a d.c. amplifier.

**LOOP GAIN**

The gain required in a negative feedback amplifier to obtain a good approximation to the above results is given in Appendix 4 as a mathematical expression

\[
\frac{AR_1}{R_1 + R_2} \gg 1
\]

Now, if the feedback loop is broken at any point and a signal injected towards the amplifier input terminal with \(v_{\text{in}}\) earthed, an amplified version of this signal appears at the other side of the break.

The gain between these two points is given by

\[
\frac{AR_1}{R_1 + R_2}
\]

and the magnitude of this is known as the 'loop gain'. If, for example, the loop is broken at \(X\), and signal \(e\) is applied to \(R_2\) at \(X\), with \(v_{\text{in}}\) earthed, \(v_1\) is given by \(eR_1/(R_1 + R_2)\) and \(v_{\text{out}}\) is \(-AR_2/(R_1 + R_2)\), which appears at \(X\). Loop gain is therefore \(AR_1/(R_1 + R_2)\), as stated above. A similar result is given by breaking the loop at \(Y\) or \(Z\) or in the middle of the amplifier circuit; in fact, anywhere in the loop, but not for instance in \(R_4\), which is outside the loop.

Note that it is the loop gain which must greatly exceed unity. This applies to all negative feedback systems regardless of circuit details; the loop is opened as described above and the gain round the loop must be large. If this condition is not obeyed, the advantages of feedback are reduced, and the inverse of loop gain is the fraction by which performance departs from the ideal.

Thus, in the present example a loop gain of 10 means that the overall gain \(v_{\text{out}}/v_{\text{in}}\) will be \(-R_2/R_1(1 + 1/10)\) instead of \(-R_2/R_1\). If tolerance in \(A\) is such that loop gain varies from 10 to 20, overall gain will vary from \(-R_2/R_1(1 + 1/10)\) to \(-R_2/R_1(1 + 1/20)\). To be certain of obtaining an overall gain which is constant to within 5 per cent, the loop gain must therefore be at least 20 so that \(A\) will be 20 times bigger than in a non-feedback circuit of similar overall gain.

It can be seen from Appendix 4 that the loop gain appears in all the important expressions: input impedance, output impedance, bandwidth, and gain.

**VIRTUAL EARTH**

This term is applied to a point such as \(v_1\) in Fig. 8.1, where, although it is a vital part of the signal circuit, the voltage amplitude present is very much less than signal voltages in the rest of the circuit. For calculating signal currents it can be regarded as being at zero potential or virtually earthed, and the error which results is negligible (e.g. \(i_{\text{in}} = (v_{\text{in}} - v_1)R_1 \approx v_{\text{in}}/R_1\), since \(v_1 \approx 0\)).

**Effect of Amplifier Input Impedance**

It is shown in Appendix 4 that the presence of a resistance \(R\) from \(v_1\) to earth changes circuit performance but not so much as in a non-feedback circuit with \(R_2\) open-circuit.

This is obvious if the feedback system is understood, because \(v_1\) is known to be much less than \(v_{\text{in}}\). Any resistance \(R\) to earth causes a current of only \(v_1/R\) to flow, which is much less than if the same resistor were placed across \(v_{\text{in}}\). Therefore a small fraction of the signal current \(i_{\text{in}}\) flows into \(R\) and the rest flows through \(R_2\) to \(v_{\text{out}}\). Gain is therefore virtually unchanged.

The above argument is plausible but can lead to wrong conclusions if the significance of \(R\) is not appreciated. If for example \(R = R_1)
then overall gain is hardly affected but the loop gain has been reduced (to half its former value if \( R_f \gg R_i \)). This means that definition of overall gain, bandwidth improvement, etc. are worse by the ratio by which the loop gain has fallen. The loop gain can always be worked out as before by breaking the loop at X, Y, or Z.

PROBLEMS IN NEGATIVE FEEDBACK LOOPS

Desirable as negative feedback is, there is always a practical limit to the amount of loop gain which may be applied. This is due to inevitable phase shifts within the loop which at some frequency, either the l.f. or h.f. end of the band or both, add up to an extra 180 degrees. Feedback at this frequency is therefore positive and if the loop gain

![Diagram](image)

Fig. 8.2 Low-frequency time constants: (a) coupling, (b) decoupling

(which always falls as the phase angle increases) is still unity or more, oscillation results at that frequency. If the gain is close to but less than unity, the system is stable, but application of a step input causes a burst of sine waves at the critical frequency which die away exponentially in the same way as in a damped resonant circuit. This effect is known as ‘ringing’. A sine-wave input test over the frequency spectrum will show a peak in the gain at the critical frequency.

These forms of complete or partial instability are caused at the l.f. end of the amplifier response band by the same components which cause the gain to fall. If no coupling or decoupling capacitors, transformers, or chokes are used and the response is constant to zero frequency, as in directly coupled amplifiers, no instability can result in this region, since 180 degrees phase reversal is impossible. In fact the presence of one simple CR network in the whole amplifier in the typical circuits of Fig. 8.2 is also guaranteed to be ‘safe’ as the maximum phase shift obtainable from such a network is 90 degrees. In theory two networks cannot produce 180 degrees phase shift but even though oscillation is impossible, ringing is likely if the angle exceeds 150 degrees with a loop gain of unity.

At the high-frequency end of the response band the components which cause the fall in gain and the accompanying phase shift are much more numerous. Transistor \( \alpha \), which falls in accordance with the equation

\[
\alpha = \frac{\alpha_o}{1 + jf/f_o}
\]

behaves like a CR network. Collector capacitance acts in the same way in conjunction with the collector load in an earthed emitter stage. Wiring strays and lead inductance complicate the issue still more.

For this reason overall feedback is rarely used in applications where the limits of frequency response for the transistors are being approached: there are so many elements contributing phase shift that after taking measures to prevent oscillation the response is greatly reduced. The designer thus generally tolerates relatively badly defined gain in the interest of simplicity and higher frequency response.

A situation which is identical from the point of view of feedback theory exists in many low-frequency amplifiers, such as chopper amplifiers or voltage stabilizers, where the response falls as the frequency increases, because of networks deliberately added. This is still a phase shift and gain drop at h.f. in that the phase lags increase and gain falls as frequency rises. In these cases the number and magnitude of such phase-shift networks is known, and the transistor effects and stray effects may be ignored if their effective time constants are very much smaller than those of the networks. As before, such a loop containing one simple CR network is free from oscillation, but two or more can ring or oscillate and the loop response must be examined.

Preventing L.F. Instability

The first step when designing the circuit detail of a feedback amplifier is to avoid all unnecessary l.f. coupling or decoupling circuits, which usually means avoiding capacitors, transformers and chokes. Direct coupling should be used where possible, and the aim

1
is to reduce the number of phase-shift-producing networks. Making capacitor values so large that phase shift is negligible within the signal band does not help, since there is always some frequency, however low, where phase shift is appreciable. (It is little consolation that the resulting oscillations are lower in frequency than the input signal band!)

If the number of phase shift networks at low frequency, more conveniently referred to as 'l.f. time constants', can be reduced to one, any amount of feedback may be applied and no l.f. oscillation can occur.

An example is given in Fig. 8.3 (a) and (b), where performance without feedback of the two amplifiers is identical over the audio band, provided that \( C, C_1, C_2, \) and \( C_b \) are large enough, \( R_b \gg R_0 \), and operating currents and voltages are correctly designed.

When feedback is added as shown, the circuit of Fig. 8.3 (b) is likely to oscillate, whereas Fig. 8.3 (a) is guaranteed to be free from l.f. oscillation.

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**Fig. 8.4 Single CR network**

If operating levels cannot be adjusted to allow direct coupling and two or more a.c. couplings have to be used, the important parameters are the ratios between the various time constants involved, as explained below.

**Effect of several time constants**

The normalized gain and phase response plots for a single \( CR \) network are given in Fig. 8.4 (a) and (b), where the frequency scale is logarithmic and expressed in multiples of \( 1/CR \).
of the network is plotted on a logarithmic scale in the form

\[ G = 20 \log_{10} \left( \frac{V_{\text{out}}}{V_{\text{in}}} \right) \]

commonly called decibels (dB or db). (The use of the term 'decibels' is strictly incorrect, since it should be used for voltage ratios only if the impedances associated with the voltages are identical. However, the use of decibels for any voltage ratio is almost universal.)

The curve is obtained by calculating

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{j \omega CR}{1 + j \omega CR} \]

so that

\[ |\frac{V_{\text{out}}}{V_{\text{in}}}| = \frac{\omega CR}{\sqrt{1 + \omega^2 C^2 R^2}} \]

and the phase angle \( \theta = \tan^{-1}(1/\omega CR) \). At very low frequencies, \( |\frac{V_{\text{out}}}{V_{\text{in}}}| \to \omega CR \), so that the plot of \( 20 \log_{10} \frac{V_{\text{out}}}{V_{\text{in}}} \) against log \( \omega \) is linear and has a slope of 6 dB per octave, meaning that a factor of 2 fall in frequency produces a factor of 2 fall in gain. At high frequencies \( |\frac{V_{\text{out}}}{V_{\text{in}}}| \to 1 \), which has been called 0 dB; and at \( \omega CR = 1 \), \( |\frac{V_{\text{out}}}{V_{\text{in}}}| = 1/\sqrt{2} \), which corresponds to -3 dB. This frequency is commonly known as the break-frequency, cut-off frequency, or 3 dB frequency of the network. The phase angle \( \theta \) approaches 90 degrees at very low frequencies, is 45 degrees when \( \omega CR = 1 \), and is 0 degrees at high frequencies.

The advantage of plotting the curves in this manner is that the effect of two or more networks on the loop gain and phase may be found merely by plotting the curves for each one separately and then adding the ordinates. The individual plots are easy to draw, since they are all identical in shape and are displaced along the frequency axis according to the ratios of the time constants.

Returning to the circuit of Fig. 8.3 (b), there are three networks causing l.f. phase shift, and it will be assumed that \( C_1 (R_5//R_6) \) is 3 msec, \( C_2 R_5 + R_6 \) is 1 msec, \( C_3 (R_{\text{in}} e//R_4) \) is 5 msec, and \( R_2 \gg R_3, R_e \gg R_3, R_4 \gg R_{\text{in}} e \). The plot corresponding to the 5 msec time constant is shown in curves (1) and (2) in Fig. 8.5: the gain is 3 dB down at \( \omega = 1/(5 \times 10^{-8}) \) with \( \theta = 45 \) degrees, and curves (1) and (2) are identical with the curves in Fig. 8.4.

The second largest time constant (3 msec) produces similar-shaped curves, but the 3 dB frequency (and the entire frequency scale) will be shifted in the ratio 5/3, as shown in curves (3) and (4), Fig. 8.5.

The last time constant of 1 msec is shifted in frequency by a factor of 5 from the first (curves (5) and (6)).

The combined effect of all three is shown in curves (7) and (8), which are obtained by direct addition of curves (1), (3), (5), and (2), (4) (6), respectively.

![Fig. 8.5 Low-frequency response for unstable feedback amplifier (Fig. 8.3b)](image)

Note that at point P the phase angle has reached 180 degrees and continues to rise as the frequency falls, finally approaching 270 degrees at an infinitely low frequency (90 degrees for each network).
At P the frequency is given by \( \omega = 1/(5 \times 10^{-3}) \) and the gain is -22 dB compared with its medium-frequency value, where the networks have little effect.

If the loop gain at medium frequencies exceeds +22 dB, which is a factor of 12-6, the circuit will oscillate at \( \omega = 1/(5 \times 10^{-3}) \), i.e. at 31.8 Hz.

This implies that if feedback were applied with a loop gain of just less than 12-6, the circuit would be on the verge of oscillation. In a practical design, with typical circuit tolerances the variation in loop gain is considerable: the variable gain of the circuits is indeed one reason for applying feedback. The designer must be certain that even under maximum gain conditions the loop gain is less than 12-6. Even this is insufficient, because a condition near to oscillation produces ‘ringing’ and response peaks as already mentioned, and a ‘phase margin’ of 30 degrees and ‘gain margin’ of 6 dB (factor of 2) are necessary to avoid these effects. Phase margin is defined as the nearness to 180 degrees when loop gain has fallen to unity; gain margin is the amount by which the loop gain falls short of unity when the phase angle has reached 180 degrees. In our example, inspection of the curves at 0 = 180 degrees gives a loop gain of -22 dB, so if the loop gain at normal frequencies is +22 dB, the gain and phase margin are zero at 31.8 Hz and the design would be unsatisfactory.

If the gain margin is made 6 dB, then the loop gain is +16 dB. The phase margin for this condition is obtained from the phase angle when loop gain is -16 dB, namely 163 degrees, so that the phase margin is 17 degrees, which is insufficient. In this example it so happens that phase margin is the more critical condition. To determine the maximum safe loop gain the figure for 150 degrees margin is found, namely -14 dB, so that a loop gain of 14 dB (factor of 5) satisfies both criteria.

Improving stability

With the time constant values in this example, the amount of loop gain which may be used is very small. Faced with this situation and a requirement that the loop gain is to be at least, for example, 10 (i.e. 20 dB) in a particular application, there are three courses open to the designer. The first is to reduce the number of time constants, as already stated; a momentary glance at the curves shows the value of removing the second (3 msec) time constant. The second is to alter the relative magnitudes of the time constants, the effect of which is to be described. The third is to add networks which favourably change the phase response of the system.

Assuming that the first method can be pursued no further, the second should be adopted; in all but the most critical applications this will succeed. Suppose that the smallest time constant is made 10 times smaller, i.e. 0.1 msec. The new plots of gain and phase are given in Fig. 8.6 (a), and the summed curves are labelled (7) and (8). This time the value of \( \omega \) at 180 degrees is 2/6/10^-3 and the gain is -38 dB, so that in the limit +38 dB loop gain could be applied. For a gain margin of 6 dB, +32 dB is permissible, but the corresponding phase margin is then only 25 degrees. Again, the phase margin criterion is the more critical, and for 30 degrees margin (150 degrees phase angle) the gain is -30 dB. The safe limit for loop gain is therefore +30 dB (a factor of 31-6) and the desired loop gain of 20 dB is quite safe.

The reason for this great improvement in stability from a permissible loop gain of 5 to 31-6 is obvious on examining the curves: the greater the ratio of the smallest two time constants, the lower has the gain fallen when the phase reaches 180 degrees. The absolute values of the time constants do not affect permissible loop gain but merely change the time-scale and the overall bandwidth of the feedback system.

The designer should therefore make the ratio of the two smallest l.f. time constants as large as possible. Having done this, a safe rule is that the loop gain may be as large as the ratio of the two smallest time constants. (For a more precise statement see Littauer, op. cit.)

The validity of this rule can be checked in the two cases considered. In the first the ratio in question was 3 and the permissible loop gain 5. In the second the ratio was 30 and the permissible loop gain 31-6. The rule is not empirical and has a sound mathematical basis, but the proof is involved and will not be attempted here.

There is a certain disadvantage in the second method which is usually tolerable. If for any reason the smallest time constant cannot be less than 1 msec in the above example, and for good definition of gain a loop gain of 10 or more is required, then the second method demands the increase of the other two time constants to 10 msec or more. This is sometimes impossible, in which case the third method should be adopted, but only after the first and second have been carried as far as possible.
The third method involves the addition of components to one or more of the phase-shifting networks and in its simplest form consists of a parallel CR combination in series with the capacitor of the main network (see Fig. 8.7). The effect of the extra components depends on the ratio of the two capacitors and of the two resistors; Figs. 8.9–8.19 show the gain and phase response curves for several ratios. The curves show that although the phase plot can be changed dramatically to give a greatly undulating phase angle, the gain curve is changed much less.

The use of these networks is best illustrated by its application to the original example with time constants of 5, 3, and 1 msec giving the curves of Fig. 8.5, where the maximum permissible loop gain is 14 dB (× 5) for a phase margin of 30 degrees. If it is required to use a loop gain of +20 dB (× 10) it is evident that the dangerous region

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**Fig. 8.6 (a) Stabilized version of Fig. 8.5 (second method)**

**Fig. 8.6 (b) Stabilized version of Fig. 8.5 (third method)**
from the point of view of loop oscillations begins when \( \theta \approx 150 \) degrees, which corresponds roughly to \( \omega = 1/(3 \times 10^{-3}) \), and ends when the gain is \(-26 \text{ dB (} \omega = 1/(6 \times 10^{-3}) \text{)}, this being deduced from the required phase margin of 30 degrees and gain margin of 6 dB.

The next step is to examine the curves for modified networks as given in Figs. 8.9–8.19 and to choose one which, when used in place of one of the original time constants, will ensure stability. As mentioned earlier, the phase plot tends to be changed much more than the gain, so initially, look for a phase characteristic with a dip towards zero extending from \( \omega = 1/(3 \times 10^{-3}) \) to \( \omega = 1/(6 \times 10^{-3}) \). If the largest (5 msec) time constant is to be modified, then \( R/X = 1 \)

![Fig. 8.7 Modifying networks (l.f.): (a) coupling network, (b) coupling network with correction added](image)

on the normalized curves corresponds to \( \omega = 1/(5 \times 10^{-3}) \), so that the phase dip must extend down to \( R/X = 0.8 \) (i.e. \( \omega = 1/(6 \times 10^{-3}) \)) and up to \( R/X = 1.66 \) (i.e. \( \omega = 1/(3 \times 10^{-3}) \)). If the 3 msec time constant is to be modified, then \( R/X = 1 \) represents \( \omega = 1/(3 \times 10^{-3}) \) and the phase dip must extend down to \( R/X = 0.5 \) and up to \( R/X = 1 \). Similarly, in the 1 msec case the phase dip must extend from \( R/X = 0.166 \) to \( R/X = 0.33 \).

Which network to modify depends partly on convenience; sometimes the required additional components would require to be very large accurate capacitors which are expensive or unavailable, and, on the other hand, it may be impossible to stabilize the system by operating on the network the designer would prefer to modify. In some designs more than one network may require to be changed.

Having decided tentatively on which network to alter—for example, the 3 msec time constant—the designer picks out a selection of curves which have the desired shape of phase response. In our example, curves (5), (6), (9), (10), and (13) look promising and if superimposed on Fig. 8.5 such that the \( R/X = 1 \) line coincides with \( \omega = 1/(3 \times 10^{-3}) \) the phase angle is reduced in the critical region.

Deciding which curve to use can be difficult: the general rule is that the less violent the change in phase, the better will be the overall l.f. response of the system. This rule does not, however, take precedence over the need for complete stability with all component tolerances. In the present case the author’s choice was curve (5) (Fig. 8.13), and this is shown in dotted lines in Fig. 8.5. The curves are now summed as before, remembering that the new curves completely replace curves (5) and (6).

The resulting phase curve now reaches 180 degrees at \( \omega = 1.7/10^{-2} \) at which frequency the gain totals \(-29 \text{ dB, so that if the loop gain is +20 dB, there is a gain margin of 9 dB. When the gain is -20 dB the phase angle is 149 degrees, which gives a phase margin of 31 degrees.} \)

The use of this network requires the insertion, in series with \( C_1 \) in Fig. 8.3 (b), of the parallel combination of a capacitor \( C_1/\sqrt{10} \) and a resistor \((R_5/R_6)\).

As indicated above, this is not necessarily the optimum network to use, but the gain and phase margins are not excessive. A safer network is curve (10) (Fig. 8.18), which gives very large margins but has a worse effect on the overall response. Again, it may be preferable to treat one of the other networks instead (e.g. curve (11) applied to the 5 msec. time constant).

**Overall response**

The totalled curves obtained after deciding the correcting networks now represent the open-loop response of the amplifier, and the overall response with feedback may be calculated. In the example given, the gain curve (Fig. 8.6 (b)) shows a loss of 12 dB at \( \omega = 6/10^{-2} \). At this frequency the loop gain is \((20 - 12) \), i.e. \( 8 \) dB (ratio of 2:5) and this implies a 3 dB loss in overall gain (overall gain \( = G/[1 + (1/2.5)] = 0.7G = G - 3 \) dB).

Had the network modification been unnecessary, the 3dB frequency for the overall system with 20 dB loop gain would have been \( \omega = 4/10^{-2} \).

**Preventing High-frequency Instability**

h.f. instability is caused and cured in the same way as l.f. instability. It is, however, much more difficult to deal with in practical transistor
circuit. The number of h.f. time constants is often outside the designer's control; there is no equivalent to the use of direct coupling which removes I.f. time constants. The time constants are often unpredictable; transistor current gain cut-off frequency often has a spread of 3 or 4 to 1. The time constants often interact and cannot then be considered separately.

Overall feedback is consequently rarely applied to amplifiers intended to work near the transistor frequency limit. The problem more usually facing the designer is how best to restrict the bandwidth in, say, an audio amplifier so that it does not oscillate in the megahertz region.

![Fig. 8.8 Modifying networks (h.f.): (a) collector load, (b) collector load with correction added](image)

The simplest solution is to follow method 2 and add just one h.f. time constant which is larger than any other in the circuit by a factor at least equal to the mid-frequency loop gain. The best solution is obtained if the time constant known to be the largest is further increased.

As indicated earlier in the chapter, similar h.f. problems can occur in servo systems, low-frequency amplifiers, or control loops where smoothing circuits, inductance of motors, and other predictable components produce a fall in gain and a phase lag as frequency rises. In such applications all of the procedures used for I.f. instability can be adopted.

There are some practical differences in their application. For instance, in the second method the important ratio is now between the largest and next largest time constant. The normalized curves are plotted in the opposite direction (which is made clear in Fig. 8.4) and the correction networks are the 'dual' equivalents of the I.f. circuits (Fig. 8.8).

**Other Causes of Instability**

It is an all too frequent occurrence that, having carefully designed a feedback loop to be free from instability, the designer finds that in practice the circuit oscillates.

Assuming that no gross error has been made either in estimating the time constants or in drawing and adding the curves, there are two common reasons for oscillation. The most usual is that the power supply rails and many other connections which are assumed to be zero impedance in most design calculations often have considerable impedance at both very high and very low frequencies. These introduce either extra feedback paths or add more time constants to the system. Much trouble can be avoided by the methods described in Chapter 16, and where the supply lines are suspected of introducing feedback the addition of extra capacitance between the lines will prove the point by changing the oscillation frequency. If this proves to be the case, then isolation between stages must be provided, preferably by means of a simple Zener diode stabilizer (Chapter 1) to supply the lower-powered stage.

The second common cause of oscillation is an unsuspected feedback loop coupled through stray capacitances. These can occur in wiring looms where high- and low-level signals may be carried on closely coupled wires and by mutual inductance between chokes or transformers. The effects can be reduced by improving cable routing and by mounting transformer and choke cores at right-angles.

If there is doubt as to whether the negative feedback loop is the cause of oscillation, reduce the gain of the negative feedback loop; if the loop is causing the trouble, any l.f. oscillation will become lower in frequency and h.f. oscillation will become higher in frequency, and the violence of oscillation will decrease; if the loop is not responsible, oscillations will usually increase and will tend to move further in frequency from the bandwidth limits of the amplifier.

**SUMMARY**

The benefits of negative feedback should be already well known to the student and most of this chapter has therefore been devoted to the methods of preventing instability. The use of separate gain phase plots rather than the composite gain-phase Nyquist diagram has the advantage that the influence of each network can be clearly seen. Corrective measures can then be taken as described and, finally, a Nyquist plot made if considered desirable.
Fig. 8.13 Network 5

Fig. 8.14 Network 6

Fig. 8.15 Network 7

Fig. 8.16 Network 8
Fig. 8.17 Network 9

Fig. 8.18 Network 10

Fig. 8.19 Network 11
9—d.c. Amplifiers

Engineering philosophers have spent many hours discussing the abbreviation 'd.c.', originally intended to mean direct current as opposed to alternating current. Among engineers it tends to be used as an adjective, leading, for instance, to 'd.c. voltage' which to an engineer means a zero-frequency signal.

When applied to an amplifier the term 'd.c.' could be taken to mean 'directly coupled' (i.e. no capacitor or transformer couplings) or 'direct current' (i.e. 1 μA input gives several milliamps output).

In the present context it is intended to mean an amplifier which operates as a voltage amplifier down to infinitely low frequencies: it is not necessarily direct-coupled nor does it necessarily amplify the input current.

DIRECT-COUPLED AMPLIFIERS

The simplest approach to d.c. amplifiers is to use the same circuit configurations as in Chapter 7. The practical difficulties involved are different in character, because changes of operating levels in the transistors caused by ambient temperature variations affect the output in the same way as if the signal were changing. Correct design of operating levels is therefore even more important.

For example, a simple earthed emitter amplifier has a gain of

$$ R_L \frac{1/g_m + R_s + R_d}{g_m R_L} $$

and for maximum a.c. amplification $R_s$ is by-passed, giving a gain of $g_m R_s$ for a zero resistance source. For d.c. amplification the by-pass capacitor must be replaced by a resistor or Zener diode or some other element which operates down to zero frequency, while at the same time the operating current must remain correct.

The base operating potential will be determined by the input signal, and the collector voltage, although it represents in amplified form the input signal, has a standing voltage level which may be undesirable.

Suppose, for example, that a signal of 0 to $-50$ mV from a source resistance of 100 Ω is to be amplified by 10, and the output level is to be 0 to $+500$ mV into a 1 kΩ load.

The direct attempt shown in Fig. 9.1 fails in at least two respects. First, the collector voltage has to be negative by 0.5 V or more for correct transistor operating, yet this is also the load voltage, which has to be 0 to $+500$ mV.

![Simple direct-coupled amplifier](image)

Secondly, the voltage gain has to be 10, so that even if $R_L$ were infinite, $R_s + 1/g_m + R_d/\beta$ would have to total 100 Ω since the gain is

$$ \frac{R_L}{1000} = \frac{R_s + 1/g_m + R_d/\beta}{R_L} $$

This implies $R_s$ of about 30 Ω and $1/g_m$ of the same order, which requires $I_s$ of 2 mA and a positive supply of $V_p = (V_{eb} + 60$ mV).

This leads to very poor temperature-stability and great variation if the transistor is changed for another of the same type; in fact, on $V_{eb}$-tolerance alone, some transistors would conduct heavily and others cut-off. There remains also the problem of collector voltage.

The above disastrous example illustrates that when d.c. gain is achieved, stability of levels is much more difficult to maintain; this is only to be expected, since the main technique for good stability is low d.c. gain.
Persevering with the simple circuit of Fig. 9.1, some of the snags can be overcome. The supply voltage requirement of \( (V_{eb} + 60 \text{ mV}) \) for \( V_p \) can be obtained as shown in Fig. 9.2 from a more readily available supply line; provision can be made for adjustment for different \( V_{eb} \), which could vary from 0-4 to over 1 V for a silicon transistor, and a change of level can be added in the load circuit \( (V_2) \).

The use of setting-up potentiometer \( (RV_1) \) is often required in d.c. amplifiers, because even when temperature stability is good, some means have to be provided to offset resistor and \( V_{eb} \) tolerances. In Fig. 9.2 \( RV_1 \) is adjusted so that with zero input voltage \( V_{out} \) is zero. The voltage gain cannot be predicted accurately, as the setting of \( RV_1 \) directly affects the gain and \( 1/g_m \) is also subject to wide variation. The first problem could be overcome by making \( RV_1 \) much lower in value (e.g. 10 \( \Omega \)), but the standing current through it (\( \approx 100 \text{ mA} \)) then represents a considerable drain on the supply.

Temperature-stability is poor, since a 10 \( \text{deg} \) rise causes \( V_{eb} \) to change 20-25 mV, having the same effect on the output as an input signal change of the same amount.

By modifying the circuit as shown in Fig. 9.3, the above difficulties are relieved, with the exception of gain variation due to \( 1/g_m \)-tolerance, which is unchanged.

In this emitter-coupled pair the emitter circuit of the second transistor acts as the emitter load of the first and the setting-up control has much less influence on the gain, since its apparent value in the emitter circuit is reduced by a factor \( B \). Temperature changes tend to cause equal \( V_{eb} \) variation in \( T_1 \) and \( T_2 \), and if these match exactly the output change is only \( \delta V_{eb} (R_L/R_e) \) rather than \( \delta V_{eb} (g_m R_L/2) \).

The signal behaviour of this circuit is identical to the a.c. performance of the emitter-coupled pair when the free base is decoupled, and is dealt with in Chapter 4.

**D.C. Amplifiers**

D.c. amplifiers suffer from two main defects, 'zero drift' and 'gain drift'.

**Zero drift**

Ideally, when the input signal is zero the output should remain constant (not necessarily at zero), but in practice temperature and voltage supply variations cause the output to drift.

The importance of this drift depends on how much signal change would have been required to produce the same effect at the output. It is customary therefore to quote the zero drift of a d.c. amplifier in terms of the equivalent input signal, and this is known as 'referring the drift to the input'.

**Gain drift**

When the input changes, the output changes by a larger amount such that \( V_{out} = GV_{in} \). 'Gain drift' refers to the change in \( G \) due to temperature, supply variation, and ageing effects.
This effect is usually not so important as zero drift, since change of $G$ always represents the same percentage error in signal. Zero drift is independent of signal and therefore represents an infinite percentage error if this signal is infinitely small.

**Calculation of drift**

Unless the amplifier is kept in a constant-temperature enclosure, the drift contributions to be considered are power supply and temperature variations, ageing effects normally being negligible. When an enclosure is used, ageing may become noticeable.

The various transistor parameter variations must be considered as in Chapter 2; usually only the first stage of an amplifier need be examined when calculating zero drift, since any subsequent variations are less significant by a factor which is the gain from the input to that point in the circuit.

The procedure is quite straightforward and for the circuit of Fig. 9.3 is as follows. $T_1$ and $T_2$ are assumed to be low-power silicon transistors, with $\beta$ variation at 25°C from 25 to 100, and $I_{eb0}$ at 50°C of 3 $\mu$A.

Input base current at 25°C can vary from $(I_b/100) - I_{eb0}$ to $(I_b/25) - I_{eb0}$, i.e. from 20 to 80 $\mu$A $- I_{eb0}$ if $RV_1$ is set to give about 2 mA in $T_1$ and $T_2$. At this temperature $I_{eb0}$ may be 0.5 $\mu$A, giving a range of 8 from 19.5 to 79.5 $\mu$A.

At 50°C, for example, $\beta$ variation will be from 150 to 37.5, assuming 2 per cent per degree C rise, and $I_{eb0}$ may be 3 $\mu$A, giving a range of 8 from 10.5 to 50 $\mu$A.

Possible base current limits are therefore 10.5 to 79.5 $\mu$A, but this represents initial transistor $\beta$ tolerances as well as temperature effects and indicates the voltage adjustment required from $RV_1$ because of $\beta$ and $I_{eb0}$.

For a given circuit, after setting up $RV_1$, drift is worst for a low-$\beta$ transistor and 8 can then vary from 79.5 $\mu$A at 25°C to 50 $\mu$A at 50°C, a drift of 20-5 $\mu$A. This causes the input base to move by 29.5 $\times$ $R_2$ $\mu$V $= 2.95$ mV (25 to 50°C).

The base of $T_2$ has a similar influence on the output as the base of $T_1$ but opposite in sign. Its maximum variation is the same as above, except that $R_2$ is replaced by the impedance between the base and ground. This is highest when $RV_1$ is central and is then 75 ohm $[(50 + 100)/(50 + 100)]$ if $R_1$ and $R_2$ are ignored, giving a maximum shift of +3.9 mV, which is equivalent to $-3.9$ mV at $T_1$ base.

Unfortunately, these drift voltages cannot be added, because it can well happen that one transistor has a high $\beta$ with no temperature coefficient and zero $I_{eb0}$. If this applies to $T_2$, then $T_2$ base variation with temperature is zero.

Input drift from $\beta$ and $I_{eb0}$ can therefore total (2.95 mV for a 25 to 50 degC rise).

Changes of $V_{be}$ with temperature tend to cancel, since if both drift exactly together, at the maximum value of $-2.5$ mV/degC rise, the effect is merely to increase the drop across $R_e$ by 62.5 mV for a 25 to 50 degC rise. This is an increase in $(T_1 + T_2)$ emitter current of 62.5/5 = 11 $\mu$A, i.e. 5.5 $\mu$A in each transistor. The input voltage which would be required to increase $I_{eb}$ by this amount is given by $2(5.5/\mu A)$ mV, or approximately 0.5 mV. It is clear that the larger $R_e$ can be, the less is the error due to equal $V_{be}$ changes; replacement of $R_e$ by a constant-current device (Chapter 6) virtually eliminates this error, but of course drift in the device has to be considered.

When the two drifts are unequal, the equivalent input drift is the difference between the two. For dissimilar transistor types the drift is therefore $\pm(2.5 - 2)$ mV, i.e. $\pm0.5$ mV/degC, giving a total drift of $\pm12.5$ mV from 25 to 50°C.

It is therefore advantageous to use similar transistor types, when this figure will be reduced to about $\pm5$ mV, and if dual transistors can be used a further reduction to $\pm1$ mV or even less will be obtained.

Other sources of temperature drift are the resistors and $ZD_1$. If the coefficient of $ZD_1$ is $+0.07$ per cent per degree C, then over a range from 25 to 50°C this is equivalent to an input drift of 70 mV/G, i.e. about $-7$ mV. Resistor drifts of up to $\pm0.02$ per cent per deg C can be calculated similarly and referred to the input.

Power supply variations also contribute to drift and the effect of the variation can be calculated by considering separately each entry point to the circuit.

For instance, if the $-20$ line changes by $\pm10$ per cent, the direct effect on the emitter circuit is to change the drop across $R_e$ by $\mp 2$ V, giving a change in each emitter current of $(1/2)(2/5-6)$ mA, i.e. $0.18$ mA. This is equivalent to an input change of $\mp 2 \times 0.18$ mV, or $\mp 0.18$ mV.

The $-20$ line affects $T_2$ base to an extent slightly dependent upon the setting of $RV_1$. If $RV_1$ is central, the change in base potential for $\pm10$ per cent in the $-20$ line is about $\mp 40$ mV giving an equivalent input of $\pm 40$ mV.
In this case a cancelling effect can be taken into account, since $R_s$ and $R_1$ are attached to the same $-20$V line, the result being a drift of $\pm 22$ mV referred to the input.

**Variations in Directly Coupled Amplifiers**

Because of the inherent cancellation of $V_{be}$, the differential amplifier is the basis of most low-drift directly coupled amplifiers. Many refinements are used to increase input impedance, to reduce the effect of changing supply voltages, and to define the gain accurately. To reduce zero drift the only approach is to use the lowest possible base currents in the transistors directly connected to the source, and to balance $V_{be}$ accurately in each half of the amplifier.

![Fig. 9.4 Use of emitter follower](image)

To this end emitter followers, preferably using planar epitaxial types of high gain, either $p-n-p$ or $n-p-n$, may be added as shown in Fig. 9.4. An even better method, due to Bénéteau (Fig. 9.5), is to replace each transistor of the differential amplifier by a complementary pair (see Chapter 10). The advantage is that, as in the two-transistor differential amplifier, only two $V_{be}$s need to be balanced, yet the base currents are as low as the emitter followers in Fig. 9.4.

Supply voltage has its biggest effect in coupling to the second base of the input pair. One method to reduce this at the cost of ‘rejection ratio’ and gain is to earth this base and adjust for zero balance by an emitter potentiometer (Fig. 9.6). To be effective the drop across $R'_s$ when the slider is at one end must exceed the possible $V_{be}$ differential when half the emitter current in $R_s$ flows through $R'_s$.

**D.C. Amplifiers**

Bootstrapping, described in Chapter 15, may be added to increase the input resistance, but it must always be remembered that the input drift of an emitter-coupled pair will be $V_{be}$ differential drift plus the voltage produced when $I_{b1}$ flows into the resistance seen by the base. If by bootstrapping or any other method the input resistance is made infinite, then the drift due to $I_b$ is $I_b R_s$ where $R_s$ is the source resistance. Removing the source then gives ‘infinite’ drift, i.e. the amplifier drifts until some limiting occurs.

![Fig. 9.5 Bénéteau's circuit](image)

![Fig. 9.6 Alternative zero set](image)

In assessing a design it is therefore important to know its input voltage drift $V_D$ with $R_s = 0$ and its current drift $I_D$ which causes a further drift of $I_D R_s$. For example, an amplifier with $V_D = 1$ mV
and \( I_D = 1 \mu A/\text{deg}C \) will drift 1 mV + 1 V/degC when driven from a 1 M\( \Omega \) source.

**Typical Application: Voltage Stabilizer**

By far the most common use for a directly coupled amplifier is the reference amplifier of a voltage stabilizer. The stabilizer is used to provide a source of stable voltage and low output resistance when the main supply and the load are variable.

A simple Zener stabilizer was described in Chapter 1, but this becomes undesirable when the load current is large compared with the rated Zener current; moreover the output resistance is much greater than the 0.1 \( \Omega \) which is often required.

![Fig. 9.7 Block diagram for voltage stabilizer](image)

The principle of operation is shown in Fig. 9.7, where the 'reference' voltage is usually a Zener diode supplied as described in Chapter 1 and preferably of low temperature coefficient. A differential amplifier such as an emitter-coupled pair is supplied from the main d.c. supply \( V_s \) and has a high-power output stage from which the stabilized output \( V_{\text{out}} \) is taken. An attenuated version of \( V_{\text{out}} \) is coupled by \( R_1 \) and \( R_2 \), and the two differential amplifier inputs are connected between this point and the reference voltage \( V_{\text{ref}} \).

The system is a negative feedback amplifier, and if correctly designed causes \( V_{\text{out}} \) to be

\[
\frac{R_2 + R_1}{R_1} V_{\text{ref}}
\]

regardless of \( V_s \) variations. The connections are such that if \( V_{\text{out}} \) is too small, the differential amplifier changes its current in the correct direction to increase \( V_{\text{out}} \), and vice versa.

In practical stabilizers the power stage is usually a simple emitter follower. If it is arranged so that any output load current tends to increase the emitter follower current the circuit is called a 'series stabilizer' and the emitter follower is known as the 'series transistor' or 'series element'. If load current tends to decrease emitter follower current, the terms are 'shunt stabilizer', etc.

**Typical Design**

A practical example of the above system is given in Fig. 9.8, where \( T_1 \) and \( T_2 \) form the differential amplifier and \( T_3 \) and \( T_4 \) enable currents of about 1 A to be delivered to the load. Phasing can be checked by imagining the loop broken at between \( R_6 \) and \( T_4 \) and levels adjusted to produce normal transistor operating conditions.

![Fig. 9.8 Practical voltage stabilizer](image)

Then move the free end of \( R_6 \) negatively. This causes \( T_2 \) to pass more current, so that its collector moves positive. \( T_3 \) emitter and \( T_4 \) emitter therefore move positive, and when \( R_6 \) is reconnected this will tend to move \( R_6 \) positive, thus resisting its negative movement.

To design such a circuit, assume that all voltage levels are at the desired level. In this case the output required is taken to be about 15 V with a current 0-1 A for \( V_s = 20 \pm 10 \) per cent, temperature 0-100°C. To make the potential of base 2 of the differential amplifier correct (5.6 V), a ratio of about 2.8 to 1 is required for \( (R_5 + R_6)/R_5 \). For reasons of drift and loop gain, to be discussed later, the values of \( R_5 \) and \( R_6 \) should be as low as possible as long as they do not drain a large proportion of the output current capability of \( T_4 \). \( R_5 = 220 \Omega \) and \( R_6 = 390 \Omega \) are suitable; lower values in similar proportions may also be used.
Since the output transistor will carry 1 A and have a $V_{ce}$ of $20 - 15$, i.e. 5 V, it must have a power rating of at least 5 W at the maximum operating temperature. If this is 100°C, then the 2N3055 in conjunction with a small heat-sink (of about 5 degC/W) is satisfactory and since its guaranteed minimum $\beta_L$ at 1 A is 50, $I_{bo}$ is 20 mA maximum. ‘Minimum’ current occurs at no-load (leaving only $R_5$ and $R_6$ as loads), when only $I_{eb}$ flows. At 100°C this is about 1.4 mA maximum, but if the full load has been applied, raising the junction temperature to 130°C, and is then removed, $I_{eb}$ may be 10 mA. The base current of $T_4$ can therefore be 20 mA flowing outwards, at one limit, or 10 mA inwards, at the other.

This is of extreme importance, as it shows the need for $R_4$; the base current requirement for $T_4$ could be inwards, and without $R_4$, $T_3$ then has no source of emitter current. $T_4$ base falls until $I_{bo} = 0$, giving $I_{bc} = \beta_4 I_{eb0}$; the loop loses control and the output remains at some unstable level between 15 and 20 V. Stabilizers have often been designed without $R_4$, and they invariably fail at high temperature with no load, especially if full load had just been applied, making $T_4$ hotter still.

The only safe design procedure is to make $R_4$ current equal to the maximum possible value of $T_4 I_{eb0}$, in this case 10 mA, so that $R_4 = 1.5$ kΩ.

$T_3$ can now be chosen; it will carry a maximum current of 10 mA (from $R_4$) + 20 mA (full load for $T_4$ with minimum $\beta_L$ of 50 and zero $I_{eb0}$), i.e. 30 mA at 5 V, a power rating of 150 mW at 50°C. A 2N696 is suitable and has a minimum $\beta$ of 25, giving a maximum $I_{bo}$ of 1.2 mA.

$R_3$ must now be designed so that when $I_{bo}$ is maximum the potential across $R_3$ is such that its more negative end never goes more negative than $(15 + V_{eb} T_3 + V_{eb} T_4)$ V, i.e. +16 V. (No great accuracy is required here so long as a large margin is allowed in the value of $R_3$.) For the first time in the design the more positive limit of $V_T$ is important, since the loop will fail completely as soon as the drop across $R_3$ is too large. This positive $V_T$ limit is +18 V, so that 2 V drop in $R_3$ can be allowed in the limit when $I_{bo}$ is maximum. This gives a limit value for $R_3$ of $(2/1-2)$ kΩ, i.e. 1.6 kΩ, and to ensure a large margin for tolerance and to avoid $T_2$ approaching cut-off under $I_{bo}$ maximum conditions, a value of 1 kΩ is selected.

Under the opposite set of conditions, where $V_T = +22$ and $I_{bo} \approx 0$, which can occur if $T_3$ and $T_4$ have high $\beta$ and high $I_{eb0}$, then to bring the output to +15 a current of $(22 - 15)/R_3 = 7$ mA must be supplied to $R_3$. (This time no allowance for $V_{be}$ was made, since in the worst case this approaches zero.) This current must therefore be available from $R_3$, the value of which must be $(V_{ref, min.})/I_{eb} = 4.3/7 = 0.6$. If $V_{ref} = 5.6 \pm 10$ per cent and $V_{eb1} = 0.7$ V, $R_3$ is chosen as 560 Ω.

$T_1$ and $T_3$ have to carry a maximum current of $(5-6 + 10/11) A = 11$ mA, and $T_1$ may at the same time have 16 V, giving a dissipation of 176 mW. This may be reduced by connecting $T_1$ collector to $T_4$ emitter, since this is just as effective a supply line as $V_T$. Dissipation then is 121 mW maximum in $T_1$ and $T_3$, and the 2N930 or BC108 are suitable.

$R_1$ may now be decided to suit Zener diode ZD1, which may be of low power rating, e.g. 300 mW, with an optimum current of 5 mA. $T_1$ base current may reach 1/3 mA, which has little effect, and the value of $R_1$ may be $(20 - 5-6)/5 \approx 2.7$ kΩ.

**Refinements and their dangers**

As in the case of $T_1$ collector, there appears to be no reason why $R_3$ should not be connected to $T_4$ emitter, with a change in value to $(15 - 5-6)/5 = 1.8$ kΩ, since this is a more stable supply than $V_T$ and will result in less variation in the reference when $V_T$ changes. In this particular design this is a good idea and should be adopted. In general this ‘gimmick’ always needs careful examination, because the result may be that the circuit never ‘starts’, the output remaining at zero. (If this should happen in a stabilizer, a momentary resistive link from $V_T$ to the reference Zener will ‘start’ the stabilizer and the link may then be removed.)

The only reason for its success in the present circuit is that $R_3$ is pulled positive by $V_T$ and this pulls the output positive, so turning ZD1 on until full stabilization is reached. An example where non-starting results is shown in Fig. 9.9, in which $T_3$ is an earthed emitter stage instead of an emitter follower. To overcome the inversion of signal thus produced, $T_3$ is supplied from $T_1$ collector instead of from $T_2$. If ZD1 is supplied from the output as shown and $V_T$ switched on, $R_A$ and $R_B$ are pulled positive, but unless the $I_{bo}$ of $T_4$ pulls $R_C$ positive enough to turn on $T_4$ and ZD1, the output never reaches stabilizing level. Momentary connection of $R_D$ or $R_E$ starts the circuit.
Compensation for line changes
Returning to Fig. 9.8, it is clear that variations in $V_1$ are coupled into the system by $R_3$, thus undesirably varying the output; the percentage variation is reduced by the loop gain (which is typically 10 in this simple circuit), but this still amounts to ±1 per cent.

One way to reduce this is to couple some of the $V_1$ variation into $T_2$ base at such an amplitude that it cancels the $V_2$ change seen at the collector. This requires a resistor $R_7$ of such a value that

$$V_2 = \frac{R_3}{R_0} \frac{R_{in}}{R_{in} + R_7} R_{g_m} = 2 = V_1$$

$R_7$ therefore depends on $g_{m2}$ so that it cannot be given an exact value and will require adjustment for the particular transistor. This is not a welcome situation, but even without exact setting a marked improvement is given and this applies to any form of $V_1$ variation, including ripple. In Fig. 9.8 $R_7$ should be about 10 kΩ. Note that in Fig. 9.9 $R_A$ and $R_B$ move together when $V_2$ varies, so that if the output resistance of $T_1$ ($\approx r_0 / \beta_2$) is much higher than $R_A$, no change of current results in $T_2$; thus, no change appears at the output.

Positive feedback to give zero output resistance
The use of positive feedback within a negative feedback loop has a surprising effect. If the positive feedback is applied to one stage of the loop amplifier to the critical point where the positive loop would normally oscillate, the negative loop behaves as if it had infinite gain. Oscillation does not occur (except for other unconnected reasons), and the output impedance of the negative feedback system is zero. If positive feedback is increased beyond what would normally cause oscillation, the system is still stable and the output impedance becomes negative.

Although this is a useful result, it is by no means the cure for all stabilizer or feedback problems. The main difficulties are, first, that the value of positive feedback for $Z_{out} = 0$ is highly critical and cannot be held with great accuracy, so that the risk always exists that $Z_{out}$ may become negative, causing oscillations with certain types of load; and, secondly, that the bandwidth over which the critical condition holds is restricted to a fraction of the normal system bandwidth.

Both these considerations make this idea useful only in narrow-band systems where negative $Z_{out}$ is tolerable—namely stabilizer circuits. Here the idea is practical provided the designer does not attempt to convert a poor performance into perfection solely by its means.

Fig. 9.9 A 'non-starter' (see text)

If, for example, $Z_{out}$ is 1 Ω and the use of positive feedback is used to reduce $Z_{out}$ to 0, then normal tolerance effect will result in $Z_{out} = 0 ± 0.1$ Ω, not $Z_{out} = 0 ± 1$ mΩ.

Fortunately, the incorporation of this scheme is very simple and can be done by adding a collector load in $T_1$ (so designed that $T_1$ cannot saturate on $I_{C_{3(min)}} = 10$ mA) and a resistor from $T_1$ collector to $T_2$ base. The value of the latter component should ideally be such that the gain round the $T_1c-T_2g-T_1e$ loop is just unity. The extra components affect $T_2$ base, and some change in $R_3$ or $R_4$ is required to reset $V_{out}$ correctly. These conditions are easily calculated: what is not easy is to estimate the long-term drift in the positive loop gain. As stated above, this 'trick' should be regarded as a final touch to an already adequate design (Fig. 9.10).
**Extra loop gain**

Using the original circuit of Fig. 9.8, the loop gain of about 10 is
often insufficient, although attention to some of the above refine-
ments will often remove the need for more. (Only isolation, not loop
gain, is required to improve performance against changes in \( V_z \);
although more gain also improves this, it is bad design to use more

![Fig. 9.11 Extra stages for higher loop gain](image)

than is required from other points of view.) The important advan-
tages of more loop gain are the stability of output against amplifier
gain variations; predictability of output as a simple ratio of the
reference voltage; and low output impedance.

The usual way to increase gain is to add a second coupled pair of

![Fig. 9.12 Alternative forms of reference amplifier](image)

the opposite type (see Fig. 9.11), and a novel method having some
advantages is given in Chapter 12.

**Combined reference amplifier**

An alternative to the Zener diode/emitter-coupled pair combina-
tion is the use of a single transistor with the reference diode in its
emitter circuit (Fig. 9.12).

This has certain conveniences counterbalanced by slightly worse
performance.

The advantages are: the saving of one transistor; and the omission
of the Zener feed resistor if the transistor emitter current is suitable.

Disadvantages are: temperature performance depends on \( V_{be} \),
matching with \( V_z \) (e.g. 6.8 or 8.2 V Zener tends to match a transistor
\( V_{be} \) in temperature coefficient), whereas in the first circuit matched
transistors and low-coefficient Zener can be obtained; change of
output current varies output transistor \( V_{be} \) and results in some change
of Zener current; variable supplies are difficult because the Zener
cannot be tapped down, as a very-low-resistance potentiometer
would be needed and if the feedback resistors are varied, minimum
output is \( V_z \), not zero; positive feedback is more difficult to add.

The first snag has been tackled by some manufacturers, who
supply a matched Zener-transistor reference amplifier.

On the whole, the single-transistor version tends to be used in
fixed supplies unless performance is to be exceptional.

**Loop stability**

Since a stabilizer is a feedback circuit the problem of loop stability
exists. The principles of controlling loop response were given in
Chapter 8.

The designer must assume that the output load may be capacitive
to any degree and so the only safe method to cure loop oscillation is
to add output capacitance to make the output load the major time
constant. Additional load capacity will then improve stability. In
many applications the a.c. performance of the loop is unimportant
and the simplest techniques may be used. If the loop has to have
good h.f. response it must be treated as a wide-band feedback
amplifier as in Chapter 8.

**SUMMARY**

Directly coupled amplifiers are designed by assuming the desired
conditions have been achieved and the values chosen accordingly.
Feedback circuits incorporating such amplifiers are equally simple
to design, the example chosen being the voltage stabilizer.

**CHOPPER AMPLIFIERS**

When amplifying a slowly moving signal such as the output of a
thermocouple by means of a conventional direct-coupled amplifier,
slow drift in the amplifier caused by ageing or ambient temperature changes are indistinguishable from the signal. Although precautions can be taken to keep amplifier drift small, some drift inevitably takes place and this sets a lower limit to the input signal level which can be measured to a given accuracy.

If, however, the input signal is treated before entering the amplifier in such a way that it can always be distinguished from other signals not so treated, then amplifier drift becomes unimportant. (Strictly, it is the ‘zero drift’ which becomes unimportant; ‘gain drift’ is still as significant as before, but this causes a percentage rather than a fixed error so that it imposes no lower limit to the useful signal level.)

One system using this principle is the ‘chopper amplifier’, which in its simplest form consists of a vibrating switch $S_2$ which alternately connects and disconnects a short-circuit across the signal; often a series resistor $R_1$ is added where a direct short-circuit could cause excessive current flow (see Fig. 9.13).

![Fig. 9.13 Chopper amplifier block diagram](image)

In this way the input $V_{in}$, which is assumed to move very slowly compared with the switch period ($T_1 + T_2$), appears at $V_1$ in the form of a square wave which moves between zero and $V_{in}$ alternately. If $V_1$ is now a.c. coupled to an a.c. amplifier, the signal is always recognizable as the peak-to-peak amplitude of the square wave, any slow drift of the mean level in the amplifier being insignificant.

After amplification to a level $AV_{in}$ sufficient for direct monitoring, the signal can now be recovered in its original form (but amplified) by simple diode peak rectification which produces an output of $(1/2)AV_{in}$. This method has the fault that input signals of either polarity always produce the same output polarity, which depends only on the polarity of the rectifier diode connections.

Usually it is desirable to reproduce the input polarity, in which case a ‘synchronous rectifier’ is used. This is $S_2$ in Fig. 9.13 and is driven in synchronism with $S_1$, so that with the positive $V_{in}$ assumed in Fig. 9.13, $S_2$ closes during the more positive ‘half’-cycle of $V_{in}$. $C_1$ therefore acquires an extra charge corresponding to $AV_{in}$ (the left terminal being the more positive), compared with the level it would carry if $S_2$ did not exist. When $S_2$ opens, $V_2$ simultaneously falls by $AV_{in}$, giving the waveform $V_3$ shown in Fig. 9.14.

Hence, $V_3$ swings from zero to $-AV_{in}$ and the average value of $V_3$, obtained by the smoothing circuit $C_2R_3$, is

$$-A \frac{T_1}{T_1 + T_2} V_{in}$$

Knowing $T_1/T_2$ and $A$, $V_{in}$ is therefore known in magnitude and sign.

Note that if $S_2$ is operated in the opposite sense to $S_1$ (i.e. closed during $T_1$), the output polarity reverses and is positive for positive inputs; the same result applies if the amplifier has a gain $+A$ instead of $-A$.

The synchronous rectifier possesses the valuable property of being insensitive to signals which are unrelated in frequency to its own switching frequency. The easiest way to understand this is to assume that no normal signal is present and that only random noise is coupled to $S_2$ through $C_{in}$. When $S_2$ closes, $C_{in}$ is being charged and discharged randomly, so that when $S_2$ re-opens the output begins at zero and follows the amplifier variations until $S_2$ closes again. Since
the noise has zero mean level, the integration of the output over many
openings of $S_2$ yields zero output from the smoothing circuit. This
applies however large the noise is, provided the amplifier does not
change the form of the noise, e.g. by cutting off, and that $S_2$ works
as a perfect switch.

Design Problems

Apart from the switches $S_1$ and $S_2$, which will be dealt with later,
a number of problems arise which are of a somewhat unusual nature.
The first of these concerns the input impedance of the a.c. ampli-
fier, which is represented in Fig. 9.13 as a resistor $R_m$. It is obvious
that this causes a reduction in overall gain, because after $S_1$ opens
$V_1$ cannot rise to $V_{in}$, as assumed previously, but only, it would
appear, to $(V_{in} R_m)/(R_1 + R_m)$. One might expect therefore that the

![Diagram](image)

**Fig. 9.15** Chopper input circuit

overall gain would be reduced in this proportion and that if $R_m = R_1$
the gain would be halved.

This very natural conclusion is, in fact, quite wrong and, as will
be shown, if $R_m = R_1$ and $T_1 = T_2$ the gain is reduced by a factor
of $\frac{3}{4}$, not $\frac{1}{2}$.

The method of calculation follows the principle given in Chapter 1:
the charge and discharge of $C_{tin}$ per cycle are calculated and assumed
equal, which must be so when equilibrium is reached. $C_{tin}$ is assumed
very large ($C_{tin} R_m \gg T_1 + T_2$), so that the a.c. waveform across $C_{tin}$
is negligible and it is assumed to have, at equilibrium, a steady voltage
$V_{Ctin}$ between its plates.

When $S_1$ is closed, $C_{tin}$ discharges through $R_m$ and since $V_{Ctin}$
changes negligibly during $T_2$ (i.e. no a.c. waveform on $C_{tin}$) the
current, assumed flowing from left to right, is constant at $-V_{Ctin}/R_m$
(see Fig. 9.15 (a)). Charge flow in the direction shown is therefore
$(-V_{Ctin}/R_m) T_2$.

When $S_1$ is open, the current is given by $(V_{in} - V_{Ctin})/(R_1 + R_m)$
in the direction shown (see Fig. 9.15 (b)) and charge flow is therefore
$T_1 V_{Ctin}/(R_1 + R_m)$.

Since both $i_C(t_1)$ and $i_C(t_2)$ were assumed to be in the same direc-
tion, the sum of these charges must be zero.

Hence,

$$(-V_{Ctin}/R_m) T_2 + T_1 (V_{in} - V_{Ctin})/(R_1 + R_m) = 0$$

i.e.

$$V_{Ctin} = \frac{V_{in}}{1 + (T_2/T_1)(1 + R_1/R_m)}$$

To calculate the peak-to-peak output across $R_m$, Kirchhoff's
second law is used to find $V_{hin}$ in each position of $S_1$, regarding $C_{tin}$
as a battery of e.m.f. $V_{Ctin}$ in the direction shown. When $S_1$ is closed
(i.e. during $T_2$) the voltage across $R_m$ is $-V_{Ctin}$ and when $S_1$ is open
(during $T_1$) the voltage across $R_m$ is

$$(V_{in} - V_{Ctin}) R_m$$

$$(R_1 + R_m)$$

The peak-to-peak voltage across $R_m$ is therefore

$$V_{Ctin} + (V_{in} - V_{Ctin}) R_m, \quad R_1 + R_m$$

which reduces to

$$V_{hin} = \frac{1 + T_2/T_1}{1 + T_2/T_1 (1 + R_1/R_m)}$$

In the special case where $T_1 = T_2$,

$$V_{hin} = \frac{2}{2 + R_m/R_1} V_{in}$$

so that if $R_1 = R_m$, $V_{hin} = \frac{3}{2} V_{in}$.

Note also that if $T_1 \gg T_2$, $V_{hin} \approx V_{in}$ provided $R_1 \gg R_m$, and
if $T_1 < T_2$, $V_{hin} \approx V_{in}/(1 + R_1/R_m)$, an attenuation equivalent to a
direct loading of $R_m$ on $R_1$.

A second difficulty appears in connection with the synchronous
rectifier. In the description of its action it was assumed that the
capacitor, when connected to earth by $S_2$, would become charged
to the signal level at the amplifier output within the closure time of
$S_2$. In practice this does not always occur, since the current required
to charge $C_1$ in this time may be more than the amplifier can supply.
In this case $C_1$ becomes only partly charged and on successive cycles
receives more charge until finally the correct level is reached. This represents a delay between the application of an input signal and the obtaining of the final output and is additional to the delay in the smoothing circuit which follows \( S_a \).

In designing the output stage it is advisable to avoid this effect. The delay in itself may be tolerable and is calculable if the maximum current capability \( I_{\text{max}} \) of the output stage is known (from \( I_{\text{max}} = C_1 \Delta V/\Delta t \)), but in most cases the value of \( I_{\text{max}} \) differs according to the direction in which \( C_1 \) is being charged. This gives unequal response times for rising and falling input signals which can cause instability if used in a servo loop (a common use for a d.c. amplifier). A more subtle consequence is that when the output stage is near cut-off, just before \( I_{\text{max}} \) is reached, any spurious ripple or noise associated with the signal becomes rectified and gives an output error, thus nullifying one of the best features of a synchronous rather than a diode rectifier.

An alternative arrangement which makes the design of the output stage simpler consists in adding resistor \( R_3 \) in series with \( C_1 \). The advantage of avoiding high current demand from the amplifier almost always outweighs the longer delay for a given value of \( R_3 \) and \( C_2 \). In practice the longer delay is slight, is easy to calculate and is independent of output direction, provided the amplifier is designed to remain linear when the load of \( R_3 \) is switched in by \( S_a \).

**Chopper and synchronous rectifier switches**

\( S_1 \) and \( S_2 \) may be either mechanical or electronic switches. Generally, the former behave more like the ideal switch in giving virtually perfect open-circuits and short-circuits, but the life and switching frequency are limited. Electronic switches can be transistors which are alternately cut-off and saturated, or may be photoconductive cells which change from low to high resistance when illuminated by a flashing light source.

**Mechanical choppers**

When amplifying input signals in the microvolt region from a high impedance source of several megohms, the present-day semiconductor chopper is unsuitable and a mechanical chopper has to be used. These have been highly developed and are now of much greater reliability than the standard astable relay which was often used in the past.

Even so, these devices should be used only when dictated by the circuit specification, since they are expensive, have short life, and usually fail to operate under military vibration conditions. Usually the contact arrangement consists of one changeover switch so that to work as \( S_1 \) and \( S_2 \) the moving contact has to be earthed.

The power consumed by the driving coil depends on whether a resonant structure is used. For a resonant vibrator the drive is of relatively low power (100 mW) but the 'over and back' time is fixed regardless of drive period: this results in non-unity mark/space of chopping if the drive period differs from twice the resonance period or if temperature changes affect the resonance. The non-resonant type is free from this difficulty and can be driven at any frequency up to its top limit, but requires several times the drive power (2 W) and is more expensive.

Other design problems when using mechanical choppers are the need to protect the chopper unit from mechanical shock and the need to screen the input chopper connections (which are close to the high-level synchronous rectifier leads) and eliminate earth currents in the lead to the common contact. It is also highly desirable to operate the chopper at a frequency unrelated to any mains fields which might enter the a.c. amplifier and be treated as a chopped signal, so that the use of the mains supply for the drive is questionable.

In some applications it is important that the signal earth should be 'floating' relative to the amplifier earth, and this can be done by using an input transformer. Specialists in mechanical choppers will supply complete chopper/transformer units, doubly screened to avoid undue stray pick-up.

**Transistor choppers**

As explained in Chapter 3, the alloy or planar epitaxial transistor can be operated as a switch by alternate cut-off and saturation. The performance of a transistor switch is not perfect, and can be represented approximately as a battery \( V_{\text{eout}} \) when saturated (Fig. 9.16(b)), and as a source of current \( I_{\text{eoff}} \) when cut off (Fig. 9.16(c)), if used in the optimum configuration (Fig. 9.16(a)).

Assuming that \( V_{\text{eout}} \) may double for a 50°C rise from perhaps 3 to 6 mV, and that \( I_{\text{eoff}} \) may be negligible at 0°C and 1 nA at 50°C, the errors caused by these imperfections are easily calculated. If \( E_{\text{en}} \) were zero, \( e_{\text{out}} \) ought to be zero also, but will be alternately \( V_{\text{eout}} \)
and $R_e I_{ebo}$. The sign of $V_{c(eonut)}$ is negative and $R_e I_{ebo}$ is positive, so that the resulting waveform has an amplitude of $(V_{c(eonut)} + R_e I_{ebo})$ peak-to-peak. This is equivalent to an unwanted input of $(V_{c(eonut)} + R_e I_{ebo})$ and this is therefore the drift caused by the chopper referred to the input. It is often referred to as the 'pedestal' of the chopper, since the input signal sits on top of it.

![Figure 9.16 Transistor chopper equivalent circuits](image)

To minimize this figure, $R_e$ must be as small as possible and a transistor with low $I_{ebo}$ specified. When only low-level input signals will be present, the effect of $I_{ebo}$ can be reduced, it would appear, to zero by driving the base only to collector potential (Fig. 9.17).

In this circuit $D_1$ cuts off when the drive goes positive, so that $T_1$ base remains at zero potential. Provided $E_{in}$ is no larger than a few tens of millivolts in the positive direction, $T_1$ is cut off (or at least presents a high resistance); and since there is no reverse bias from base to emitter, no $I_{ebo}$ can flow. This is an over-simplification, because transients caused by hole storage in $T_1$ and $D_1$, and by transistor and diode capacitance, momentarily reverse $T_1$ base-emitter during which time $I_{ebo}$ flows.

One idea to reduce the effect of $V_{c(eonut)}$ is to introduce some of the drive waveform into the collector circuit of the chopper transistor.

![Figure 9.17 Circuit to reduce effect of $I_{ebo}$](image)

This can be done by transformer coupling or by resistive injection from an inverted version of the drive. These methods do not help temperature drift of $V_{c(eonut)}$, so that where temperature changes of 50 degC are likely, the total zero offset at one temperature extreme would normally be halved. A more useful compensation method for use under these conditions is to back-off the $V_{c(eonut)}$ of two identical transistors driven simultaneously (Fig. 9.18). In the circuit illustrated, a differential amplifier such as an emitter-coupled pair must be used to amplify the difference signal from $T_1$ and $T_2$ emitter. This may be overcome by transformer-coupling the drive (Fig. 9.19); if the transistors are matched for $\beta$ and $V_{c(eonut)}$, the emitter 1 to emitter 2 voltage at saturation can be less than 100 $\mu$V over a 50 degC temperature range. Suitably matched transistors are available from several manufacturers, and in some cases they are encapsulated with a transformer and called solid-state choppers.

The circuit of Fig. 9.19 has further useful properties. Consider the state condition when no base drive is applied, and assume $T_2$ emitter is earthed and a potential is applied to $T_1$ emitter. If this signal is very small (a few millivolts) then no forward current or reverse current flows. If the signal is large and positive, $T_1$ conducts,
causing $T_3$ base and collector to move positively. This cuts off $T_3$, since its emitter–base junction is reverse-biased and its collector and base are at the same potential, so that $I_{ebo}$ flows from the signal source. Conversely, if the signal is negative, $T_1$ cuts off and $I_{ebol}$ flows into the source.

The chopper is therefore open-circuit in the static condition for all signal voltages up to the $V_{dd}$ reverse breakdown level. This can be advantageous when the drive mark/space is designed to be small in such a direction that the short-circuit switch time is short, since the removal of zero frequency component by the transformer then causes the excursion in the cut-off direction to be very small.

**Photoconductive choppers**

A photoconductive element of, for example, cadmium sulphide has the property of increasing its conductance when illuminated,

![Photoconductive cell used as a chopper](image)

without generating internal voltages or currents. Such an element may be used as a chopper, provided its ratio of high/low resistance is sufficiently large and that its values of resistance are suitable for the signal source. The behaviour of this chopper can be represented as a perfect switch $S$ with series and parallel resistors $R_{re}$ and $R_p$ (Fig. 9.20). A lamp (tungsten or neon) is placed near the element and made to flash at the desired chopping rate, and the output signal is then chopped version of the input. The peak-to-peak output is less than the input signal by an easily calculated amount which depends on source resistance $R_s$, $R_{re}$, and $R_p$. This quantity varies considerably with ambient temperature and with lamp drive voltage but represents only a gain drift, not zero drift, so that overall feedback (as described later) masks the drift to any desired extent.

Zero drift would be non-existent if there really were no voltage generation in the element. Any asymmetry in its construction does, in fact, lead to some e.m.f., although this is only in the microvolt region even for quite inexpensive elements.

**Practical difficulties in the use of photo-choppers.** Two of the best features of transistor choppers over mechanical and photoconductive types are that they may be operated at a few kilocycles per second before transient effects ruin the performance. This implies that the overall bandwidth of the chopper amplifier can be a few tens of cycles after smoothing the output. The disadvantage of transistor chopping is the generation of unwanted e.m.f.s and currents.

The photo-chopper is more or less free from these defects but only the expensive materials have a fast response, cadmium sulphide being limited to about 100 Hz. As the drive frequency is increased, the element is unable to reach its high- and low-resistance values within the half-cycle, so that peak-to-peak chopper output falls and, since the waveform becomes non-rectangular, further loss occurs in the synchronous rectification.

Unless expensive elements (e.g. lead telluride) are used, the overall response is limited therefore to a few cycles per second. A further difficulty, directly related to speed of response, is that the chopped waveform lags several milliseconds behind the drive waveform. This occurs also in mechanical choppers but is much smaller (a few microseconds) in transistor choppers. If no correction is applied and the synchronous rectifier is driven by the same drive waveform, reduction of output results. Since the phase shift is not predictable and varies with lamp intensity (which changes by ageing), any compensation by extra phase shift in the drive is approximate only.

Another problem is the flashing light source, which may be a constantly lit source separated from the photo-element by a motor-driven rotating shutter, or may be a lamp driven on and off. The motor method has been used successfully in a range of commercial instruments. The flashing tungsten lamp has a limited life and a neon lamp (although spectrally suitable) requires higher voltage drive than is readily available in a transistor instrument.

Transformer coupling from the drive generator to obtain a high voltage requires a bulky transformer owing to the low drive frequency. One solution, devised by A. Errington (formerly of B.A.C., Stevenage), is to use the drive waveform to start and stop a high-frequency oscillator (100 kHz) the output of which is transformed up in voltage, rectified, and coupled to the neon lamp.

This last arrangement appears to be the best driving system, and although apparently complicated the cost and bulk of components is low and no special power supply is required.
Field effect transistor choppers

Field effect transistors behave in a similar manner to photo-conductive cells when used as choppers. The source-drain channel resistance switches from its normal value (between 10 Ω and 5 kΩ) to a very high value (several megohms) when the gate voltage is driven to cut-off. No c.m.f. is generated and gate-source leakage is very low.

Unlike the photo device, the isolation between drive and switch electrodes is not perfect due to gate-source and gate-drain capacitance which couples the drive waveform to the switch connections during each transition. Similarly ‘back-wash’ is coupled from the switching electrodes back to the gate. (In a photo-conductive cell it is naturally impossible for voltages present across the cell to modulate noticeably the brightness of the lamp.) Even though the isolation seems at first sight adequate (only a few picofarads) its presence gives rise to spike problems as in the transistor chopper. This is due to the large gate voltage swing needed to effect cut-off—usually 10 V more negative than both gate and source potentials for an n-channel depletion type such as the 2N3824.

Because of the spike problem the field effect transistor is often used as a modulator rather than a switch. This is achieved by driving the gate with a sine wave superimposed on a d.c. level in such a way that neither cut-off nor full conduction is reached. This attractive-sounding solution is not ideal since a special drive waveform is required, having a.c. and d.c. levels suitable for the particular field effect transistor sample. Another snag is that the resulting amplitude of the modulated input d.c. signal is very much less than that obtained by using the field effect transistor as a true switch. A more subtle problem is that the field effect transistor gate-source capacitance causes a greatly attenuated version of the gate waveform to appear across the source-drain terminals in quadrature to the main signal (in quadrature because it is coupled by a very small capacitance). If the output chopper drive is exactly in phase with the input chopper drive and if no phase shift occurs within the amplifier then this quadrature signal produces no output error from the synchronous rectifier, unless it is so large as to cause limiting. If, however, a phase error does exist then the quadrature ‘leakage’ contributes to the output in the same way as if an input d.c. were present. This point has been emphasized because generally a slight phase error has no significance in a chopper amplifier, merely resulting in a slight loss of gain. Consequently great care must be taken to check this point if it is proposed to modify an existing chopper amplifier by using a field effect transistor modulator.

For most applications the field effect transistor, used as a switch, should be considered if a transistor is inadequate due to its $V_{\text{equiv}}$. It is easier to drive than a photo-conductive element but is less convenient to drive and more expensive than a transistor.

Synchronous rectifier switches

The requirements here are less difficult to meet than in the input chopper. Signals levels are much higher, so that a switch 'pedestal' as high as several tens of millivolts is normally tolerable. The only extra problems arise in the use of a transistor switch where the base drive must exceed, in the cutting-off direction, the largest signal peaks, so that the series diode method for reducing $I_{\text{beo}}$ cannot be used.

If spurious signals are present which the synchronous rectifier is intended to ignore, then the base potential for cut-off must exceed the total possible signal peak (i.e. the sum of wanted signal and spurious signal); if not, the spurious signal becomes rectified and the resulting output is equivalent to an input error.

Overall Feedback in Chopper Amplifiers

Zero errors and zero drift in the input chopper cannot be improved by signal feedback, since no sensing element following the input chopping can distinguish between a true input with a perfect switch and zero input with an imperfect switch.

Feedback can be used with advantage in the reduction of the variations of gain, which occur when the mark/space changes, and also, in photoconductive choppers, if illumination changes. These changes apply also to the synchronous rectifier, and further overall gain changes occur due to variation in the gain of the a.c. amplifier and in its input impedance.

Whenever gain accuracy of better than a few per cent is required, it is therefore advisable to add overall feedback as shown in Fig. 9.21. As in any feedback system, the improvement obtained depends on the amount of loop gain, i.e. gain 'thrown away' in feedback. The designer must therefore assess the likely change in gain without feedback and deduce the amount of feedback required; then add the extra gain and apply the feedback remembering that the extra gain may also change.

Where the amplifier is to handle relatively large signals such as a
few volts, but is to have very high input impedance (hundreds of megohms) an alternative feedback connection can be made (Fig. 9.22). Here the output signal after rectification smoothing and attenuation by \( n \) to 1, is connected to the input chopper in such a way that the input switch oscillates between input and attenuated output.

![Diagram of chopper system with overall feedback](image)

**Fig. 9.21** Chopper system with overall feedback

With correct phasing the system settles in the state where the chopper switch output is very small and the output is \( n \) times the input to a degree of accuracy which again depends on the loop gain. If the gain from input chopper to smoothed output is \( A \), then the loop gain is \( \frac{A}{n} \) and the input impedance is of the order of \( R_{in} A/n \). Since \( R_{in} \) can easily be 1 M\( \Omega \) either by bootstrapping (Chapter 15) or by the use of low-current planar transistors, and \( A/n \) can be 1000, an overall input resistance of 1000 M\( \Omega \) with a gain accuracy of 0.1 per cent may be achieved. Zero drift is still present, so that the overall gain equation is of the form \( V_{out} = nV_{in}(1 \pm 0.1) \pm nV_0 \) where \( V_0 \) is the zero drift referred to the input.

This feedback system is particularly effective with photochopping, since \( V_0 \) is then negligible.

**Detailed Design of Chopper Amplifiers**

The problems described above are mainly concerned with the chopper system rather than individual circuits: the details of circuit design follow normal considerations.

To take a practical example, suppose the output of a thermocouple of 100 mV maximum is to be amplified by 20 with an accuracy of \( \pm 10 \) per cent, with permissible drift of 10 mV referred to the input. (This zero drift must always be specified, as well as percentage accuracy.) Source resistance is 100 \( \Omega \) and the source may be short-circuited without damage. Ambient temperature range is 0–50°C.

**Choice of switches.** The possibility of using a single-transistor chopper should be considered first, since this is the least expensive arrangement. The zero drift of a transistor chopper in the earthed collector configuration (Fig. 9.16) is given by \( V_{cesat.1} + I_{eb0}R_e \), where \( V_{cesat.1} \) is the value of \( V_{ee} \) for heavy saturation with zero emitter current, and \( I_{eb0} \) is the reverse emitter leakage at 50°C.

Manufacturers seldom quote the value of \( V_{cesat.1} \) under the conditions mentioned above: they quote a value of perhaps 0.1 V for 10 mA \( I_e \) and 1 mA \( I_b \) in the earthed emitter configuration. This is useful in designing multivibrators and binary circuits but gives no indication of chopper performance. Fortunately, a very simple static test can be made to convince the reader that the quoted 5 mV figure is correct. Using the circuit of Fig. 9.23, \( V_{cesat.1} \) can be measured directly at several base currents, showing that \( I_b \) is not critical for a \( V_{cesat.1} \) of 5 mV. Generally, a high-\( \beta \) transistor has lower \( V_{cesat.1} \), provided it is of epitaxial or alloy construction.

Returning to the example, \( I_{eb0}R_e \) is only 3 mV, so that a normal-drive waveform may be used, i.e. no series diode need be added (Fig. 9.17). The synchronous rectifier switch can be another transistor operated in the same mode. It is likely that the resistance in its emitter circuit will be higher than 100 \( \Omega \), so that its \( I_{eb0} \), will cause much more than 3 mV drift; however, the significance of any drift will be reduced by a factor of 20, so the rectifier may contribute 60 mV or more to the drift and the design will still remain within specification. This point must be confirmed when the design is complete.
Drive waveform. The simplest arrangement is a conventional multivibrator of unity mark/space, and if the two switches are of the same type (e.g. $p$–$n$–$p$) and are driven from opposite collectors of the multivibrator, a signal inversion in the a.c. amplifier will give an overall gain inversion.

A possible circuit is given in Fig. 9.24, where a ‘standard’ symmetrical free-running multivibrator is operated between $+5$ and $-10$ V supplies, giving a total period of $1.4 CR = 1.4 \times 0.1 \times 18 \times 10^{-3} \approx 2.5$ msec. The coupling resistor to $T_3$ of $4.7 \, \text{k} \Omega$ gives a base current of $2 \, \text{mA}$, which is adequate for good saturation at the maximum input current ($100 \, \text{mV}/100 \, \Omega$, i.e. 1 mA). The value of $R_8$ which supplies $S_0(T_4)$ cannot be determined until $R_9$ and $R_9$ are fixed.

Synchronous rectifier components. These are so chosen that $R_9$ does not unduly load $R_8$, giving direct loss of gain; at the same time, $R_8$ cannot be so large that significant error is caused by the leakage of any emitter follower which may be added at the output; also, $R_8$ cannot be so low that closure of $T_4$ causes the amplifier to limit. The last condition is usually the most important, so it will be assumed that $10 \, \text{mA}$ is the maximum current which the amplifier can supply when $T_4$ closes. Since maximum peak signal is at least $2 \, \text{V}$ (to produce $2 \, \text{V}$ output), $T_4$ would have to pass $2/R_8 \, \text{A}$ if the input jumped to maximum within one switching period. This leads to a value for $R_8$ of about $200 \, \Omega$, and to allow for output losses which increase the amplifier signal, $R_8$ should be about $270 \, \Omega$.

Note that in this application this is an extremely conservative value, since in the steady state (i.e. fixed input) $T_4$ would pass only enough current to make up for the output drain $V_{out}/R_L$. This current would be $2V_{out}/R_L$ and is much less than $2/R_8$. One point in favour of using the conservative figure is that no further protection against excessive amplifier current from the destructive point of view need be taken and sudden application of maximum signal by a switched input connection cannot cause damage.

$C_4$ can now be designed on the basis that it must charge to the correct voltage within the closure time of $S_2$. The conservative value is given by assuming that to charge fully in $1.25$ msec. could occupy a time of $4C_4R_8$, i.e. $C_4 = \frac{1.25}{4 \times 270} \times 10^{-3} \approx 1 \, \mu\text{F}$.

During the open-circuit interval of $S_2$, $C_4$ must not discharge appreciably so that $(R_8 + R_9)C_4 \gg 1.25$ msec., i.e. $R_8 + R_9 \gg 1.25 \, \text{k} \Omega$, e.g. $12.5 \, \text{k} \Omega$, giving $R_9 \approx 12 \, \text{k} \Omega$.

If $R_L \gg 50 \, \text{k} \Omega$, which is likely if the load is a pen-recorder or voltmeter, the attenuation $R_9/R_L$ is small. If $R_L \ll 10 \, \text{k} \Omega$ then it is advisable to add a unity-gain amplifier between $R_9$ and $R_L$ (see Fig. 9.25). One emitter follower could be used, but its $V_{oc}$ variation of $125 \, \text{mV}$ from $0$ to $50^\circ \text{C}$ represents $6-25 \, \text{mV}$ zero drift and so two complementary transistors are preferable, as shown.

$C_5$ is now given by $C_5R_9 \gg$ switching period, i.e. $C_5R_9 \gg 2.5 \times 10^{-3}$ or $C_5 \gg 0.2 \, \mu\text{F}$, e.g. $50 \, \mu\text{F}$. This would have to be of the electrolytic type, but as its accuracy is unimportant and voltage rating less than $6 \, \text{V}$, this is acceptable.

Returning to the base drive for $T_4$, the maximum emitter current for $T_4$ is $10 \, \text{mA}$, so that a base drive of $2 \, \text{mA}$ would appear to be adequate, because this would cause near-saturation, though not to the millivolt level. When first turned on after an increase in signal
from zero to maximum, $T_4$ would therefore saturate to perhaps 100 mV and on the next closure the emitter current would be only 100 mV/270 or 0.4 mA. On this closure, saturation to a few millivolts would now occur.

This reasoning is, however, incorrect, as it fails to take into account the reversed emitter current which flows if the input signal is suddenly disconnected. In this situation 10 mA reverse current flows and if $\beta_L$ is $< 5$, $S_2$ does not close to saturation level. This gives an overall delay in amplifier response which differs according to whether the input is increasing or decreasing, an effect which $R_8$ was intended to prevent.

Two solutions are possible; either $T_4$ is a symmetrical transistor ($\beta_L \approx \beta > 20$) or the base drive is increased to $10/2 = 5$ mA. In either case the 'normal' saturation level is worse than for a very asymmetrical type with very small forward emitter current. In this design a 2S323 will be specified and the base drive increased by adding emitter follower $T_2$ and putting $R_6 = 2.2$ kΩ.

Temperature drift. Drift in the output circuitry is caused by $T_b$ base current flowing in $R_9$ so that $I_b R_9/20$ must not exceed a few millivolts in either polarity; if $|I_b| R_9/20 \leq 2 \times 10^{-9}$, then $|I_b| \leq 3.3 \mu$A. The leakage component alone for a germanium transistor at 50°C (30 μA) would give excessive drift, so that a silicon planar type 2N930 should be used, with an emitter current of no more than 100 μA, giving $R_{10} \approx 100$ kΩ. $T_6$ may be silicon alloy or planar type, e.g. OC202, 2S323, or 2N2906, etc., with emitter current of no more than 1 mA (so that base current $\leq 30 \mu$A), giving $R_{11} = 5.6$ kΩ.

Drift caused by the synchronous rectifier $T_4$ consists of its $V_{cet(sat.)}$ and its $I_{eb0} V_{cet(sat.)}$ when divided by the gain of 20 will be negligible for any transistor suitable for chopping. $I_{eb0}$ is more difficult to assess: when $S_3$ opens, $I_{eb0}$ flows into $R_8$ in parallel with $C_4$ and $R_8$. If $C_4$ were infinite and $R_8$ were zero, no error would be caused, but in fact an immediate step of $I_{eb0}(R_8/R_8)$ occurs followed by a positive rise as $I_{eb0}$ charges $C_4$.

If $I_{eb0}$ is 1 μA, then the step error is $(12 kΩ/270) \times 10^{-6} = 0.16$ mV, or 9 μV referred to the input. By the end of the open period for $S_3$, $C_4$ has charged by $(I_{eb0}T/2)/C_4$ which is

$$\frac{10^{-6} \times 2.5 \times 10^{-3}}{10^{-6} \times 2} = 1.2 \text{ mV}$$

or 60 μV referred to the input. The sum of errors is tolerable, so that $T_4$ may be a 2S323 transistor.

**a.c. Amplifier**. The circuits which are peculiar to the chopper have been dealt with and it only remains to specify the a.c. amplifier, which may then be designed using conventional techniques.

The required a.c. gain depends on whether overall feedback is to be used and this depends on the conditions of operation and installation. There are several causes of error: for example, mark/space of chopping, where the circuit is reasonably stable against ageing and temperature variation but where the absolute conditions have a wide spread. In such cases adjustment could be provided (such as one base resistor in the drive multivibrator or a gain control within the amplifier) to set this right in the factory. On the other hand, this involves extra labour and may be more costly than incorporating feedback which, especially in the rather wide gain tolerance permitted here (± 10 per cent) removes the need for any adjustment.

If feedback is to be incorporated, the expected maximum errors which would occur without feedback must be assessed. Experience helps to obtain the right order of tolerance and if in this example a variation of ± 50 per cent in the overall gain, including the amplifier, is assumed, this will be a conservative estimate. This assumes that components which directly affect the gain are of ± 5 per cent tolerance: this includes multivibrator resistors, $R_3$ and $R_4$, and presupposes that the a.c. input resistance of the amplifier is to be much greater than $R_a$.

Given this expected variation, a gain of 5 must be 'thrown away' in feedback to convert the ± 50 per cent into ± 10 per cent. Amplifier gain must therefore be $20 \times 5 \times$ (chopper losses); chopper losses include a factor of 2 in synchronous rectification and further small losses such as $R_3$: $R_4$, loss in $T_3$ and $T_4$, input resistance of the amplifier loading $R_a$, and non-unity mark/space. These would amount to no more than 30 per cent with reasonable design, giving an amplifier gain requirement of 130.

Input resistance must be much greater than $R_a$, e.g. 1 kΩ, and the output resistance must be much less than $R_a$, e.g. 20 Ω, or alternatively included in $R_a$. Output current capability must be greater than 10 mA.

Bandwidth must be adequate to pass a square wave of 2.5 m sec period without great deterioration, e.g. 50 Hz-3 kHz. Design can now proceed as described in Chapter 7 using two earthed emitter
stages followed by one or two emitter followers, the final one having a standing current in excess of 10 mA. This gives adequate gain and no overall signal inversion. The lower 3 dB cut-off frequency should be about 50 Hz.

**FEEDBACK CONSIDERATIONS**

Negative feedback is added by connecting RF from final output to the input chopper $S_1$, the value being $20 \times R_s$, i.e. 2 kΩ. Since the loop gain is about 5, the system will be free from h.f. oscillations if the ratio of the two major lagging time constants is greater than 5. The main lags are $R_sC_5$, i.e. $12 \times 10^3 \times 50 \times 10^{-6} = 0.6$ sec, and one period of chopping, i.e. 2.5 msec (since no mean output change could occur faster than this even if $R_sC_5$ did not exist). The ratio is 240:1, so the system is stable.

**SUMMARY**

Design of a chopper amplifier is straightforward provided the principles are understood; the system can then be divided into individual circuits each designable by techniques discussed in earlier chapters. There is a large amount of choice in the design of every circuit involved; this gives the designer a good opportunity to test his judgement.
10—Complementary circuits

Complementary circuits' is the name generally given to those circuits which employ both n-p-n and p-n-p transistors in such a way as to exploit their opposite bias polarities.

Very often in 'standard' circuits using two n-p-n or two p-n-p transistors, one transistor can be changed for its opposite type. The result is a circuit which in many examples is more economical in components, and often a performance improvement is also obtained.

Other complementary circuits are unique, not being derived from any 'normal' circuit.

The examples which follow are by no means exhaustive but are intended to assist the designer in inventing his own circuits and variations on the ones discussed.

It is often profitable when examining critically a newly completed design to consider the effect and possible improvement which would follow a change of any transistor in the circuit for its complementary version.

COMPLEMENTARY BISTABLE NO. 1

This circuit is derived from the two-state circuit given on page 69 and reproduced below.

If we replace $T_2$ in Fig. 10.1 by a p-n-p transistor, remembering to reverse the polarities applied to collector and base relative to emitter, the circuit of Fig. 10.2 is obtained. As in the original circuit, this arrangement has two stable states.

Assume, for example, $T_1$ is 'on' and saturated. Then its collector is near zero potential, so that the current in $R_6$ turns on $T_2$ to saturation (with correct design). The saturation of $T_2$ brings its collector potential to zero, so that current in $R_1$ maintains saturation of $T_1$. 

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This is therefore a stable state. Now assume $T_1$ is cut off, its collector potential being highly positive. With correct values the base of $T_2$ is now positive, cutting off $T_2$. Its collector now falls negative, thus cutting off $T_1$. This is also, therefore, a stable condition.

The derived circuit has two stable states, as did the original version; the changes in polarity, however, result in several important differences which are discussed below.

**Properties**

First, instead of each transistor being ‘off’ while its partner is ‘on’, both are ‘off’ or ‘on’ together.

Secondly, instead of each output alternately and oppositely going from earth to positive (or negative if $p-n-p$ types are used), both outputs are near earth in one state, and in the other state one is highly positive, the other highly negative.

Thirdly, the output waveforms of the two outputs of the original circuit each have a sharp fall and slow rise (the opposite for $p-n-p$ types), whereas the complementary circuit has from $T_1$ a sharp fall, slow rise, and from $T_2$ a slow fall and sharp rise (Fig. 10.3).

Fourthly, the original circuit always requires a negative base drive to turn off either ‘on’ transistor, and the new circuit requires a negative drive on $T_1$ base to turn off $T_1$ or a positive drive on $T_2$ base to turn off $T_2$. 

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**Fig. 10.1** Conventional two-state circuit

**Fig. 10.2** Complementary bistable No. 1

**Fig. 10.3 (a) Waveforms for conventional two-state circuit (Fig. 10.1)**

**Fig. 10.3 (b) Waveforms for complementary bistable No. 1 (Fig. 10.2)**
There are many consequences of the above properties.
The first result can be very useful where the circuit normally stands in its 'off state' with very low drain on the supply, yet, when turned on, delivers heavy current to two separate loads, each saturated collector-emitter path. This same action also has its snags, since to turn off such a circuit two heavily saturated transistors have to be cut-off, which requires roughly twice the drive normally required to turn off the 'on' transistor in a normal two-state circuit. Moreover, the drain on the supply varies greatly in the complementary circuit according to the state of the circuit; in the original circuit supply drain is constant except during the transition between states.

The second action of producing opposite polarity outputs is most useful and would probably be the main reason for using this circuit. Many additional components would be required to achieve this result in other ways, and the simultaneity of the output waveforms would generally not be so exact.

The third effect, whereby simultaneous fast edges of opposite polarity are generated, is another valuable feature, enabling, by differentiation, fast positive- and negative-going pulses to be obtained, a frequent requirement in logic circuitry.

The fourth characteristic of differing turn-off drives is merely different from normal and may be useful or a nuisance according to circumstances.

COMPLEMENTARY BISTABLE NO. 2

This complementary bistable is not derived from a conventional bistable but uses direct collector-base mutual connections between the two transistors (see Fig. 10.4).

If $T_1$ is assumed to be cut off, then its collector current will be small ($I_{cbo1}$). Provided $(I_{cbo1} + I_{cbo2})R_2$ is too small to turn on $T_2$, then $T_2$ also is cut off, its collector current having the value $I_{cbo2}$. Provided $(I_{cbo2} + I_{cbo1})R_1$ is insufficient to turn on $T_1$, then $T_1$ is cut off, as originally assumed, confirming that a stable state exists where $T_1$ and $T_2$ are both cut off and $V_{out}$ is near zero volts ($I_{cbo1}R_2$).

On the other hand, if $T_1$ is conducting such that $I_{cbo1}R_2$ turns on $T_2$, and if the current in $T_2$ collector is such that $I_{cbo2}R_1$ turns on $T_1$ further, equilibrium is reached when $T_1$ or $T_2$ saturates, i.e. a second stable state exists with both transistors conducting. $V_{out}$ is now almost equal to $V_p$.

PROPERTIES

Since this circuit, like the previous example, has two transistors which turn off and on simultaneously, some of the points still apply; for instance, low quiescent current in the 'off' state, and the possibility of using either polarity of trigger for either changeover.

The main feature, however, is the ability of the circuit to supply heavy loads with low transistor dissipation. This comes about because at least one transistor is saturated and the other very nearly so, and within limits a heavier load current results in still heavier available drive from $T_1$, since $T_1$ itself receives more base drive from the load current passing through $T_2$. This useful effect applies until eventually with very low values of $R_L$ the load current is so great that the $p_b$ of $T_1$ and $T_2$ fall. Ultimately, saturation is no longer maintained and transistor dissipation rises causing catastrophic failure.

COMPLEMENTARY EMITTER FOLLOWERS

The emitter follower is one of the most widely used circuit configurations, and is particularly helpful in isolating a voltage amplifier from its load.

The isolation produced by a single emitter follower is, however, very often insufficient, and it is common practice in such a case to use two emitter followers in cascade (see Fig. 10.5).

Although this circuit is usually quite satisfactory, there are certain conditions, for instance, in d.c. amplifiers, where the input-to-output d.c. voltage drop, now equal to two base-emitter voltage drops, constitutes a serious disadvantage. The amount of this combined
voltage drop can be as high as 2 V and the associated temperature coefficient will lie between 4 and 5 mV/degC.

An alternative arrangement which does not have this fault, and has another important advantage, is shown in Fig. 10.6 (a) and (b).

![Cascaded emitter followers](image)

**Fig. 10.5** Cascaded emitter followers

In the simple case where $R_2$ is absent, $T_1$ collector current and $T_2$ base current are equal. Hence, $T_1$ emitter current is $1/\beta_2$ of $T_2$ collector current, implying that most of the supply current $I$ flows into $T_2$ and $1/\beta_2$ of it flows into $T_1$.

![Complementary emitter follower](image)

**Fig. 10.6** (a) Complementary emitter follower (1); (b) complementary emitter follower (2)

The d.c. input-to-output drop consists of the $V_{be}$ of $T_1$ only and calculation shows that in most respects the performance of the circuit is at least equal to that of Fig. 10.5.

The main disadvantage of the complementary version in practice is its liability to oscillation by the feedback loop $T_1$ emitter–collector, to $T_2$ base and collector, back to $T_1$ emitter. As in any feedback system, a total phase shift round the loop of 180 degrees will lead to oscillation if the loop gain then exceeds unity. It is shown in Chapter 8 that the most likely conditions to satisfy these criteria occur when the phase-shifting circuit elements each have similar phase-frequency characteristics.

Since $T_1$ operates, so far as the feedback loop is concerned, in grounded base, and $T_2$ is in grounded emitter, high-frequency oscillation is most likely to occur if the phase characteristic of $\alpha_1$ approximates to that of $\beta_2$, i.e. $f_{T_1}$ equal to $f_{T_2}/\beta_2$.

Using transistors of similar $f_T$ is therefore marginally safe, but it is preferable if $f_{T_1}$ is much greater than $f_{T_2}$.

As in many circuits where direct collector-base coupling is used, the addition of $R_2$ is desirable. $R_2$ is designed so that $(I_{e01} + I_{e02})R_2$ is insufficient to turn on $T_2$. It can often be omitted if $T_1$ and $T_2$ are silicon.

This circuit has been used extensively in audio output stages since, in the form shown in Fig. 10.6 (b), the circuit as a whole behaves like a $n-p-n$ transistor (i.e. it tends to turn on when driven positive), yet the main output current passes through $T_2$, which is a $p-n-p$ type. In effect, therefore, a $n-p-n$ power transistor (in itself a rare and expensive device for commercial audio equipment) has been obtained from a low power $n-p-n$ and a power $p-n-p$ transistor (both less expensive).

In this application $T_2$ would often be a germanium power type and therefore $R_2$ would have to be very small to ensure correct high-temperature operation ($I_{e01}$ could be 10 mA at 50°C and $V_{eb}$ for turn-on could be 0.05 V, giving $R_2 \approx 5 \Omega$). Fortunately, high-temperature working is not normally important in these cases and a relaxation can be permitted. Some commercial designs are, however, question-able on this design point, and fail catastrophically even at 35°C.

The best arrangement is to return $R_2$ to a higher potential than $T_2$ emitter either by using an additional supply rail for $R_2$ or by adding diodes or a Zener diode in series with $T_2$ emitter (Fig. 10.7). The value for $R_2$ now becomes larger, since the voltage which $I_{e02}R_2$ must not exceed is not 0.05 V, but $(0.05 + V_1)$ where $V_1$ can be typically 1 to 4 V, giving $R_2$ a value from 20 to 80 times its value in Fig. 10.6 (b).

Another difficulty which arises in the design of emitter followers
occurs in stabilizer and power amplifier output circuits. Using the simple configuration of Fig. 10.5 to drive a heavy load requires the first transistor to carry $1/\beta_2$ of the load current, and a mean collector-emitter voltage which is the same as for $T_2$. Typical values for a transistor $\beta_2 = 2$, $V_{ce} = 15$ V; $T_2$ carries $I_2 = 2/\beta_2$ A, e.g. 80 mA, at 15 V, so that the power dissipated by $T_2$ is 1.2 W.

In such a case $T_1$ must therefore be more than a small-signal transistor, and, in fact, power transistors are often used for both $T_1$ and $T_2$. Apart from the obvious disadvantages of size and cost, a more subtle difficulty arises: the previous stage, probably the collector circuit of an amplifier, now has to drive $T_1$, the leakage current of which is likely to be 10 mA at an ambient of 50°C. Thus, yet another emitter follower may have to be used to drive $T_1$.

The circuit of Fig. 10.8 overcomes this problem. Here the current in $T_1$ is still $I_{ce}/\beta_2$, or 80 mA in the above example, but the $V_{ce}$ for $T_1$ is only the base-emitter drop of $T_2$, which may be 0.3 V. The power dissipated in $T_1$ is now only 24 mW and a low-power type having a maximum leakage current of perhaps 100 μA at 50°C, if germanium, or 5 μA, if silicon, can be used. Hence, the two snags outlined above for the normal circuit are overcome.

The difficulties which have now been solved are replaced in part by the problem of designing $R_1$ in Fig. 10.8. In order that $T_1$ should never cut off, $R_1$ must always pass a current greater than $I_{ce}/\beta_2$, under any condition of supply voltage. If $V_{out}$ is, for example, 40 V and $V_n$ varies from 50 to 60 V (quite normal for a stabilizer output circuit), then for a load current of 2 A and $\beta_2$ of 25, the current in $R_1$ must exceed 2/25 A, i.e. 80 mA when $V_n$ is at 50 V. $R_1$ must therefore be less than $(50 - 40)/20$ kΩ, i.e. 125 kΩ. At the other limit of $V_n$ (60 V) $R_1$ will therefore pass $(60 - 40)/125$ A, i.e. 160 mA.

This results in $T_1$ having to pass a maximum current of, in this example, twice the amount normally to be expected. Fortunately, power rather than collector current determines the physical size and therefore leakage currents for a transistor, so that in many instances this circuit is advantageous.

A second, less serious difficulty occurs when the emitter-base voltage drop of $T_2$ is insufficient for the type of transistor intended to be used for $T_1$. This is readily overcome by the addition of a diode in series with $T_2$ emitter, as shown in Fig. 10.9.

In compensation for these difficulties in fixing the operating conditions of $T_1$, the circuit has the property that temperature rise, which

![Fig. 10.7 Modified form of Fig. 10.6 for high-temperature operation](image)

![Fig. 10.8 Power-drive circuit](image)

![Fig. 10.9 Modified version of power-drive circuit (Fig. 10.8) to increase $V_{ce}$ for $T_1$](image)
above all causes the $I_{eb0}$ of $T_2$ to increase, tends to turn $T_1$ on rather than off.

**COMPLEMENTARY AMPLIFIERS**

Naturally, complete amplifiers may well contain a mixture of $p-n-p$ and $n-p-n$ transistors; the intention in this section is to deal with circuits where $p-n-p - n-p-n$ pairs are used to obtain a special advantage.

The compound emitter follower configuration already described by Fig. 10.6 can also be used as a voltage amplifier by regarding the emitter of the second transistor as the collector of a new transistor, as illustrated in Fig. 10.10. The ‘transistor’ thus synthesized has approximately the same $g_m$ as a single transistor run at the same current, but its apparent current gain is equal to $\beta_2\beta_1$.

This is the basis of a well-known differential amplifier first described by Bénétou and reproduced in its simplest form in Fig. 10.11. As is evident from the above comments on the compound transistor of Fig. 10.10, this circuit behaves like a normal emitter-coupled pair with high input impedance, as high as if emitter follower drivers were added but without the inconvenient $V_{be}$ drops which would then result. The further development of the circuit by replacing $R_E$ with a constant-current device and by coupling into a further similar stage results in exceptional performance with regard to gain and drift.

A particularly useful feature of the compound transistor is the ease with which a single-transistor stage can be changed into the compound form. The need for this can arise if the input resistance of a single stage has to be increased using the minimum of extra components and a simple emitter follower cannot be added because of its additional $V_{be}$ drop. This can occur following a modification to specification or from a previous design error. Fig. 10.12 illustrates the necessary reconnection.

Another method which exploits the interconnection shown in Fig. 10.6 to form an amplifier is shown in Fig. 10.13. This circuit is easily proved to have a gain of approximately $(R_1 + R_2)/R_3$. Transistor parameters have little influence on the gain, so that although the
input and output are in phase, the circuit behaves like a normal feedback amplifier having a loop gain of $\beta_2$.

Hence, the output impedance is $R_1/\beta_2$ and input impedance is $\beta_1\beta_2 R_2$. These are approximations which may be deduced from equivalent-circuit analysis (see Chapter 4).

**Complementary Differential Amplifier**

If one transistor of the emitter-coupled amplifier described in Chapter 4 is replaced by a complementary type, the circuit of Fig. 10.14 results.

![Complementary feedback amplifier](image)

**Fig. 10.13** Complementary feedback amplifier

As it stands, this will not operate in a linear manner, since both transistors are cut off. Returning $T_1$ base to a more positive potential than $T_2$ base ensures current flow but the collector current is then undefined. A simple method of ensuring correct biasing is shown in Fig. 10.15.

![Basic differential complementary amplifier](image)

**Fig. 10.14** Basic differential complementary amplifier

be true and the offsetting disadvantage is that inputs must be a.c. coupled unless the signal sources happen to sit at two suitable d.c. levels.

Design procedure is simple: $R_{1,2,3}$ are chosen so that the drop across $R_2$ is large compared with $V_{be}$ but not so large that $T_1$ or $T_2$ bottoms on large output swings. $R_e$ is chosen to give correct emitter current to supply the required load $R_{L1}$ and $R_{L2}$. For maximum available output $R_e$, $R_{L1}$, and $R_{L2}$ are chosen so that $V_{C1} (V_{C2})$ are halfway between $V_{B1(0)}$ and $V_{p(earth)}$. If only one output is required, the other load can be short-circuit without noticeably affecting 'push-push' rejection.

The choice of $C_e$ is dictated by the same considerations as in a normal earthed emitter amplifier, but care must be taken in a wide-
band amplifier that $C_e$ is made no larger, physically, than is necessary. This is important because at high frequencies the stray capacitance from this large component to earth causes degradation in the ‘push-push’ rejection.

Another design point is that $C_1$ and $C_2$ must be so large at the lowest operating frequency that each input signal reaches its transistor base circuit without appreciable loss. Clearly, this criterion would be applied as a matter of course in order to avoid loss of gain, but the issue is more critical than this. If $C_1$ and $C_2$ are not large enough and if $T_1$ and $T_2$ are not identical (a likely condition, since $n-p-n$ and $p-n-p$ types rarely match precisely), then unequal signals will reach $T_1$ and $T_2$ base, even if identical inputs are applied to Input 1 and Input 2.

If this amplifier were designed to be 3 dB down (at the low-frequency end) at 50 Hz and this falling response were achieved by allowing loss across $C_1$ and $C_2$ at this frequency, then push-push rejection of hum signals would be degraded very severely owing to unequal losses in $C_1$ and $C_2$. A better way of producing the fall-off would be to design $C_2$ sufficiently small for the purpose. This is still not recommended, however, since slight degradation is still caused (push-pull gain falls but push-push gain is unchanged), and the 3 dB point is badly defined (circuit is 3 dB down when

$$X_e = \frac{1}{g_{m1}} + \frac{1}{g_{m2}} + \frac{R_{e1}}{r_1} + \frac{R_{e2}}{r_2}/R_e$$

which depends greatly on transistor parameters).

The best procedure is to degrade the frequency response in a later stage where push-push rejection is no longer a problem. This is also the best procedure for the minimizing of circuit noise produced by $T_1$ and $T_2$.

The virtues of this circuit were brought to the author’s attention by John Murray of Marconi Instruments Ltd. in connection with a low-level transformerless balanced amplifier.

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11—Wide-range voltage-controlled oscillator

The frequency of either sine-wave or square-wave oscillators can be varied by changing the value of one of the frequency determining elements. When the frequency is to be varied over a wide range it is necessary in many circuits to vary two or more of these elements simultaneously in order, first, to achieve the required frequency swing and, secondly, to maintain constant amplitude at various frequencies.

When there is a requirement for a remotely controlled oscillator or where a feedback system calls for an oscillator whose frequency is to be controlled by a d.c. level (as in a frequency lock or a phase-lock loop) the normal methods of frequency control become unwieldy or impossible.

The voltage-controlled oscillator to be described is essentially a multivibrator using emitter circuit timing, so that the main output is a square wave, ideally suited to driving mixers. Because of the emitter circuit arrangement, however, a triangular waveform of constant amplitude is also available and this in turn can readily be shaped to a sine wave. The oscillator frequency is directly proportional to control-voltage over a wide range and the circuit has good temperature-stability.

**BASIC CIRCUIT**

This circuit has the unique property that an attempt to calculate component values without appreciating exactly how it operates usually results in a circuit which produces twice the expected output at all four output terminals for two quite different reasons.

To appreciate the operation of Fig. 11.1, assume that $I_1$ and $I_2$ are perfect constant-current sources and that $T_1$ is conducting and $T_2$ cut-off, i.e. $T_2$ emitter is negative with respect to $T_2$ base. Assume
further that $R_3$ and $R_4$ are much smaller than $R_{1,2,3,4,5}$, and that voltage excursions are such that bottoming does not occur.

At this time $T_2$ collector current is zero, so that $T_2$ collector voltage is near zero and $T_1$ base is $V_pR_0/(R_1 + R_0)$ positive. Since $T_1$ is conducting, $T_1$ emitter potential is just above $V_pR_0/(R_1 + R_0)$.

Consider now the current in $T_1$ emitter. It is not merely $I_1$, but $(I_1 + I_2)$, since, $T_2$ being cut-off, $I_3$ must be charging $C$ and adding to $I_1$ in $T_1$ emitter. Hence, $T_1$ collector current is $(I_1 + I_2)$ and $T_1$ collector potential is $(I_1 + I_2)R_3$ volts positive.

![Fig. 11.1 Basic circuit of emitter-coupled multivibrator](image)

A simple Ohm's law calculation shows that $T_2$ base potential is $V_pR_0/(R_5 + R_0) + (I_1 + I_2)R_3R_5/(R_5 + R_0)$ volts positive.

These are therefore the conditions which exist whenever $T_2$ is cut off, and similar expressions apply for $T_1$ cut-off (see Fig. 11.2).

Continuing the train of events, $C$ is being charged by $I_2$ and since its $T_1$ emitter connection is fixed, just positive to $T_1$ base, $T_2$ emitter rises linearly at a rate of $I_2/C$ volts/sec. (since $Q_C = I_2t = CV$). When $T_2$ emitter becomes just positive to $T_2$ base, $T_2$ begins to conduct, causing $T_2$ collector and, hence, $T_1$ base to rise sharply. Since $T_2$ is conducting, $T_2$ emitter cannot rise and, hence, $T_1$ emitter cannot follow the sharp rise of $T_1$ base, because $C$ would have to charge instantly. $T_1$ therefore cuts off.

As soon as $T_1$ cuts off, its collector potential falls to zero, a change of $(I_1 + I_2)R_3$ volts, and $T_2$ base therefore falls by $(I_1 + I_2)R_3R_5/(R_5 + R_0)$, causing $T_2$ emitter and because of $C$, $T_1$ emitter to fall by the same amount.

![Fig. 11.2 Waveforms for circuit of emitter-coupled multivibrator (Fig. 11.1). Magnitudes are approximate and assume $R_5 \ll (R_5 + R_0)$, $R_4 \ll (R_1 + R_2)$](image)

Also, when $T_1$ cuts off, the current $(I_1 + I_2)$ now flows into $T_2$, raising its collector by $(I_1 + I_2)R_4$ and therefore raising $T_1$ base by $(I_1 + I_2)R_4/(R_1 + R_2)$.

Hence, immediately $T_1$ cuts off, its emitter and base potentials (which were virtually equal during the conduction of $T_1$) have moved in opposite directions by the amounts given and at the end of the transition differ by a voltage of

$$(I_1 + I_2)R_3R_5/(R_5 + R_0) + (I_1 + I_2)R_4R_1/(R_1 + R_2)$$
C now charges linearly from \( I_1 \) at a constant rate of \( I_1/C \) V/sec until \( T_1 \) emitter potential again becomes just positive to \( T_1 \) base. Since neither base potential changes during this charging period \( C \) has to charge by the voltage quoted above.

Thus the amplitude of the emitter waveform with \( T_1 \) cut-off is
\[
(\text{error}) = \frac{(I_1 + I_2)R_3R_5}{R_3 + R_6} + \frac{(I_1 + I_2)R_4R_5}{R_1 + R_2}
\]
and the time for which \( T_1 \) remains cut-off is
\[
C(R_3R_5/R_3 + R_6) + R_4C_1(R_1 + R_2)(I_1 + I_2)/I_1
\]
Similarly, the amplitude for \( T_2 \) cut-off is the same and the time is
\[
C(R_3R_5/R_3 + R_6) + R_4C_1(R_1 + R_2)(I_1 + I_2)/I_2
\]

Special Case

The above expressions apply for any values which are in accordance with the assumptions made. In practice they are approximate because \( R_4 \) is not negligible compared with \( R_1 \) and \( R_5 \), but having appreciated the function of the circuit the designer can readily modify the calculations by writing for \( R_4 \) the parallel combination of \( R_3 \) and \( R_5 \), plus \( R_6 \), a substitution very simply done arithmetically but cumbersome algebraically.

Careful examination of the results reveals that for a symmetrical design where \( R_3 = R_4, R_1 = R_5, R_2 = R_6 \) and \( I_1 = I_2 = I \), the voltage sweep on \( T_1 \) or \( T_2 \) emitter is \( 4IR_3R_5/(R_1 + R_2) \), the half-cycle time is \( 4C_1R_3R_5/(R_1 + R_2) \), and the output voltage swing from \( T_1 \) or \( T_2 \) collector is from zero to \( 2IR_3 \).

The most striking feature of the above result is that the half-cycle time is independent of the value of \( I \), yet this is the current which directly determines the charging rate of \( C \). The reason is that \( I \) also determines the voltage by which \( C \) must charge, and since both this and charging rate are linear functions of \( I \) the charging time for \( C \) remains constant.

Two Common Difficulties

There are two difficult points in the action of the circuit which can lead if misunderstood to expecting half the output voltage actually obtained.

The first concerns the emitter current of the 'on' transistor, which, as pointed out, is \( (I_1 + I_2) \), not merely \( I_1 \) or \( I_2 \). If this is difficult to accept, consider the moment when both transistors are conducting, a condition which exists transiently during the switching-over at the end of each period. Now, assume one or other transistor, e.g. \( T_2 \), cuts-off because its base is driven suddenly positive. It is clear that \( T_2 \) emitter cannot follow a sudden change, since an enormous current would be required to charge \( C \) rapidly and its \( T_2 \) end is anchored to \( T_1 \) emitter. Hence, \( T_2 \) emitter suddenly appears to become open-circuit. \( I_2 \) continues to flow and its only path is through \( C \) into \( T_1 \), adding to \( I_1 \). Even though \( C \) now charges, \( I_2 \) remains constant, and the current in \( T_1 \) remains equal to \( (I_1 + I_2) \) until the end of that period.

If this point is not appreciated and \( I_1 \) equals \( I_2 \), the output voltage of \( 2IR_3 \) is twice the (wrongly) expected \( I_1R_3 \).

The second point which can easily be missed even though the first is understood, arises when considering the emitter waveforms. Taking the case when \( T_2 \) is off because its emitter is more negative than its base, \( T_2 \) emitter is rising linearly because \( I_2 \) is charging \( C \) and reaches the level when \( T_2 \) conducts.

Two things now happen. First, because \( T_2 \) conducts, \( T_1 \) base potential rises rapidly to the level at which it will remain throughout the ensuing period; this leaves \( T_1 \) emitter at the potential \( T_1 \) base had previously remained at while \( T_2 \) had been 'off'. Secondly, because \( T_1 \) cuts off, \( T_2 \) base falls to the level at which it will now remain, pulling \( T_2 \) emitter and, because of \( C_1, T_1 \) emitter, more negative by the same amount.

The two sudden changes, the upward step of \( T_1 \) base and the downward step of \( T_2 \) base and emitter and \( T_1 \) emitter are equal if the circuit is symmetrical, and a common mistake is to suppose that the potential now existing between \( T_1 \) base and emitter is only the magnitude of one such step instead of two.

This leads to underestimating the voltage waveforms on \( T_1 \) and \( T_2 \) emitters and also the timings by a factor of 2.

If both the above errors are made, the timing calculation is wrong by a factor of 4.

Voltage Control for the Circuit of Fig. 11.1

The above discussion of the circuit of Fig. 11.1 has shown that although the charging rates for \( C \) are proportional to the values of \( I_1 \) and \( I_2 \), the cycle time tends to remain constant.

If, however, the voltage steps on \( T_1 \) and \( T_2 \) collector are made constant, the voltage excursions of \( C \) are fixed and timing can be varied by controlling \( I_1 \) and \( I_2 \).
This is readily achieved by catching the collectors as shown in Fig. 11.3. Provided \((I_1 + I_2)R_5\) and \((I_1 + I_2)R_4\) exceed the clipping voltage, the collector swings will now be fixed for all higher values of \(I_1\) and \(I_2\).

The symmetrical case is of special interest, since the unity mark/space square wave available from each collector is ideal for driving a mixer for use in a beat-frequency oscillator or in a frequency or phase-lock loop. In this case each base excursion is \((V_D + V_Z)R_1/(R_1 + R_5)\), where \(V_D\) is the forward drop of \(D_1\) or \(D_2\) giving emitter swings of \(2(V_D + V_Z)R_1/(R_1 + R_5)\) and a total cycle time of \(4(C/I)(V_D + V_Z)R_1/(R_1 + R_5)\), or a frequency of \(I(R_1 + R_5)/4C(V_D + V_Z)R_1\) Hz.

Frequency is therefore directly proportional to \(I\), and it only remains to provide voltage-controlled high-impedance (i.e. 'constant-current') sources of current. Figure 11.4 shows a suitable circuit in which the collector currents of two transistors \(T_3\) and \(T_4\) are defined by the emitter resistors \(R_7\) and \(R_7\) (equal for symmetrical outputs) and the applied voltage \(V_C\).

Provided \(V_C\) greatly exceeds \(V_{eb}\) for \(T_3\) and \(T_4\), the value of \(I\) is \(V_C/R_7\) and the frequency is \(V_C(R_7 + R_5)/4CV_D^2R_3R_7\) Hz. The range of control begins at the value of \(V_C\) which produces sufficient current in \(R_3\) and \(R_4\) to reach the clipping level \(V_Z\), and ends when either \(V_C\) is so large that \(T_3, T_4\) bottom or the value of \(I\) produced by \(V_C\) is so large that transistor or clipping diode ratings are exceeded. Linearity of control will normally be lost before ratings are exceeded, since at high current levels the base currents of \(T_1\) and \(T_2\) will affect the voltages on the bases in an unpredictable manner (\(\beta\) has a wide tolerance).

**Performance of the circuit of Fig. 11.4**

The practical performance of this circuit is as predicted; the ratio of frequency limits depends on the linearity required, but typically 20:1 frequency swing with \(\pm 5\) per cent linearity with applied voltage can readily be achieved.

The square-wave constant-amplitude output from the collectors is obviously useful, but the peculiar emitter excursions would appear to have little application. Between the two emitters of \(T_3T_4\), i.e. across the capacitor, however, is a linear triangular waveform having no noticeable spurious features. Since only a small amount of shaping is required to convert a triangle into a sine wave, the circuit can be used to produce a controlled sine-wave oscillator by adding a difference amplifier between the emitters of \(T_3\) and \(T_4\) and shaping the output. This is particularly useful at low frequencies such as 1 Hz and slower, since a normal 'tuned' oscillator cannot change its frequency in a time comparable with one cycle of oscillation, whereas the circuit used here can change its frequency at any time during a cycle (because changing \(I\) immediately changes the rate of rise of capacitor voltage).

Temperature effects in Fig. 11.4 which affect frequency most are \(V_{eb}\) variations in \(T_3T_4\) which cause a linear increase in frequency corresponding to the same variation of \(V_C\), i.e. 2 mV to 2.5 mV/degC.
Other effects are resistor drift and change in $V_Z$ and in forward drop of the clipping diodes. Transistor collector leakage current in $T_3T_4$ and emitter leakage in $T_1T_2$ change the charging current of $C$, and $\beta$ changes in $T_1T_2$ cause changes in their base potentials when conducting.

Most of these effects are small in a good design. $V_{eb}$ changes in $T_3T_4$ are best balanced by another semiconductor junction; one method is shown in Fig. 11.5, the additional transistor having also the useful effect of reducing the loading on the control source $V_e$. Resistor drift need not exceed a few per cent and in a critical design precision types could be used giving 0.1 per cent. Drift in $V_Z$ and the catching diodes more or less cancel each other if the Zener diode has a positive temperature coefficient of about 2 mV/degC, so its voltage should if possible be 6.8 or 8.2 nominally. Effects of transistor leakage depend on the values of $I$ and resistors $R_1$, $R_2$, $R_3$, $R_6$; at maximum operating temperature the $I_{eb}$ of $T_{1,2}$ must be such that the voltage movement it produces on $T_{1,2}$ base, namely $I_{eb}(R_6//R_5)$, is negligible and the $I_{eb}$ of $T_{3,4}$ must be much less than the minimum value of $I$; these requirements often dictate the use of silicon transistors. $\beta$ should be so high that base currents in $T_1$, $T_2$ produce negligible voltage changes at $T_1$, $T_2$ base at the lowest temperature of operation.

![Fig. 11.5 Use of extra transistor for $V_{RB}$ compensation](image)

Attention to the above points will normally reduce temperature effects to a frequency change of a few per cent over a 50°C temperature range.

Refinements. If the oscillator is required to operate at frequencies above 100 kHz it is advisable to add coupling capacitors in parallel with $R_3$ and $R_6$ to offset the input capacitance of $T_1$, $T_2$ and ensure sufficient loop gain during the transition. The time constant associated with these capacitors should be short compared with a half-cycle of oscillation or, alternatively, very large, so that the collector excursion is coupled in full to the opposite base. Another method is to replace $R_3$, $R_6$ by Zener diodes to ensure good coupling at all frequencies, but since these need to operate at a few milliamps to be effective, emitter followers are then required as shown in Fig. 11.6.

It is desirable in some applications to have a means of setting up the clipping level of the collectors of $T_1$, $T_2$. The simple replacement of the Zener diode by an adjustable resistor chain is usually insufficient in practice, because the clipping level will rise as the value of $I$ is increased; since the mean current into the clipping point is changing, decoupling the resistor chain does not affect a cure. In such a case the use of transistors instead of clipping diodes is necessary in order to avoid a very-low-resistance chain causing excessive drain from the supply (Fig. 11.7).

Transistor types. It is to be noted that $T_1$ and $T_2$ experience large reverse emitter–base potentials and should therefore be of alloy
construction. Alternatively, protection diodes may be added in series with the emitters of $T_1$ and $T_2$. In other respects almost any transistor will be satisfactory, high-gain types being preferred for $T_1$, $T_2$ in order to have small base currents.

**Frequency limitations.** The lowest usable frequency is reached when (1) the leakage currents of the transistors begin to affect appreciably $I_1$, $I_2$, a practical lower limit for $I_1$, $I_2$ being 50 $\mu$A when operating up to 70°C; and (2) the value of $C$ is so large that the required accuracy for the application cannot be obtained. Note that if an electrolytic capacitor is used, it must be of the reversible type. The upper frequency limit is reached when the transition time becomes comparable to the half-cycle time, when amplitude and waveform become unpredictable. This is mainly limited by transistor $f_T$ and collector base capacitance, both of which are partially offset by the use of coupling capacitors.

**Triangular and sawtooth outputs.** As pointed out at the beginning of the chapter, this circuit is useful in having a triangular waveform available across the timing capacitor $C$. This waveform is free from significant spurious content; in particular, no noticeable kink is present at the peak.

Because of the peculiar manner in which this triangle is generated, however, there are difficulties in converting this floating waveform into an unbalanced output, i.e. with one output connection to supply-common.

![Fig. 11.7 Transistors used for clipping](image)

Figure 11.2 shows the two emitter waveforms, and it is clear that the difference between the two forms a triangle waveform with no vertical jumps at the transition, even in the asymmetrical case shown: the lack of symmetry merely causes the up and down slopes to be different. The problem arises in attempting to obtain the exact difference.

As was discussed in Chapter 4, a practical difference amplifier has certain deficiencies, the most important of which is the presence of 'push-push' as well as 'push-pull' gain. That is to say, the output will consist of the wanted signal, which is a multiple of the difference between the two input signals, and an unwanted signal which is a different, usually smaller, multiple of the sum of the two input signals.

For two sine wave input signals of the same frequency, the practical result is that two large signals give a larger output than two small input signals, even though the difference between the two inputs is the same in each case. Thus, the magnitude of the output is in error but no waveform distortion results.

The same result occurs, in fact, whenever the two inputs have the same shape at the same times, only the amplitudes being different: the output may be wrong in amplitude but is not distorted in shape.

In the case of the two emitter waveforms for $T_1$ and $T_2$, this condition does not hold, even in the perfectly symmetrical circuit, since when one emitter moves the other is stationary. The waveform likely to result from passing these signals through a difference amplifier of poor 'push-push' performance is shown in Fig. 11.8. Here the push-push and push-pull gains are assumed to be $A/10$ and $A$, respectively, and the output is seen to contain vertical steps at the triangle peaks, the magnitude of these steps being 1/10 of the peak-to-peak voltage of the triangle.

When examining this triangle waveform using a single input oscilloscope, these steps sometimes appear owing to the large capacitance which may exist between the oscilloscope 'common' and the transistor circuit power supply common through the mains wiring. This puzzling effect is avoided by operating the circuit from batteries or by the use of an oscilloscope with a balanced input, i.e. a differential amplifier.

The above special case in which the push-pull to push-push gain ratio is 10 to 1 may at first seem an improbable condition, since a typical rejection ratio for a straightforward emitter-coupled
difference amplifier is 50 to 1, which would give only 2 per cent steps. In designing such an amplifier for extracting the triangular waveform, however, it soon becomes clear that unless special measures are taken the magnitude of the capacitor signal, usually several volts, will severely overload the emitter-coupled pair. Attenuating resistor networks cannot normally be used, because resistive loading on the capacitor will cause the charging current to vary, giving non-linear sides to the triangle.

The simplest solution is to add emitter resistance to the coupled pair (Fig. 11.9); correct values of all three emitter resistors then ensure that neither transistor cuts off at extremes of input swing. Unfortunately, this necessary modification to the emitter-coupled pair leads to a very poor rejection ratio, quite often only 2 to 1, for reasons explained in Chapter 4.

Hence, it is further necessary to replace R in Fig. 11.9 by a constant-current source which restores the rejection ratio to typically 500 to 1. The magnitude of the step is now likely to be only 1/500th of the total triangle. This differential amplifier is shown in Fig. 11.10.

Properties of the triangular waveform (given by the circuit of Fig. 11.10 combined with Figs. 11.4, 11.5, 11.6, or 11.7). (a) Each half-cycle is linear immediately after the transition, i.e., no rounding of corners. (b) Frequency is variable by change of C, variation of \( V_e \), or by change of \( R_7 \) and \( R_8 \). Amplitude remains constant unless \( V_e \) or \( R_7 \) or \( R_8 \) are so chosen to cause emitter current(s) to reduce below critical level, at which clipping of collector waveforms takes place. (c) The two slopes are respectively inversely proportional to \( R_7 \) and \( R_8 \), a ratio of 20:1 being readily obtained. (d) The rectangular
collector waveforms of $T_1$ and $T_2$ correspond in their vertical edges to the two peaks of the triangle. (e) The frequency can be changed very rapidly (e.g. in a few microseconds at any frequency), since change in $V_o$ produces a rapid change in charging currents as soon as $T_3$, $T_4$ respond.

**Improvements**

Where great precision and stability of frequency is required, the circuits described in Chapter 16 of *Circuit Consultant’s Casebook* (Business Books, 1970) may be used instead of the simple constant current transistors $T_3$ and $T_4$.

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12—Ultra-high gain in one stage

Very often the open-loop gain required in a feedback amplifier is more than can normally be obtained in a single stage of transistor amplification. The use of two conventional stages brings with it several disadvantages: in a feedback amplifier the two separate phase shifts thus produced at high frequencies, each phase shift reaching 90 degrees, cause difficulty in loop stability (see Chapter 8); ripple on the supply lines and slow supply variations enter the input of the second amplifier through the collector loads of the first, and appear in amplified form at the output; owing to coupling losses the resulting gain is rarely as high as the square of the gain of one stage; finally—a very practical point—inclusion of the extra stage in an existing equipment containing only one stage is difficult to carry out.

There is therefore good potential usage for a circuit which gives very high stage gain by simple means.

The circuit to be described, although by no means of universal application, is often suitable for use within feedback amplifiers and particularly in d.c. stabilizer loops where very high gain is often required.

Since a detailed examination of the circuit reveals it to have several incidental advantages, which in many stabilizer applications are even more useful than the high gain, its use may well become standard practice in stabilizer design.

**BASIC IDEA**

The approximate voltage gain for an earthed emitter amplifier is $g_m R_L$. The only obvious way to increase the gain when using a particular transistor is therefore to increase $R_L$. With normal supply
voltages, however, raising $R_L$ implies lowering the transistor operating current to maintain a suitable collector voltage. When this current is reduced, $g_m$ falls, and below a few hundred microamps $g_m$ is directly proportional to the operating current. This means that as $R_L$ is increased and $I_c$ reduced appropriately the gain tends to remain constant. Finally, at low currents, $g_m$ falls more rapidly than current and the gain falls.

What is required, therefore, in order to make the idea practical is a collector load of high incremental resistance but low static resistance. A suitable load would be the collector circuit of a second transistor, since provided current directions are correct it could pass, for example, 1 mA and drop 1–20 V and yet present a high incremental resistance in the region of a few megohms. It is shown in Appendix 4 that the incremental resistance obtained varies from $r_c/\beta$ to $r_c$, depending on the external emitter load $R_e$ of the transistor; values of $R_e$ exceeding a few kilohms give very nearly $r_c$ for typical small-signal transistors, and $r_c$ has a magnitude of a few megohms.

To suit current flow the amplifier and load transistors must be of complementary types, so that no resistor is required to supply the operating currents, since this would shunt the load circuit. The basic circuit is then of the form shown in Fig. 12.1. The gain to be expected from this arrangement is not $g_mI_{eoa}$ as the approximate formula would suggest, since that formula applies only when $R_L \ll r_c/\beta$, but is approximately $r_c/r_n$, as shown in Appendix 4, which is typically 750 for most small-signal transistors.

**Fig. 12.1** Basic idea for ultra-high gain amplifier

In the form shown in Fig. 12.1 the circuit is impractical for two reasons. First, any slight inequality between the collector currents of $T_1$ and $T_2$ will cause one or other transistor to bottom; secondly, any normal load placed at the output will reduce the load, and therefore the gain, considerably.

**Practical Circuit**

To make the circuit of Fig. 12.1 usable, the common collector potential must be defined and buffering must be provided between the collector junction and external loads. This can be achieved as shown in Fig. 12.2 by the use of a compound emitter follower in conjunction with a d.c. feedback loop. The operating current of $T_2$ is set by $R_1$, $R_2$, and $R_3$. If it is assumed for the moment that $T_1$ is in normal conduction and that its base current can be neglected in comparison with the current in $R_4$, then the current in $R_4$, which is approximately equal to $V_n/R_4$, must flow through $R_5$ so that $R_3$ drops $V_nR_5/R_4$ volts and $T_4$ emitter sits at $+V_nR_5/R_4$. The $T_1/T_2$ collector junction will be slightly more positive, and since this can happen only if $T_1$ is in normal conduction and passing a current equal to that in $T_2$ (less the base current in $T_2$), the first assumption is correct. The second assumption (small base current in $T_1$) is also correct provided the current in $R_4$ is made large compared with the current defined in $T_2$ emitter divided by $\beta_2$.

Thus, the $R_4$, $R_5$ feedback circuit fixes the working currents and...
voltages correctly, and T₃, T₄ provide buffering. In Fig. 12.2 the gain from input 1, i.e. from the base of T₁, to the output at T₄ emitter is about 750. On the other hand, the gain from input 2, i.e. through R₄ is about R₄/R₅, provided R₄/R₅ ≪ 750, and the circuit has the properties of a typical feedback amplifier having an open-loop gain of several hundreds.

Applications
It appears from the above paragraphs that to make any practical use of the original idea no less than four transistors per stage are required. One could well argue that the transistors would be better used in a simple two-stage amplifier with interstage buffering. In general this is true, but there is one particular application where the circuit is economic and, moreover, investigation reveals it to have many virtues quite apart from high gain.

![Fig. 12.3 Conventional stabilizer](image)

This application is the direct voltage stabilizer. Figure 12.3 shows a conventional circuit which has already been discussed in Chapter 9. It will be noticed that this circuit bears a strong resemblance to that of Fig. 12.2, in that a feedback loop defines the collector voltage of T₂, and emitter followers T₃ and T₄ provide buffering between T₂ and the output load. Conditions are therefore ideal for replacing the amplifier collector load R₃ by a suitably biased p-n-p transistor. This is shown in Fig. 12.4, where R₃R₅ and R₇ determine the current in T₅; T₉ is forced by the loop to take the same current as T₇ (less the current in T₇ base), since if these currents were unequal either T₂ or T₉ would bottom. R₂ is preferably designed to pass twice the current of T₅, since this will result in equal currents in T₇ and T₂, giving best balance between them for temperature drift (see Chapter 2). It may, of course, be necessary in stabilizers for heavy currents to add another emitter follower between T₇T₉ and T₉ or to convert T₉ into a compound emitter follower by adding a p-n-p transistor (see Chapter 10).

In the circuit shown in Fig. 12.4 a gain of about 750/2 is achieved in the T₃ stage (an emitter-coupled pair of similar transistors has half the gain of the corresponding earthed-emitter stage) as compared with about 25 in the conventional circuit.

**Fig. 12.4 Stabilizer—first modification**

<table>
<thead>
<tr>
<th>Typical values</th>
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<tbody>
<tr>
<td>R₁</td>
</tr>
<tr>
<td>R₂</td>
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<tr>
<td>R₃</td>
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<td>R₄</td>
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<td>R₆</td>
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<td>D₁</td>
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<tr>
<td>T₁,2,3,4</td>
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<tr>
<td>Vₑ₂</td>
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<tr>
<td>Vᵦᵱᵣᵳ</td>
</tr>
<tr>
<td>Vᵦᵱᵲᵳ</td>
</tr>
</tbody>
</table>

**Self Balancing Stabilizer**
Before considering the final development of this circuit, it is useful to recall some of the difficulties in designing stabilizers having widely variable output voltage.

One method which can be used to control the output of the circuit shown in Fig. 12.3 is to vary the ratio R₄/R₆. This directly varies the output voltage, which is equal to (R₄ + R₆)Vₑ₂/R₄. This system has two disadvantages, however, one being that Vₑ₂ can never be less than Vₑ in magnitude, the other, that the amount of feedback varies so that many of the properties of the stabilizer, such as transient response, output resistance, and stabilization against input variations, will change according to the selected output voltage.

The more straightforward method of varying Vₑ is therefore preferable, and this can be achieved by adding a potentiometer as shown in Fig. 12.5. Although the loop gain is now more constant for different output settings, this method still has a serious snag. When a low output is required, Vₑ is small, so that current in R₆ is small. The
voltage drop across $R_3$ is, however, large, which implies that $T_2$
collector current is large. Hence, $T_2$ emitter current is required to be
greatest just when its supply from $R_3$ is least. In a practical design
control will in fact be lost as $V_{out}$ is reduced, as eventually all the
current in $R_3$ is taken by $T_2$ and $T_1$ cuts off. Even over the range
where control applies, the sharing of current between $T_1$ and $T_2$ is
changing. This is of special importance in low-drift stabilizers, where
$T_1$ and $T_2$ are matched with respect to $V_{be}$ temperature coefficient,
since such matching applies only for equal operating currents.

![Fig. 12.5 Variable stabilizer]

If variable $V_Z$ control is applied to the circuit of Fig. 12.4, some
improvement is obtained as compared with the normal circuit,
because the current in $T_9$ remains virtually constant for all output
settings, but since the current in $R_9$ does vary, $T_1$ emitter still falls as
$V_{out}$ is reduced, so that balance exists at only one setting of $V_{out}$.

With the further modification of Fig. 12.6 the above difficulty is
overcome. Here, the current for $R_9$ is provided by $T_1$ collector in
stead of $R_8$. If $T_1$ collector current has a value $I_1$, then $R_9$ drops
$I_1 R_9$ volts and $T_5$ emitter current is $I_1 R_9 / R_7$ if the $V_{eb}$ drop of $T_5$ can
be neglected in comparison with $I_1 R_9$. Since $T_2$ collector current equals
$T_5$ emitter current (ignoring $T_3$ base current), the ratio of the currents
in $T_2$ and $T_1$ is simply $R_9 / R_7$. Thus, if $R_7$ and $R_9$ are made
equal, $T_1$ and $T_2$ take equal shares of the current supplied by $R_2$
and remain balanced at all output settings. The low-output limit is
now reached only when the current in $R_3$ falls to leakage current
level; then $T_1$ and $T_2$ gains fall and $T_3$ base current cannot be
ignored.

![Fig. 12.6 Stabilizer—second modification]

Another advantage of this arrangement (Fig. 12.6) is that the
$T_1$, $T_3$ stage has twice the gain of the Fig. 12.4 version. This comes
about as follows. A signal applied to $T_3$ base which causes a rise in
$T_3$ collector current of $\delta I$ also causes a fall of $\delta I$ in $T_1$ collector
current, since the total emitter current supply is constant for any
one setting of $V_Z$. This fall of $\delta I$ causes the emitter current of $T_5$ to
fall by $\delta I (R_9 / R_7)$. Thus, a load attached to the common collector
point of $T_3$, $T_5$ experiences a current change of $\delta I (1 + R_9 / R_7)$, or
$2\delta I$ if $R_7 = R_9$.

It might be wrongly deduced that a larger ratio than unity for
$R_9 / R_7$ would give even higher gain, but this would be true only if
the load presented to $T_9$ collector were constant. However, as $R_7$
becomes smaller the effective load on $T_3$ becomes less, and in the
limit when $R_7$ is zero the incremental collector resistance of $T_5$—
which is the load for $T_2$—becomes $r_c / \beta_5$ instead of $r_c$. At the same
time, the current change for $T_3$, $T_5$ is $(\beta_3 + 1) \delta I$, so that the voltage
gain is similar to that obtained in the circuit of Fig. 12.4. Even this
is optimistic, since $T_1$ and $T_2$ now share the total emitter current in
the ratio $1 / \beta_3$ rather than 1/1, resulting in lower gain and worse
temperature drift.

The optimum value for $R_9 / R_7$ is therefore near unity.

**Improvement for precision stabilizers**

In examining the circuit of Fig. 12.6, the effect of the $V_{eb}$ of $T_3$ has
so far been ignored. Its presence means that the voltages across $R_9$
and $R_7$ are unequal, and the inequality varies with temperature.
This causes the sharing of current between $T_1$ and $T_2$ to vary also, a characteristic which can be important in precision stabilizers of wide output range.

The first effect can be made small, as already indicated, by making these voltage drops large compared with $V_{eb}$; it can be made smaller still by making some allowance for $V_{eb}$ in the choice of $R_6$ and $R_9$. For instance, if $\frac{1}{2}I_6R_6 = V_{eb} + \frac{1}{2}I_6R_7$, where $I_6$ is the current in $R_6$, then the currents in $T_1$ and $T_2$ will be equal. However, this equality will hold at only one setting, because $I_6$ changes with setting, and at only one temperature, because $V_{eb}$ changes with temperature.

A better system is to include a forward-biased diode in series with $R_9$. Although this does not ensure exact equality of $T_1$ and $T_2$ currents, it is consistent at various settings of $V_2$, since the diode and $T_5$ currents vary similarly. Its temperature coefficient also cancels that of $T_5 V_{eb}$ to a large extent. Figure 12.7 shows the improved version.

Another improvement for better drift and absolute accuracy is made by using a dual transistor for $T_1$ and $T_2$, e.g. the 2N3680 or 2N2644.

Effect of line variations

In the circuits considered here no capacitors are shown, so that slow line variations and ripple have the same amount of influence on the output voltage.

Returning to Fig. 12.3, line variations enter the system through $R_1$ and $R_3$. Chapter 9 showed how the effect of these variations may be reduced, and it is noted there that the path through $R_3$ is just as troublesome as the more obvious route through $R_1$.

On examining Fig. 12.6 or 12.7, it will be seen that in the new circuit, although the $R_3$ route is unchanged, the $R_9$ path is replaced by $R_1$ and $R_9$. Since line variations affect $R_1$ and $R_9$ equally, and $T_1$ collector circuit is of high incremental resistance compared with $R_3$, $T_5$ is not subjected to any base-emitter change and its collector current is therefore unaffected. In other words, the new circuit lets in line changes only by the $R_1$ path.

Use of positive feedback

Positive feedback in the $T_1/T_2$ stage, which was shown to lead to increased performance, in particular to zero output resistance, in a circuit similar to 12.3 (see Chapter 9), can still be applied to the circuit of Fig. 12.6 with similar results. As the gain in the stage comprising $T_1/T_2/T_5$ is very high, the actual resistor between $T_1$ collector and $T_3$ base necessary to produce zero output resistance from the stabilizer is much higher than in the conventional circuit and so has less upsetting effect on transistor operating currents.

SUMMARY

The idea of using a transistor collector circuit instead of a resistor as an amplifier load can be made practical. Owing to the special arrangements required, the circuit is uneconomic except in a voltage stabilizer loop. In this application the circuit is found to have the following advantages:

1. A voltage gain of about 750 can be obtained from a typical stabilizer loop amplifier by adding one transistor and one resistor ($T_3$ and $R_9$) or in some cases two transistors and two resistors ($T_5$, $R_9$, $T_3$, $R_8$).

2. The circuit configuration is such that $T_5$ can often be added to an existing layout, giving an improvement of about 30 times in loop gain.

3. The circuit automatically keeps $T_1$ and $T_2$ in current balance at all output settings.

4. In a precision stabilizer, where $T_1$, $T_2$ are dual transistors matched for $V_{eb}$ temperature drift; the equality of currents in $T_1$ and $T_2$ ensures matched $V_{eb}$ at all output settings.
(5) Even better matching of $T_1$, $T_2$ currents can be obtained by adding a diode in series with $R_9$ to cancel the (small) effect of the $V_{eb}$ of $T_6$.

(6) Line variations enter the circuit to a lesser extent than in the normal circuit, since the direct path through the collector load of $T_2$ is eliminated. This is quite distinct from the improvement obtained in stabilization and ripple due to the extra gain of the loop.

(7) Positive feedback can still be employed, and less is required to reach zero output resistance than with a conventional stabilizer.

13—The transistor pump

The title refers to a circuit described by J. Willis and P. L. Burton* (p. 293) as an improved version of the well-known diode pump, which is often used as a ‘staircase’ generator and as a counting-type frequency discriminator.

In order to appreciate fully the transistor circuit it is essential to understand the operation of the apparently simple diode pump.

THE DIODE PUMP AS A STAIRCASE WAVEFORM GENERATOR

When used as a staircase generator the diode pump circuit is as shown in Fig. 13.1.

Since the output waveform, neglecting initial transients, will be independent of the d.c. component of the input waveform, it will be assumed for convenience that the input square wave starts at zero volts and falls periodically by $V_{in}$ volts (Fig. 13.2), and that initially $V_{out}$ is zero.

When the input falls to $-V_{in}$, $D_1$ conducts and $C$ charges by $V_{in}$ volts, provided the forward voltage drop of $D_1$ is negligible compared with $V_{in}$. Referring again to Fig. 13.1 the left-hand connection

* Wireless World, March 1958, 'Some Unusual Transistor Circuits'.
of \( C \) is now at \( -V_{in} \) and its right-hand connection approximately at zero potential; \( C \) has therefore acquired a charge of \( CV \) coulombs.

When the input returns to zero the right-hand connection of \( C \) rises and \( D_2 \) conducts, connecting \( C \) to \( C' \). Again ignoring forward diode voltage drops this connection of \( C \) to \( C' \) occurs immediately the input rises, so that the rise of \( V_{in} \) in volts is shared between \( C \) and \( C' \) giving a rise of \( V_{in}C/(C + C') \) at the output.

On the second cycle of operation, conditions are different. The input falls to \( -V_{in} \) and \( C \) charges as before, but when the input rises towards zero, \( D_2 \) does not conduct until the input has moved by \( V_{in}C/(C + C') \), bringing the left-hand connection of \( D_2 \) equal to

\[
\begin{align*}
\text{Output} & \quad \frac{V_{in}C}{(C + C')} C \frac{C}{(C + C')} \\
\text{Input} & \quad -V_{in} \\
\end{align*}
\]

Fig. 13.2 Waveforms for diode pump staircase generator (Fig. 13.1)

\( V_{out} \). Thereafter the remaining change in \( V_{in} \), namely \( V_{in} - V_{in}C/(C + C') \), which is \( V_{in}C/(C + C') \), is shared between \( C \) and \( C' \) as before so that the output rises by

\[
[V_{in}C/(C + C')][C/(C + C')]
\]

to a new voltage of

\[
V_{in}C/(C + C') + [V_{in}C/(C + C')][C/(C + C')],
\]

that is

\[
[V_{in}C/(C + C')][1 + C'/C'(C + C')].
\]

Succeeding cycles continue to increase \( V_{out} \) in increments to give the output waveform shown in Fig. 13.2. It is shown in Appendix 5 that after \( n \) input cycles the output voltage is

\[
V_{in}[1 - (C'/C + C')^n]
\]

which for small values of \( n \) or small ratios of \( C/C' \) approximates to

\[
\star = V_{in} C(C + 2C')/(C + C')^2
\]

The Transistor Pump

\( V_{in}C/(C + C') \). Under these conditions, therefore, the output rises by \( V_{in}C/(C + C') \) for every input cycle.

When the above conditions for \( n \) and \( C/C' \) are not obeyed, examination of the expression for \( V_{out} \) shows that the steps of output voltage become successively smaller and that the maximum output after an infinite number of input cycles is \( V_{in} \).

Applications

Within the limitations just mentioned this circuit is useful in the generation of a staircase waveform; these limitations mean, however, that if all steps of the staircase are to be substantially equal, either the number of such steps will be small, or capacitor ratio \( C/C' \) will be small, implying that the final value of \( V_{out} \) will be much less than \( V_{in} \).

Fig. 13.3 Diode pump frequency discriminator

This circuit can be converted into a frequency divider by adding a trigger circuit at the output, which operates an electronic switch arranged to discharge \( C' \) when \( V_{out} \) reaches a certain voltage level. The accuracy of the triggering level and of the capacitor ratio, and the magnitude of \( V_{in} \) determine the maximum frequency division ratio at which operation will be reliable. Because of the successive reduction in each step discussed above, the limitations are severe and a frequency division of 10 to 1 is near the reliable limit for normal input voltages.

A more commonly used form of the diode pump is shown in Fig. 13.3, where a resistive load \( R \) is added in parallel with \( C' \). In this circuit \( C' \) is made very large in comparison with \( C \), so large that virtually no voltage steps are present across \( C' \) compared with the magnitude of \( V_{in} \).

In this circuit the capacitor \( C \) charges to \( V_{in} \) each time the input goes negative and discharges into \( C' \) as before, so that \( C' \) must acquire charge each positive input half-cycle. Since the values of \( C' \)
and $R$ are so large that no significant waveform is visible across $C'$, the implication is that $C'$ must lose this acquired charge at some time during the cycle. The only path for discharge of $C'$ is through $R$, and for such a discharge to occur $C'$ must carry a steady potential, since it is known to carry no alternating waveform.

For an input of frequency $f$ and magnitude $V_{in}$ there is therefore a d.c. output $V_{out}$ the magnitude of which is calculable by equating the charge gained and lost by $C'$ during a complete cycle.

The charge gained by $C'$ can be calculated as follows. When the input goes negative by $V_{in}$, $C$ acquires a charge $CV_{in}f$ since, as in the staircase circuit, $D_1$ conducts. When the input now rises by $V_{in}$, $D_2$ conducts and charge flows from $C$ into $C'$. At the end of this half-cycle, when the input is about to fall, the right-hand connection of $C$ is at a potential $V_{out}$, since it has been assumed that $V_{out}$ is unchanged during a cycle. Hence, the charge on $C$ is now $CV_{out}$ since the input is at earth potential. The charge lost by $C$ is therefore $CV_{in} - CV_{out}$ and this is therefore the charge gained by $C'$.

The charge lost by $C'$ is simply $V_{out}/RF$ because $C'$ continuously discharges by a current $V_{out}/R$ throughout the cycle and the period of a cycle is $1/f$. The equilibrium equation is therefore $CV_{in} = CV_{out}$, giving $V_{out} = CV_{in} RF/(1 + CRf)$. This implies that for values of $CRf$ much less than unity, $V_{out} = CRfV_{in}$, i.e., the circuit of Fig. 13.3 produces a d.c. output proportional to input frequency.

The above condition, that $CRf \ll 1$, is equivalent to $V_{out} \ll V_{in}$. If the condition is not satisfied, then although the output voltage still rises with input frequency, the relationship is non-linear, the ratio $V_{out}/f$ becomes less and eventually $V_{out}$ approaches the value $V_{in}$.

This circuit is often used as a direct reading-frequency meter by replacing $R$ by a milliammeter. Provided $V_{in}$ is more than a few volts in magnitude, the non-linearity is no more than a few per cent with a typical meter voltage drop of 100 mV.

**SUMMARY**

The diode pump can be used as a staircase generator, as a frequency divider and as a frequency discriminator.

For most applications where linear characteristics are desirable it is necessary to accept an output voltage which is always much less than the input level, whichever form of the circuit is being used.

This forms a severe limitation at high frequencies, where large inputs may be difficult to provide and at any frequency if exceptional linearity is desired.

**TRANSISTOR PUMP**

In looking for a method of improving the diode pump, it is helpful to recall that the non-linear characteristic and low maximum output level are both caused by the influence of the output level on the charge and discharge of $C$.

The clearest approach to improve this situation consists in isolation of the charge and discharge circuits for $C$ from the output voltage.

**First Version**

One method is shown in Fig. 13.4, where $D_2$ becomes the emitter-base diode of a transistor and $C'$ is placed in the collector circuit.

![Transistor pump staircase generator (1)](image)

When the input goes negative, $C$ charges to $V_{in}$ and when the input returns to zero, $C$ discharges completely into the emitter of $T_1$. This current flows out of the collector and charges $C'$ to a voltage of $V_{out} = CV_{in}/C'$ ($C'V_{out}$ must equal $CV_{in}$ since all the charge from $C$ transfers to $C'$).

This action is repeated on every cycle, since the change taking place at $T_1$ collector has virtually no influence on the charge/discharge process.

Hence, a staircase output waveform results in which each step is equal until the transistor bottoms, after which no further change
occurs. The maximum output is therefore limited not by the magnitude of \( V_{tn} \), but by the supply voltage, which in turn is limited by the maximum permissible \( V_{be} \) for the transistor and the available supply voltage.

The staircase is therefore equally stepped, of magnitude limited only by transistor ratings, and can therefore be used as a frequency divider for reliable division even at high ratios, 50 to 1 being quite practicable.

Consider now the addition of a resistor \( R \) in parallel with \( C' \), \( C'R \) being very large compared with an input period (see Fig. 13.5). For an input frequency \( f \), \( C \) charges to \( V_{tn} \) on every negative input swing and discharges its charge of \( CV_{tn} \) into \( T_1 \) emitter on every positive swing. Consequently, \( C' \) receives a charge \( CV_{tn} \) on every cycle and during the cycle loses a charge \( V_{out}/Rf \) into the load \( R \), assuming \( C'R \) is so large that no change of voltage occurs within any one cycle.

Hence, \( CV_{tn} = V_{out}/Rf \) or \( V_{out} = CRfV_{tn} \), so that \( V_{out} \) is directly proportional to input frequency until, as for the staircase, \( V_{out} \) is so large that \( T_1 \) bottoms.

The transistor circuit therefore forms a linear discriminator with an output voltage which can be considerably larger than the input swing and is limited only by transistor rating and available supply voltage.

The circuits of Fig. 13.4 and 13.5 therefore correct the main deficiencies of the simple diode pump by the principle of preventing the output voltage from influencing the charging of \( C \).

**Second Version**

The circuit of Fig. 13.6 shows the form of transistor pump usually quoted. In this circuit the first negative swing charges \( C \), through the base-emitter diode of \( T_1 \) to \( V_{tn} \) and the succeeding positive swing to zero causes \( C \) to pass charge to \( C' \), so that \( V_{out} = V_{tn}C/(C + C') \).

The second negative swing charges \( C \) to \((V_{out} + V_{tn}) \) volts, since the right-hand connection of \( C \) is caught at \( V_{out} \) by \( T_1 \). When the input voltage starts to return to earth for the second time \( D_1 \) conducts immediately (unlike the diode pump where the input had to rise by \( V_{tn}C/(C + C') \) before \( D_1 \) would conduct). The full rise of

![Fig. 13.5 Transistor pump frequency discriminator (1)](image)

![Fig. 13.6 Transistor pump staircase generator (2)](image)

![Fig. 13.7 Transistor pump frequency discriminator (2)](image)

\( V_{tn} \) is therefore shared by \( C \) and \( C' \) as on the first positive swing and the output therefore rises, as before, by \( V_{tn}C/(C + C') \).

This process is repeated so that each step is \( V_{tn}C/(C + C') \) and the staircase has a linear characteristic as in the first version of the transistor pump.

The addition of a load resistor \( R \) (Fig. 13.7) with large \( C' \) still results in a performance equivalent to that of the first transistor
Comparison Between the Two Circuits (Fig. 13.4, 13.6)

The main practical differences between the two circuits are as follows.

1. If only a negative supply rail is available, each circuit requires a p-n-p transistor; the first version gives an output which is positive-going with respect to the negative rail and the second gives an output which is negative-going with respect to earth.

2. Similarly a positive rail requires an n-p-n transistor; the first version gives a negative-going output relative to the rail, whereas the second gives a positive-going output relative to earth.

3. In the first version used as a discriminator (Fig. 13.5) C' may be returned to earth rather than the supply rail, thus confining the input signal current paths away from the supply rails. This is advantageous when used at frequencies above 1 Mc/sec., and cannot be achieved in such a simple manner in the second circuit (Fig. 13.7).

4. In Fig. 13.4, step magnitude is \( V_{in}C/C' \); in Fig. 13.6, \( V_{in}C(1 + C') \).

Further development of the transistor pump

If both polariies of supply are available, a two-transistor pump enables simultaneous staircase generators or discriminators to be obtained with opposite polariies, as shown in Fig. 13.8 and Fig. 13.9. By omitting one or other resistor in Fig. 13.9 one output can be used for frequency measurement and a staircase is produced at the other output terminal.

Design of pump circuits

Although basic design of these circuits consists in satisfying the simple formulae for \( V_{out} \), there are a few further points which should be numbered.

First, it has been assumed throughout that C charges and discharges to a steady state between each input swing and that the input is a square wave. The essential point is that C shall always complete its charge and discharge before the input reverses. In a practical circuit there will exist some source resistance \( R_s \) and \( D_1 \) and \( D_2 \) (or \( T_1 \) in the transistor pump) will have forward resistance \( R_D1 \) and \( R_D2 \). Provided the total series R is such that \((R_s + R_D1)C\) is less than one-

fifth of the duration of the negative input step and \((R_s + R_D2)C\) is less than one-fifth of the duration of the positive step, the errors will be less than one per cent. A further factor of 2 in these time constants will make errors from this cause negligible.

It is not necessary that the input should be a symmetrical square wave; so long as the input remains in each state long enough for C to charge to the correct level, this is sufficient.

The turn-on voltages required by the diodes and transistor have been ignored in the analysis. In practice these voltage drops cannot be ignored and their effect is equivalent to a drop in the magnitude of \( V_{in} \). This equivalent drop varies with temperature at a rate of
-2 to -2.5 mV/degC and can thus cause considerable error in an equipment operating over a wide temperature range. The effect can be minimized by using a large input voltage, by arranging a controlled input variation with temperature or, for extreme stability, by oven-controlling the temperature of the diodes.

Temperature errors are also caused by diode and transistor leakage currents which add to or subtract from the charging currents to C'. In the discriminator circuits a steady output of $I_{eo}R$ results. This error is reduced by using large values of $C_i$, giving lower values of $R$ for the same output voltage, but demanding more current from the signal source $V_{in}$. The use of silicon transistors having low leakage is often necessary when designing for wide temperature range variation.

Another parameter ignored in the analysis is the transistor current gain. For a $\beta$ of 33, this gives an error of about 3 per cent in the absolute magnitude of the output and a variation of about 0.03 per cent per degree C. The effect is reduced by using a high-$\beta$ transistor.

Finally, it should be remembered that any of these circuits can be changed by reversing each diode and type of transistor from $p-n-p$ to $n-p-n$ or vice versa, provided the power supply polarity is also changed.

The diode pump is a useful circuit with, however, severe limitations in output and linearity. The transistor pump circuits almost completely eliminate these faults and greatly extend the use of this type of circuit.

14—The transistor cascode

The circuit to be described in this chapter is a transistor version of the well-known valve cascode arrangement. Although not all cascode characteristics are common to both the valve and transistor circuits, it will be useful to recall the problems of low-noise high-frequency amplification which led to the development of the valve cascode.

PROBLEMS IN HIGH-FREQUENCY VALVE AMPLIFICATION

At high frequencies it becomes difficult to prevent oscillation in a
triode-tuned amplifier because of relatively large grid-anode capacitance giving unwanted coupling. A pentode stage overcomes the difficulty since direct anode-grid coupling is very small; moreover, higher stage gains are obtained. However, a pentode, because of its multielectrode structure, produces a high degree of partition noise.

There appears in such stages therefore to be a choice between a triode with poor gain and stability or a pentode with bad noise performance.

The cascode, which uses two triodes in a special configuration (Fig. 14.1), provides, at the cost of an extra valve, the stability and gain of a pentode and the low noise of a triode.

Examination of the circuit shows that the output electrode is well-screened from the input, and since the anode load of $V_1$ is low compared with the $r_a$ of $V_1 (1/g_m \ll r_a)$, $i_{a1} = g_m i_{v1}$ provided $X_c \ll 1/g_m$. Also, $i_{a2} = i_{a1}$, so that $V_{out} = g_m i_{v1} R_L$, which is the same result as if $V_1/V_2$ were replaced by a single pentode valve with a $g_m$ equal to $g_m 1$.

**TRANSISTOR CASCODE**

At first sight the transistor version of Fig. 14.1 appears to offer little advantage over a single-transistor amplifier stage, since in many respects a transistor has pentode characteristics, and, as for the pentode, its stage gain is $g_m R_L$. Partition noise does not occur, as no fourth electrode exists and so the cascode connection of two such devices could not be expected to have better noise performance than one.

However, input to output capacitance still exists in a simple transistor amplifier and the cascode connection will provide isolation as in the valve circuit. Although this alone often justifies its use, the cascode offers many other advantages which are not immediately obvious and which come about because the circuit requirements are divided between the two transistors.

Before examining the properties of the circuit of Fig. 14.2, it is desirable, first, to recall the effect of $f_a$ on the high-frequency response of an amplifier and, secondly, to note the way in which base circuit conditions affect the permissible collector-emitter voltage rating of a transistor.

First, the base-to-collector current gain $\beta$ is 3 dB down at a frequency $f_a/\beta$ and the emitter-to-collector current gain $\alpha$ is 3 dB down at a frequency $f_a$. Hence, an earthed emitter amplifier is 3 dB down at 1/\beta of the frequency at which an earthed base amplifier is 3 dB down provided the source resistance is high (see Chapter 4).

Secondly, the collector-emitter voltage rating of a transistor usually depends on the base conditions; the permissible voltage is higher if the base resistance is low and is often halved if the base resistance becomes as high as 1 kΩ.

Returning now to Fig. 14.2, it is clear that if the transistors are similar, the $f_a$ of the output transistor $T_2$ has little influence on the high-frequency performance of the circuit and can have a value as low as $f_a 1/\beta$ before beginning to contribute to the frequency fall-off.

Hence, for good high-frequency performance $T_1$ must be chosen as if the circuit were a normal earthed emitter amplifier stage, and $T_2$ can have an $f_a$ which is roughly $\beta$ times worse.

With regard to voltage ratings, $T_1$ must have a $V_{ce}$ rating of at least $V_1$ under its particular base resistance conditions, and $T_2$ must have a rating of $(V_2 - V_1)$ under zero base resistance conditions (i.e. the favourable case).

Since $V_1$ can be small in view of the small signal amplitudes on $T_1$, and $(V_2 - V_1)$ may have to be large where large output voltage swing
is required, $V_{ce}$ rating can be low for $T_1$ but must be high for $T_2$. The same applies to the power ratings since currents are equal.

These results are useful because the division of duties between $T_1$ and $T_2$ proves to be ideal: $T_1$ must have good $f_a$ and $T_2$ must have high voltage and high power rating. For a given application this implies that two inexpensive transistors can replace one expensive type which in a simple amplifier would require not only high voltage and power ratings but also good high frequency performance.

Although the above suggests that the cascode is primarily of interest as a high frequency amplifier it must be remembered that the term 'high frequency' applies whenever the frequency of operation approaches the $f_a$/$\beta$ for the transistors used.

**Effects Caused by Transistor Capacitance**

A simple transistor stage suffers from output to input (collector-base) capacitance in the same way as a simple triode amplifier. For

![Figure 14.3](image)

**Fig. 14.3** Miller effect: (a) connection of $C_{cb}$, (b) equivalent effect of $C_{cb}$

a stage gain $-G$ the effect of collector-base capacitance $C_{cb}$ is equivalent to the addition of a capacitor of value $(1 + G)C_{cb}$ in shunt with the base-emitter path (see Fig. 14.3). This is known as Miller effect, and the above result is easily calculated.

This is often the major cause of falling high-frequency response in a transistor amplifier and conversion to a cascode connection in such a case will bring about a dramatic improvement.

Another high-frequency problem is that of designing an isolating stage to prevent, for instance, an oscillator being detuned by changes in the load circuit. At low frequencies a natural choice for the isolating stage would be an emitter follower, since any change of load current in the emitter would cause a much smaller change in the base current. At high frequencies, however, transistor base-emitter capacitance forms a low impedance connection from source to load and isolation is poor.

Again, the cascode is useful as no direct back-coupling exists and buffering is effective to several hundred megahertz.

**Variations of the Cascode**

The addition to the basic cascode of a resistor $R'$, as shown in Fig. 14.4, enables the operating current of the two transistors to be

![Figure 14.4](image)

**Fig. 14.4** Cascode with different emitter currents for $T_1$ and $T_2$

different. Thus $T_1$ can be run at high current, making its $g_{m}$ high, and $T_2$ can pass a small current, equal to $I_{T1} - (V_2 - V_1)/R'$, so that $R_L$ can be large. In this way high gain is obtained at the cost of less accurate determination of $T_2$ collector voltage. This occurs because $T_2$ collector current, which determines this voltage, is the difference between $T_1$ current and the current in $R'$, two independent variables. Changes in either of these currents therefore produces a larger percentage change in $T_2$ collector current. In practice $T_2$ current can rarely be made less than one-third of $T_1$ current without introducing too large an uncertainty in $T_2$ current.
Fig. 14.5 illustrates the use of a multiple cascode circuit as an adding amplifier. The standing currents in $T_1$, $T_3$, $T_4$, etc. all add in $T_2$ emitter so that in a $k$-input symmetrical design each $g_m$ will be $1/k$ of the value which would be obtained in a single-input stage (since $g_m$ is proportional to current). Hence, the gain from any input is $1/k$ of that of a single-input stage running at the same $T_2$ collector current. This circuit is particularly useful in having very good isolation between the several input terminals.

Fig. 14.5 Cascode as an adding amplifier

Fig. 14.6 shows how another form of multiple cascode enables large voltage output swings to be obtained from transistors of much lower rating. The network of resistors and capacitors maintains sharing of output voltage between the transistors; capacitors are necessary for transient operation and must be much larger than the input capacitances of the associated transistors. Since saturation of any transistor would upset this capacitor–resistor potential divider, precautions, such as prelimiting, must be taken to prevent overload. Switch-on and switch-off transients which often cause difficulty in the operation of voltage-sharing circuits are in this case harmless since the capacitors ensure slow rise and decay of the applied voltages.

A complementary version of the cascode is shown in Fig. 14.7.

The major disadvantage of this version is of course the need for resistor $R_e$ to supply the total current of $T_1$ and $T_2$.

Fig. 14.6 High-voltage cascode

Fig. 14.7 Complementary cascode

**SUMMARY**

1. In high-frequency high-output applications the transistor requirements are split; the input transistor must be high frequency but can be low voltage, and the output transistor must be high voltage but can be $\beta$ times worse in frequency response.
(2) Miller effect, often the major cause of falling high frequency response, is virtually eliminated.
(3) Because $T_2$ base is fixed to a low impedance point the $V_{cb}$ rating of the transistor can be applied (not merely the lower $V_{ce}$ rating).
(4) The cascode is a much better isolator at high frequencies than an emitter follower or single-transistor amplifier.

Part three
Useful techniques
15—Bootstrapping

The technique known as 'bootstrapping' involves a positive feedback loop which causes a point in the circuit to be 'pulled up as if by its own bootstraps'. This principle is often used in linear sweep generation (see Chapter 5) and also in high input impedance amplifiers, in driver circuits for power stages, and in reducing the effects of stray capacitance.

In this chapter are discussed the last three applications; design procedure and also practical difficulties are given.

HIGH INPUT IMPEDANCE BY BOOTSTRAPPING

When an amplifier is required to have a high input impedance of, for example, 1 MΩ, the obvious circuit configuration to consider is the emitter follower either in its simplest form or compounded. In defining the transistor operating conditions, however, there is a maximum value which the input base resistor may be given without the risk of excessive temperature drift due to base current variations. In practice this value can rarely exceed a few kilohms for germanium, or some tens of kilohms for silicon transistors, so that even if the circuit is inherently capable of providing high input impedance, the external base resistor spoils its performance.

In some applications the resistance of the source may be sufficiently low to act as the base resistor, so avoiding the need for the shunting effect of a separate resistor, but in many cases the source must be capacity coupled. A very common use for a high input impedance amplifier is in conjunction with a crystal transducer, such as an accelerometer, a record pick-up, or microphone; such a transducer is roughly equivalent to a source of e.m.f. in series with a
capacitor of several hundred picofarads, shunted by hundreds of megohms. In this and many other applications there is no escape from the use of base resistors.

In such circumstances the bootstrap technique can be used to make the low-valued base resistor appear to have a much higher value from the point of view of the input signal.

To understand the principle involved here it is helpful to consider the true meaning of high input impedance. It means that when the input signal changes by a small amount the change of current taken by the circuit from the source is small. In the circuit of Fig. 15.1, assume for the moment that connection AB is not made. The input impedance at T1 base is \((R_{b1} + R_{b2})\) in parallel with the impedance of the transistor circuit, and the current taken by \((R_{b1} + R_{b2})\) is clearly \(\delta V_b/(R_{b1} + R_{b2})\) when T1 base voltage \(V_b\) is changed by \(\delta V_b\).

![Fig. 15.1 Simple bootstrap feedback](image)

Now consider the effect of connecting link AB, assuming that \(C_b\) is very large and that \(R_{b1}, R_{b2}\) cause no appreciable loading on T2. It is easy to write a set of equations involving \(V_b, V_{out}\), etc. and this is essential in discovering some of the side effects, but the best way to see the object of the feedback is to assume that \(V_b\) is moved by \(\delta V_b\) and deduce roughly what change in input current takes place.

If the gain \(V_{out}/V_b\) is exactly unity and if \(C_b\) is so large that \(V_A = V_{out}\), then \(V_A\) will change in the same way as \(V_b\) i.e. \(\delta V_A = \delta V_b\). The change in current in \(R_{b1}\) is therefore zero, since there is no nett change across it; here \(R_{b1}\) behaves as if it were of infinite size in terms of its loading effect on the source.

**BOOTSTRAPPING**

This is the principle of bootstrap arrangements designed to increase the apparent impedance of a component: a method is found which makes that connection of the component which is remote from the source vary as nearly as possible in sympathy with the source. The apparent impedance is raised since the current taken from the source is reduced.

In an actual design of the circuit of Fig. 15.1, the gain \(V_{out}/V_b\) will in general be slightly less than unity and can be represented by \((1 - \lambda)\), where \(\lambda\) is a small fraction. The result now of an input change \(\delta V_b\) is to produce a change \(\delta V_a = (1 - \lambda)\delta V_b\) so that the current taken by \(R_{b1}\) is \(\lambda\delta V_a/R_{b1}\). The apparent resistance of \(R_{b1}\) is therefore \(\lambda R_{b1}\), which, as was shown previously, tends to infinity as \(\lambda\) tends to zero.

The apparent magnitude of an impedance is therefore multiplied, by applying bootstrap feedback, by a factor which is the inverse of the fraction by which bootstrap gain falls short of unity.

If the emitter followers of Fig. 15.1 are replaced by a circuit having a gain of more than +1, e.g. \((1 + \lambda)\), \(R_{b1}\) now appears to be \(-R_b/\lambda\). For gains slightly in excess of unity, the bootstrap therefore converts the bootstrapped component into a higher impedance of opposite sign. A capacitor would under these conditions appear to be inductive and vice versa, and a resistance becomes a negative resistance. Note that the presence of 'negative resistance' does not necessarily imply that the circuit will oscillate, since other parallel resistances (possibly the source) may produce overall positive resistance.

**D.C.-COUPLED BOOTSTRAP***

Where high input resistance in a circuit such as that of Fig. 15.1 is required down to zero frequency, \(C_b\) can be replaced by a direct coupling network. The point to remember is that \(R_{b1}\) appears to be large not only to the signal source but also to the transistor base current; if no source is connected, the temperature drift and quiescent behaviour of the circuit will be just the same as in a simple (i.e. non-bootstrap) circuit, but with \(R_{b1}\) replaced by its large bootstrapped apparent value.

For a particular transistor type, run under stated conditions, the drift/input resistance ratio at zero frequency is unaltered by bootstrapping; in this context 'zero frequency' implies the very low frequencies at which temperature changes occur.
Special Design Points for the Circuit of Fig. 15.1

The design for this circuit is affected by several considerations.

(1) As shown, only $R_{b1}$ is bootstrapped, so that other components of the input impedance, not normally important, may cause appreciable lowering of the predicted value for $Z_{in}$.

(2) For d.c. bias purposes the base resistor is $(R_{b1} + R_{b2})$, but only the $R_{b1}$ component becomes bootstrapped. Hence, an amplifier gain of 
\[
\left(1 - \frac{1}{30}\right)
\]

leads to a raising of $Z_{in}$ of only 15 times if $R_{b1} = R_{b2}$.

(3) The amplifier has to supply its normal load and $R_{b2}$ in parallel. Strictly, the bootstrapped value of $R_{b1}$ should also be included, but this will be of no significance in most circuits.

(4) When fed from a capacitive source, as shown, the bootstrap loop has a gain $V_{out}/e_s$ which always exceeds unity at a frequency
\[
f = \frac{1}{2\pi \sqrt{(C_b C_b R_{b1} R_{b2})}}
\]

(neglecting $R_b$). The gain at this frequency is given by
\[
V_{out}/e_s = \sqrt{\frac{1 + \frac{C_b R_{b1} R_{b2}}{C_b (R_{b1} + R_{b2})^2}}}
\]

which can be very large (e.g. 20 times!). The presence of resistance between $V_b$ and earth, and of $R_b$, makes little difference to this result; nor does the fact that $V_{out}/V_b$ is less than the unity value used in the calculation. These results are derived in Appendix 6.

Unfortunately, when this circuit is used for its most usual purpose, namely audio amplification from a crystal transducer, the peak usually occurs within the audio band, and its magnitude is often 2 or 3 times the mid-band gain of unity. A second unlucky point is that a designer who is being particularly careful will naturally give $C_b$ an overlarge value to ensure that the bootstrapping will be effective at low frequencies, where, indeed, the high input resistance is especially needed when using a capacitance source. As the equations show, this leads to an even higher peak than normal, and at a lower frequency.

There are four basic approaches to make this circuit usable in spite of this problem. One is to make $C_b$ less than normal design would suggest; this reduces the peak and tends not to upset low-frequency gain because of the presence of this remaining peak. Low-frequency phase response is, however, greatly affected. Another is to add a network which damps the effect: the frequency equation shows that the whole amplifier, following $C_b$, if replaced by an inductor of magnitude $R_{b1} R_{b2} C_b$, would produce the same equation. This suggests that a parallel resistance from $T_1$ base to earth would damp the peak—this works but degrades the input impedance. An alternative damping idea is to add a capacitor and parallel resistor in series with the bootstrap feedback (e.g. between points A and B), so that at very low frequencies feedback is reduced. The third method is to make $C_b$ so large that the peaking frequency is below the low frequency cut-off required by the specification. This is the usual remedy but can be very dangerous. If $C_b$ cannot be similarly increased (because it is outside the designer’s control), the peak can be extremely large. Even if peak frequency signals are not passed to the output, various troubles now arise. A change of input source by switching, or the turn-on of amplifier supplies, can result in violent saturation or cut-off in the preamplifiers for a quarter-cycle at the peak frequency, i.e. several seconds. If $R_{b1}$ and $R_{b2}$ are returned to a supply line (rather than supply common), the peculiar gain characteristic can lead to very-low-frequency oscillation through a loop which includes the supply lines. Often this effect leads to a damped oscillation, or ‘ring’, and is accepted as being the time required by the circuit to settle after switch-on; it is, however, likely to recur even after a momentary disturbance. The fourth idea is to make $R_{b1}$ as small as possible consistent with a high enough $Z_{in}$.

The point is that the peak gain depends on
\[
(C_b/C_b)[R_{b1} R_{b2} / (R_{b1} + R_{b2})^2]
\]

$(R_{b1} + R_{b2})$ is constant at a value determined by thermal drift. $C_b R_{b2}$ is constant for a given low-frequency performance, so that peak gain depends on $R_{b1}$, which should therefore be small. The snag this time is that lowering $R_{b1}$ requires more gain ($\lambda$ smaller) for a specified $Z_{in}$, and the constancy of $Z_{in}$ becomes worse, since, the nearer the gain is to unity, the greater is the effect of a slight change of gain.

**Design Procedure**

Except for the calculation of the response peak, which should be done as a matter of course with this capacity-fed circuit, design is
simple. $T_2$ operating conditions are chosen according to the load and output voltage swing, giving $R_{eb}$; $T_1$ current must exceed $I_{bo}$ for $T_2$ and be sufficiently large to ensure that $\beta_2$ is adequate. $(R_{b1} + R_{b2})$ can be determined as usual, although some advantage can be taken of the fact that $T_1$ and $T_2$ can be allowed to drift more than normal in this circuit before waveforms are affected.

The only 'bootstrap' design points are the correct splitting of $R_{b1}$ and $R_{b2}$, and the choice of $C_b$. If $R_{b1}$ is made much smaller than $R_{b2}$, then the bootstrapped value of $R_{b2}$ may be insufficiently large; on the other hand, if $R_{b2}$ is very small it adds considerably to the amplifier loading, so that its reflected value at $T_1$ base $(\approx R_{b2}/\beta_2)$ may seriously reduce the input impedance; the unwanted peak gain will also be higher than for low $R_{b1}$. A small value of $R_{b2}$ may also cause $T_2$ to cut-off on positive signal swings if the current in $R_{b2}$ is too small to supply the extra load of $R_{b2}$; then $R_{b2}$ has to be reduced and this again reduces the input impedance. The practical solution is a compromise and the optimum ratio $R_{b1}/R_{b2}$ usually lies in the region 1/4 to 3/4. The optimum as far as input impedance is concerned occurs when the bootstrapped value of $R_{b1}$ and the reflected value of $R_{b2}$ are equal as seen by the signal source, but the graph relating $Z_{in}$ to $R_{b1}/R_{b2}$; has only a flat maximum.

Having decided $R_{b1}$ and $R_{b2}$, $C_b$ is now designed to give adequate bootstrap action at the lowest frequency of interest, but is made no larger than necessary, i.e. $\omega C_b R_{b2} \approx 1$. The peak frequency and gain $V_{out}/V_b$ are now calculated, and, knowing that the gain at zero frequency is zero and the gain at medium frequencies is almost unity, the approximate response curve can be predicted. If unsatisfactory, $C_b$ or $R_{b1}/R_{b2}$ may be changed until a suitable value is found giving the smallest peak while maintaining adequate low-frequency performance.

Typical Design

To illustrate the procedure, assume the circuit is to be driven from a crystal pick-up having 500 pf capacitance and into a load of 10 kΩ, the required low-frequency cut-off being 1 kHz. Supply rails of ±10 V are available and maximum signal output voltage is 1 V peak. The current in $T_2$ must be at least $V_{out}/R_b$, i.e. 100 μA. In addition $T_2$ has to supply $R_{eb}$, which the designer can only estimate at this stage as being no less than perhaps 5 kΩ, i.e. another 200 μA.

Finally, $T_2$ must supply its own supply resistor $R_{eb}$, which for 300 μA would be 30 kΩ, requiring another 33 μA. To allow wide tolerance margins, $R_{eb}$ may be 18 kΩ, giving a supply current to $T_2$ of roughly 10/18 = 0.55 mA; ignoring $V_{be1}, V_{be2}$ and the drop in $R_{b1}$ and $R_{b2}$, the current required is still 100 μA for $R_1$, 200 μA for $R_{b2}$ and now 1/18 = 55 μA for $R_{eb}$, totalling 10/35 mA. Hence, 18 kΩ is satisfactory for $R_{eb}$. Now $R_{b1}$ current must exceed $I_{bo}$ for $T_2$ (which could be 10 μA at 50°C) and should be sufficient to give reasonably high $\beta_1$ in $T_1$. Here $R_{b1}$ is 39 kΩ.

$(R_{b1} + R_{b2})$ must be low enough to cause negligible effects due to drift. The base current of $T_1$ is highest at low temperatures in the outward direction and is roughly

$$[(V_1/R_{eb2})/β_2 + V_1/R_{eb1}]/β_1,$$

i.e. $(0.55/β_2 + 0.25)/β_1 \approx 10 \mu A$ if $β_1|_{T=25}$ is 25. The $I_{bo}$ component of base current flows inwards and can be 10 μA, so that the drift of $V_b$ is from $+10(R_{b1} + R_{b2})\ μ V$ at low temperature (0°C) to $-10(R_{b1} + R_{b2})\ μ V$ at high temperature (50°C). The tolerable drift in this circuit is determined mainly by the danger of $T_2$ cutting-off if the drift is positive; in the negative direction a drift of even 5 V would be harmless. $(R_{b1} + R_{b2})$ can therefore be 5/100 MΩ, i.e. 50 kΩ. Giving a drift from $-0.5$ to $+0.5$ V, $V_{be}$ drift affects this by another $-5$ mV/degC, giving a total drift at $T_2$ emitter of $-0.5$ to $+0.75$ V from 0 to 50°C.

The ratio $R_{b1}/R_{b2}$ can be taken as approximately unity at this stage, giving $R_{b1} = R_{b2} = 22\ kΩ$. Note that this value for $R_{b2}$ is 4 times higher than the value used in calculating $R_{eb}$, thus improving the safety margin considerably.

$Z_{in}$ must now be calculated assuming no loss in $C_b$, the first step being to find $V_{out}/V_b$. From the transistor equivalent circuit given in Part 1, $T_2$ emitter behaves like a source of e.m.f. $V_b$ with series resistance $[(1/g_{m2}) + (1/β_2 g_{m2})]$ when fed from a zero impedance source $V_b$ (which is the required condition to calculate $V_{out}/V_b$). $g_{m2}$ is approximately 50 Ω at 0-55 mA and $g_{m1}$ is 100 Ω at 0-25 mA, giving a total of about 54 Ω if $β_2$ is 25. This effective source resistance from $T_2$ emitter is loaded by $R_{eb}/R_{eb2}/R_{eb1}$, i.e. 18/10/22 ≈ 5 kΩ. Hence, $V_{out}/V_b = 5/0.05$, giving $λ = 0.01$, or $1/λ = 100$. The input impedance is therefore 100 $R_{eb} = 2.2$ MΩ, in parallel with the $r_0$ of $T_1$ ($≈ 0.5$ MΩ) and the reflected values of all loads, i.e. $(5β_2 R_{eb2})/3 = 3$ MΩ ($R_{eb2}/R_{eb1}, β_1 = β_2 = 25$) and $β_1 R_{eb1} = 1\ MΩ$. Total input
impedance is \(2\Omega/6\Omega/3\Omega/1\Omega = 260\,\Omega\), and since \(C_s = 500\,\mu\text{F}\),
the response is 3 dB down at \(\omega 500 \times 10^{-12} \times 260 \times 10^3 = 1\),
i.e. \(f = 1225\,\text{Hz.}\)

\(C_b\) would normally be calculated so as not to affect this figure,
but, as shown previously, this leads to excessive peaks in the response.
Hence, \(C_b\) should be chosen so that the input resistance is beginning
to fall at 1225 Hz; if the coupling from \(C_b\) to \(R_{b2}\) is allowed
to fall 3 dB (which is reasonable because the peak in the response
will keep the gain higher) then \(C_b\) is given by 
\[
C_b = \frac{1}{2\pi 1225 \times 22 \times 10^8} = 0.006\,\mu\text{F}.
\]

The peak must now be calculated, and, if unsatisfactory, the value
of \(C_b\), and possibly \(R_{b1}/R_{b2}\), may require to be changed.

Peak frequency
\[
f_p = \frac{1}{2\pi \sqrt{(R_{b1}R_{b2}C_sC_b)}} = \frac{1}{2\pi 22 \times 10^8 \sqrt{(500 \times 10^{-12} \times 0.006 \times 10^{-6})}}
= 4.18\,\text{kHz}.
\]

Peak gain
\[
G_p = \sqrt{\frac{1 + C_b \frac{R_{b1}R_{b2}}{C_s(\frac{R_{b1} + R_{b2})}^2}} = \sqrt{1 + 12/4} = 2
\]

This being unsatisfactory, a ratio for \(R_{b1}/R_{b2}\) of 1/4 can be tried,
giving \(R_{b1} = 10\,\Omega\), \(R_{b2} = 39\,\Omega\).

Input impedance is now \(1/6/5/3/4/1 = 233\,\Omega\), a 10 per cent
drop, which is tolerable. Low-frequency 3 dB point is given by 
\[
\omega \times 500 \times 10^{-12} \times 1 = 1, \text{ i.e. } f = 1350\,\text{Hz}.
\]

\(C_b\) is given by
\[
C_b = \frac{1}{2\pi 1350 \times 39 \times 10^8} = 0.003\,\mu\text{F}
\]

\[
f_p = \frac{1}{2\pi \sqrt{(10 \times 10^8 \times 39 \times 10^8 \times 500 \times 10^{-12} \times 0.003 \times 10^{-6})}}
= 6.58\,\text{kHz}.
\]

\[
G_p = \sqrt{1 + \frac{10 \times 39}{49 \times 49}} = \sqrt{2} = 1.4
\]

These figures show what the designer is up against—the lowering of
peak gain is relatively slight, and the peak frequency is much higher
than the lowest signal frequency.

An alternative, as indicated earlier, is to make \(C_b\) very large, so
that the peak frequency is below the signal band in the hope that
the high gain is unimportant.

If \(C_b = 1\,\mu\text{F}\), and the original choice for \(R_{b1}/R_{b2}\) is taken, then
\[
f_p = \frac{1}{2\pi 22 \times 10^8 \sqrt{(500 \times 10^{-12} \times 10^{-6})}}
= 323\,\text{Hz}.
\]

and \[
G_p = \sqrt{\left(1 + \frac{10^{-6}}{500 \times 10^{-12}}\right)} = 22.35
\]

This may well be the best solution, but the designer must be
certain that any following circuits have large attenuation at 323 Hz
and that the bootstrap stage cannot receive a step input (or an input
at 323 Hz) more than one twenty-secondth of its normal (mid-band)
overload level; in the present design this would restrict step
inputs to a few hundred millivolts.

Refinements to Fig. 15.1

As mentioned earlier, the circuit of Fig. 15.1 increases only that
part of the input resistance due to \(R_{b1}\). Evidently there are other
contributors to input resistance which now become significant.
Referring to Fig. 15.1, the base-collector resistance of \(T_1\) appears
directly in shunt with the input signal and therefore adds a parallel
component \(r_{e1}\) to the input resistance. The resistance due to \(T_1\) base
circuit is also important and is given by \(\beta_1/g_{m1} + \beta_1R_{b}\), where
\(R_{b}\) includes all \(T_1\) emitter loading. This in turn is given by
\[
R_{E1}/\beta_1/g_{m2} + \beta_2(R_{E2}/R_{b2}/R_{L})
\]

Unless further stages are added, nothing can be done to reduce the
effect of \(R_L\) or \(R_{b2}\), since in the limit the input has to supply
1/\(\beta_1\beta_2\) of the load current and \(R_{b2}\) and \(R_L\) are equally to be
considered as loads.

The influence of \(R_{E1}\) is basically due to the change of current
which takes place in it when the input varies; the input source then
has to supply 1/\(\beta_1\) of this current. If the voltage across \(R_{E1}\) is
maintained constant, this part of the input loading will therefore vanish.
Similarly, if $T_1$ collector is made to follow $T_1$ base, no current will flow in the collector–base path and its effect will also vanish. As in all bootstrapping systems, imperfection means that the improvement produced is finite; typical factors of improvement are 20–100.

Figure 15.2 shows how $R_{E1}$ and $T_1$ base-collector can be bootstrapped to raise the input impedance.

As before, the bootstrapped elements are split to enable the feedback to be applied, so that the maximum benefit is somewhat lower than might be anticipated. In design $R_L$ and $R_{E1}$ are made large enough not to overload the output, but small enough to maintain normal bias conditions on $T_1$.

![Fig. 15.2 Multiple bootstrap](image)

Note that the apparent collector–base capacitance in $T_1$ is reduced for exactly the same reason that $r_e$ is increased, namely that the current flowing into these elements due to input signal has been reduced by the bootstrap feedback.

An even simpler way to avoid the loading of $R_{E1}$ is to omit this resistor; this can be done provided that the base current for $T_2$ is always outwards, even at high temperature, when $I_{eb0}$ could exceed $I_{ES}/\beta_2$. If this is not so, then $T_1$ will cut-off. Even when this condition is met, $T_1$ emitter current may be so low that $\beta_1$ is poor. In practice $R_{E1}$ can usually be omitted if $T_2$ is silicon (low $I_{eb0}$) and $T_1$ is planar (high $\beta$ at low current).

Further improvements can be made to $Z_{in}$ if $R_L$ is increased, thus allowing $R_{E2}$ to be higher, raising not only the reflected values of these resistors but also the bootstrap gain. Although $R_L$ can normally not be changed, a further buffer stage (emitter follower) interposed between $T_2$ and $R_L$ will produce the same effect. Using this arrangement, $Z_{in}$ can be made as high as 1000 MΩ.

**Bootstrapping in Power-drive Stages**

The principle described in this section is worth bearing in mind whenever swings of voltage have to be obtained which are comparable with the supply rail voltage.

The designer’s problem in such cases is shown in Fig. 15.3, where $T_1$ collector rests at about +11 V in the quiescent state and is driven between bottoming and cut-off by $V_{in}$. With only light loading on $T_2T_3$, $V_{out}$ is therefore a square wave of maximum level about +20 V, and minimum level about +2 V.

When $R_L$ is 100 Ω, the +2 V level remains unchanged, but when $T_1$ cuts off and $T_1$ collector rises, $T_2$ emitter takes current and $T_2$ base current increases, causing a voltage drop in $R_L$. $V_{out}$ does not reach +20 V but has a maximum value governed by the equation

$$20 - \frac{R_L(V_{out(max)})}{\beta R_L} = (V_{out(max)})$$

i.e.

$$V_{out(max)} = \frac{20\beta R_L}{R_3 + \beta R_L}$$

To put this into words, the load requires a peak current of about 90 mA, implying a $T_2$ base current of about 3 mA. This would cause a drop in $R_3$ of 6-6 V, so that the required positive swing will
not be attained. This difficulty often occurs in the design of audio power output stages; in other circumstances the simplest solution is to connect $R_3$ to a more positive rail, but in audio design there is often no such rail available. It is here that bootstrapping becomes useful.

In Fig. 15.4, $R_3$ has been split and the junction bootstrapped by the output voltage through $C_3$. If $R_{3A}$ and $R_{3B}$ are each equal to 1 kΩ, then the voltage at their junction normally rests at about $+15.5$ V and $T_1$ collector at $+11$ V, the current in $R_{3B}$ being about $4.5$ mA. When $T_1$ cuts off, $T_1$ collector voltage rises, but because of the bootstrap connection and the near-unity gain of $T_2$, the voltage across $R_{3B}$ remains at $4.5$. This means that $R_{3B}$ still carries $4.5$ mA, and since $T_1$ is cut-off, this current must flow into $T_1$ base. Assuming $\beta$ is 30, as before, $T_2$ emitter current can reach $120$ mA. This cannot occur unless $V_{out}$ rises to $+23$ V (from its initial $+11$ V), and so $V_{out}$ will reach $+20$ and limit.

This circuit configuration is similar to the bootstrap sweep generator referred to previously. Because this circuit is to carry signals of each polarity from the quiescent state, $R_{3A}$ cannot here be replaced by a diode since this would load $V_{out}$ on negative swings. However, a diode can with advantage be put in series with $R_{3A}$, thus reducing the loading on $V_{out}$ on positive swings.

Although square wave input was considered above, this was only for simplicity and the principle applies equally well for sine waves. The only design points are: (1) that the quiescent voltage across $R_{3B}$

Figure 15.4: Power output circuit with bootstrap

must be such that the current in $R_{3B}$, when multiplied by $(\beta - 1)^*$ is greater than the maximum load current; (2) $C_3$ must be such that the voltage drop across it while bootstrapping is negligible compared with the quiescent voltage drop across it.

* $(\beta - 1)$ rather than $\beta$, because $R_{3B}$ is itself part of the load.
16—Prototype testing

Even the experienced designer usually finds it necessary to construct and test his designs before pronouncing them fit for production.

There are many reasons why this is necessary: first, he may wish to evaluate the performance of the circuit in certain respects which were not directly controlled in the design (e.g. a particular d.c. level which has no specific limits put on it but of which it is useful to have some knowledge); secondly, in an involved circuit it is easy to make mistakes of arithmetic or to forget the influence of a certain transistor parameter; thirdly, since the circuit as drawn cannot be achieved in practice because of the unwanted resistance, inductance, and capacitance of connections and components, it is often necessary to specify a particular layout after optimizing this by practical experiment.

The following notes are intended to help the designer who lacks practical experience to avoid the most common errors in construction and testing, and also to suggest test procedures for the various types of circuit considered in the previous sections.

GENERAL PRINCIPLES

Although failure to meet a specification (or complete failure of a circuit to operate at all) is sometimes caused by bad layout, especially at high frequencies, experience has shown that the usual cause of failure is simply that the constructed circuit does not agree with the circuit design.

It is strange that an engineer, after spending some hours in designing a circuit with great care, will often make an elementary wiring blunder, so that the circuit fails to operate. He then immediately assumes the design was faulty and wastes much time in re-checking the arithmetic (usually finding an error!); finally the wiring mistake is discovered, usually by another person who does not even understand the circuit.

This sequence is, unfortunately, quite common among all engineers and is analogous to the inability of many mathematicians to make arithmetical calculations. The simple operations of wiring and arithmetic are subconsciously regarded as being too trivial to justify much expenditure of time. This attitude, and eagerness to test the new creation, combine to produce mistakes.

In another class altogether are those wiring errors where the connections are correct but where the moving of a wire by an inch along the same conductor means the difference between success and failure. This problem is by no means limited to high-frequency circuits and is often the reason for excessive hum level or loop oscillation in stabilizer circuits and audio amplifiers. The principles described in the following sections will be found helpful in curing these troubles, but in high-frequency circuits considerable practical experience is often necessary to optimize the layout.

It would be useful if in the initial stages of testing several sets of components could be used to ensure that the design is not marginal owing to miscalculation or to component spreads outside expected tolerances. This is rarely feasible because of the time required for such tests, and the designer must usually await the first production run. However, two tests which to a large extent give the same information are the effects of temperature and supply variation.

A good general rule in testing any circuit is, then, to subject it to changes of about 10 per cent in each individual supply rail (or more if required by the specification) and to ambient temperature changes of at least 10 degC. If the circuit is designed for smaller changes in these conditions, the tests may put the unit outside its normal performance specification. This is to be expected; on the other hand, complete failure to operate under these slightly changed conditions usually indicates unsound design. A circuit so marginal in operation that a 10 degC rise or 10 per cent supply change causes failure is equally likely to fail when a different set of components is used.

Even if measurements indicate that a circuit is operating correctly, it is always advisable to check transistor operating voltages and waveform amplitudes throughout the circuit. This does not normally require accurate measurements, and a direct-coupled oscilloscope is extremely useful in checking how near to bottoming or cut-off a circuit is operating by examining d.c. + a.c. levels.
It is always important to examine signal waveforms even in a circuit which is intended to contain none, since it often happens that a d.c. stabilizer circuit will oscillate without disturbing the mean d.c. levels. Thus, normal d.c. tests would indicate correct operation and the next unit constructed could well oscillate much more violently, causing failure or excessive ripple.

When designing a circuit it is usually assumed that the d.c. power supply, often a transistor-stabilized rectified supply, will behave like a perfect battery. That is, it is assumed to have negligible internal resistance or inductance and to be purely d.c. In practice the power source may be a dry battery having appreciable internal resistance, and even if a highly stabilized transistor supply is used, the long connecting leads can have enough inductance or resistance to cause trouble; moreover, unwanted ripple may be present.

The precautions to be taken here are, first, to design the circuit so that supply resistance and ripple have little effect, usually achieved by decoupling-circuits; and secondly, to take care that prototype testing is done with various possible supply lead lengths and ripple content to confirm that these are not critical. If these factors are critical, then the circuit must be modified to accept any desired length of supply lead and ripple content.

**Failure at Switch-on or Switch-off**

Another, often infuriating, problem is transistor failure at the moment of switch-on or switch-off. The difficulty here is that no practical evidence can be obtained about the cause, except that surge voltages or currents must be responsible. Even more annoying is the circuit which sometimes survives switch-on and switch-off transients but once in a while fails; this is often missed in prototype testing but can be relied upon to show up in the entire production batch which follows.

There are two main causes of this type of transient failure. The switching on of supplies can initially cause excessive voltages or currents to flow before the intended operating conditions are reached, often dependent upon the order in which the various rails are connected. On the other hand, although the supplies themselves may cause no harm, the initial action of the circuit itself can be the cause of excessive transients.

Concerning the first of these possibilities, it is often recommended that for test purposes the supplies should be either switched simultaneously by a single on-off switch, turned on in a particular sequence known to be safe or turned on at low level and brought up smoothly. The on-off switch method is best avoided (as a cure for transistor failure) since inevitably some time interval exists between the closure of various poles of the switch and this time changes with usage. Sooner or later the interval can be so long that simultaneity is lost, and failure will recur if one is unlucky in the resulting sequence of connection.

The other methods are usually successful and can be justified when measurements have to be made to confirm quickly that the circuit is basically correct. Matters must not, however, be allowed to rest there, and since circuit modifications which will follow are likely to affect normal performance, it is unwise to take detailed accurate measurements until they are incorporated.

These switch-on-off problems are not confined to transistor failure; in many circuits capacitors receive surges of several times their running voltage at these instants; in others, switching surges cause no permanent damage but cause faulty circuit operation, especially in trigger circuits.

Curing the surge troubles is not always easy and sometimes requires a completely different approach to part of the circuit. It is naturally important to understand how the damage can be caused, and this can be done easily provided the designer understands the basic principles of CR charging circuits. (Occasionally inductance is involved but, in general, capacitors which have to change charge to reach operating level are the cause of destructive transients.)

To analyse switch-on behaviour it is assumed that each capacitor in the circuit is fully discharged, having zero volts between its plates. It is then assumed that the supply line is switched in zero time to its operating value and the transistor conditions are examined just after this value is reached. For switch-off analysis, each capacitance is assumed to be at its normal circuit operating value. The supply voltage is then removed in zero time and immediately afterwards transistor conditions are again examined. In some subtle cases switch-on can be damaging only if the capacitors are partly discharged so that there is a critical interval between switch-off and switch-on which leads to failure. Since each capacitor is likely to have its own discharge rate, the calculation required here to be confident of safety can be tedious.

One example is given here to show how even the simplest circuit
can be difficult to assess. Figure 16.1 illustrates a simple pulse-operated light relay. The transistor OCP71 is a "photo-transistor", which is basically a normal p-n-p alloy germanium with a clear envelope which enables light to reach the collector-base junction. Light here produces the same effect as heat in that it increases transistor $I_{eb0}$, and since with the base open circuit the collector current is $\beta I_{eb0}$, the collector voltage varies considerably according to light falling on the junction.

In Fig. 16.1 the light source is intended to be present whenever the unit is operating so that the collector voltage of the OCP71 is about zero (i.e. there is enough light to cause $T_1$ to saturate). $T_2$ emitter and base potentials are therefore also zero, and so is $T_3$ base. $C$ therefore has zero charge and relay A/2 is not energized, implying that its contact A1 is in the position shown.

![Fig. 16.1 Pulse-operated light relay](image)

If the light beam is interrupted, the current in $T_1$ falls below saturation level, so that $T_1$ collector potential falls and $T_2$ base and emitter fall. Since $C$ cannot immediately change its charge, $T_3$ base also falls, turning on $T_3$ and energizing relay A/2. This causes A1 to change over, holding the relay in.

Return of the light beam now results in $T_3$ cutting off but relay A/2 remains energized because of the connection of A1. Other contacts of A/2 can therefore be used to give a permanent alarm signal indicating that the beam was broken. This circuit can be used for intruder detection, critical level detection, and similar applications.

From this rather detailed account it is clear that when the supply is switched on it is essential that $T_3$ should not conduct even momentarily, since contact A1 would then turn on the relay for all time.

In a particular application the lamp and circuit were energized from the same supply and sometimes the relay would lock over at switch-on; at the other times correct action would be obtained, depending upon the distance between lamp and photo-transistor.

This apparently mysterious and random effect was readily explained by circuit examination on the lines suggested above. Assume C is not charged; now apply $-10$ V to the circuit instantaneously, and also energize the lamp. To assess the voltage build-up on C, the conditions on $T_1$ and then $T_2$ must be found. There are two possibilities here: (1) the lamp output causes $T_1$ to saturate before the $-10$ V succeeds in making $T_1$ collector negative: (2) the lamp is slow to give enough output to saturate $T_1$ immediately, so that $T_1$ collector initially falls towards $-10$ V.

![Fig. 16.2 Improved light relay](image)

It is now clear that in the event of (2) the voltage on $T_2$ base and emitter falls and this is communicated through C to $T_3$, turning on the relay. In the case of (1), however, $T_2$ emitter remains at earth potential and no trouble arises.

The problem therefore resolves itself into whether the lamp or the $-10$ V supply arrives first at $T_1$. Clearly, a supply line which is fast to rise can give trouble, especially if the lamp is distant, and unless this is recognized in the design of the supply the relay is likely to operate.

Although this is not a case of destruction caused by switching, the difficulties which arise are typical of switch-on transients. The cure in this case can be achieved either by adding a time constant to the supply line to limit its rate of increase or by a re-design of the circuit which inverts the effect of the supply and the lamp (Fig. 16.2).
The operation of this circuit is as follows. In the quiescent condition with the lamp lit and the \(-10\) V supply connected, \(T_3\) is saturated, so that \(T_1\) emitter is at almost \(-10\) V. The base current for \(T_2\), which is approximately \(10/R_3\), is designed to be sufficient to make \(T_2\) bottom, thus cutting off \(T_2\) (provided \(V_{EC\text{sat}}\) for \(T_2\) is less than the voltage needed to turn on \(T_3\)). Hence, the relay is not energized and contact \(A_1\) is in the position shown.

If the light beam is interrupted, \(T_1\) emitter rises, causing \(T_3\) base to rise above \(T_2\) emitter. \(T_2\) therefore cuts off and, provided \(R_3\) is correctly designed, \(T_3\) now bottoms, turning off the relay. Contact \(A_1\) closes and holds in the relay. When the beam is restored, \(T_1\) saturates, bringing \(T_2\) back into conduction and \(T_3\) to cut-off. However, \(A_1\) keeps the relay held in.

Thus, in normal operation, this circuit behaves in the same way as the first.

Consider now the switch-on sequence. As in the original design of Fig. 16.1, there are two possibilities: (1) the light reaches \(T_1\) before \(-10\) V is applied; (2) the lamp comes on after \(-10\) V has been applied.

In the first case \(T_1\) will saturate the instant that any voltage is applied, so that, as the \(-10\) V line rises, \(T_1\) emitter immediately takes up the potential of this line. As the line rises, \(T_2\) is therefore made to conduct because of the current from \(R_2\) and from \(C\). \(T_3\) therefore remains off and the relay de-energized, so that correct conditions are established.

In the second case \(T_1\) remains at first cut-off or slightly conducting. Application of \(-10\) V then causes \(T_2\) to saturate immediately because of the current in \(R_2\). When the lamp lights, \(T_1\) saturates, bringing \(T_2\) even harder into saturation until \(C\) becomes charged (eventually supporting about \(10\) V potential difference). Again, correct starting conditions have been obtained.

The reason for the success of this circuit change is that initial transient currents have been made to assist, rather than oppose, the desired starting conditions. In particular, the charging current required by \(C\) is in such a direction as to turn on \(T_2\) even harder whether lamp or supply line appears first.

This second circuit is fundamentally better than the modified form of the first which can be obtained by adding a time constant in the supply circuit (Fig. 16.3), since no real tolerance can be put on the effective rise time of the light source. This time depends on the applied lamp voltage, the particular lamp used, the distance between lamp and photo-transistor and the sensitivity of the photo-transistor. All the designer could do in these circumstances would be to use a value for \(R'\) and \(C'\) (Fig. 16.3) so large as to be adequate under any conditions. This does lead to a safe design, but the waiting time before correct quiescent conditions are reached is then excessive.

It is hoped that the above example illustrates the approach required when a circuit has switch-on difficulties and shows how a re-design can sometimes be the only satisfactory solution. There is therefore good reason to assess such transient behaviour as soon as the geometry of a new design is envisaged, and before working out component values.

![Fig. 16.3 Alternative method to improve light relay](image)

In addition to the above general points regarding circuit testing, a number of difficulties arise which apply to particular types of circuit, and these are dealt with below.

**Power Circuits and Stabilizers**

Special care must be taken in the construction of circuits involving high currents, because even short lengths of interconnecting wire can cause potential differences of many millivolts. If a high-gain amplifier is used in the circuit, these potential differences may be amplified, thus giving large unwanted outputs.

The main precaution to be taken is correct routing of the wiring so that although the unwanted potential differences may occur, they do not become coupled to sensitive parts of the system. A second and more obvious precaution is to choose wire of adequate gauge to reduce the magnitudes of these potentials. This second method should not be carried too far, however, since in production it may be undesirable to use really heavy wire; in fact, printed circuit track may
have to be used. The third, much quoted, precaution is to keep all leads short, again to reduce potential drops. This is even more dangerous if taken too literally, since in the great majority of cases it is better to ensure correct routing, even if this requires longer wires.

![Fig. 16.4 Power supply with load remote from reservoir capacitor](image)

In Fig. 16.4, which represents a simple unstabilized d.c. supply, a quite common method for capacitor connection is shown. The arrangement is not really satisfactory, however, because of the resistance of the capacitor connecting leads. The load voltage waveform is not then the normal sawtooth charging waveform but has added to this the peaky capacitor current waveform, as shown in Fig. 16.5.

![Fig. 16.5 Ripple waveforms for Fig. 16.4](image)

Apart from the mental worry of the designer in observing this waveform instead of the expected sawtooth, the consequences of this effect are slight, because with practical values of ripple peak current and connection resistance, the extra voltage peaks are only of the same order as the sawtooth. For example, a 250 mA supply using a 5000 μF capacitor with 50 Hz bridge rectification will have a peak-to-peak ripple given approximately by $v = it/C$, where $t$ is $\frac{4}{3}(1/50)$ sec, i.e. 10 msec. Ripple voltage is therefore $\frac{4}{3}(10 \times 10^{-3})/5 \times 10^8 \times 10^{-6}$, that is 0.5 V. If the resistance of AB and CD totals 100 mΩ (representing 10 in of typical printed circuit track), the additional ripple is 0.1 $I_{\text{peak}}$ where $I_{\text{peak}}$ is the peak charging current for C, typically 10 times the mean load current, or in this case 2.5 A. Thus, the spurious ripple is (0.1) (2.5), or 0.25 V, raising the total ripple to 0.75 V peak-to-peak instead of 0.5 V.

Real trouble, however, can occur when several circuits which are powered by such a supply are attached at various points on AB or CD. This is a very common problem with stabilizer circuits and can result in the stabilized output having a higher hum content and worse stability against load changes than the original supply!

![Fig. 16.6 Simple stabilized supply](image)

In Fig. 16.6 the actual connecting points are to be taken as if the lines represent actual wires and the stabilizer connections are deliberately placed very badly.

In the normal operation of this circuit the load voltage $V_{CD}$ is stabilized by the following action. A fraction of $V_{CD}$, namely $V_{BG}$, is applied between $T_1$ base and earth, and a stable Zener voltage $V_{AH}$ is applied between $T_1$ emitter and earth. Should any change in load or supply line cause $V_{AH}$ to differ from $V_{BG}$ (ignoring $V_{BG}$ for $T_1$), then $T_1$ collector current will change, modifying $V_{CD}$ in the right direction to restore $V_{AH}$ to equal $V_{BG}$.

For correct action therefore it is essential that $V_{BG}$ is a simple fraction of $V_{CD}$, where $V_{CD}$ is the actual voltage to be maintained constant. In the drawing of Fig. 16.6 it is evident that any potential drop along BC will change this situation. Since the wire BC carries
the ripple current of C, $V_{RC}$ could well have a peak-to-peak ripple magnitude of several millivolts and a d.c. value also of several millivolts, since all the load current passes along BC. Thus—and this is the all-important point in the understanding of these problems—if the loop performs correctly, it will cause $V_{RB}$ (not $V_{CD}$) to be held constant, so that $V_{CD}$ will contain all the unwanted d.c. and a.c. variations of $V_{RC}$.

To remedy this fault, the temptation is to shorten BC or to use thicker wire, but although this will improve the performance, the real cure is to remove the connection between $R_1$ and B and connect this wire direct to C.

The situation now is that $V_{CG}$ is a true fraction of $V_{CD}$, and so if the loop keeps $V_{CG}$ constant the output $V_{CD}$ is also constant. Further examination reveals, however, that the situation is still not ideal because $ZD_1$ is connected to 'earth' at A whereas the load and potential divider $R_1R_2$ are connected at C. The loop will therefore endeavour to maintain $V_{AH}$ equal to $V_{AC} + V_{CD}$, so that again the output is made dependent not only on the intended reference voltage but also on the ripple and d.c. value obtained along a connecting wire. The cure is again to reconnect the leads by detaching $ZD_1$ from A and reconnecting to C.

Examining now the remaining supply connections it is clear that any ripple voltage between the new zero reference point (or earth connection C) and J causes injection of ripple through $R_4$ to the base of $T_2$ and, hence, to the output. If no loop were present, the magnitude of this output ripple would be roughly equal to that existing at J, i.e. the full ripple on reservoir $C_1$. Although this effect will be reduced by the gain in the loop $T_1T_2$, it can still not be ignored, so that the ripple at J should at least not be allowed to be any larger than that of $C_1$ at F (V$_{EF}$). Resistor $R_4$ should therefore be removed from J to F, and CE should be minimized in length.

Ripple injected at K will have even less effect, but again its effect can be reduced by reconnecting to F.

The new circuit layout is shown in Fig. 16.7. It will be realized that, as suggested by the drawing, many of the wires will be greatly increased in length by these changes. This is quite permissible provided either that the currents in the long wires are negligible (RC, SC, TD, UF) or that the voltage drops have no significant effect (PC, QF).

This example has been given in considerable detail to show how simple is the reasoning behind correct layout in such cases. The sceptic finds it difficult to believe that these effects are even measurable, let alone of great significance, but a practical test on a stabilizer designed for, e.g. 30 V, 1 A with 1 mV ripple, soon shows up all these dangers, spurious drifts and ripple levels of many tens of millivolts being quite usual with wrong layout.

![Fig. 16.7 Correct routing of connections for circuit of stabilized supply](image)

**Amplifiers**

Similar problems arise in all types of amplifier but predominate where high currents are involved, as in power audio amplifiers and high-frequency amplifiers.

The same principles with regard to correct routing still apply, but in the case of high-frequency amplifiers at frequencies in the tens of megacycles there is an additional difficulty: routing has to be correct but wires must also be short. The reasons for shortness are twofold: first, stray capacitance between wires, or to ground, can cause feedback or signal loss to occur because a few picofarads is a very low impedance at these frequencies; secondly, the inductance of even a centimetre of wire has considerable reactance at high frequencies, causing loss or, worse, resonant effects. This is the basic reason why high-frequency practice still has to be regarded as an art. Only intuition and long experience will lead to the optimum arrangement of wiring, although the general principles of dealing with unwanted signal paths are still applicable.

Since amplifiers require similar treatment to the stabilizer previously discussed, only one example will be given (Fig. 16.8).

This is a simple amplifier with emitter follower output, and at first
sight there is no difficulty in layout; indeed, in most applications of this circuit little thought need be given to cable routing. If, however, this circuit is used at high frequencies or for driving heavy loads, the situation is quite different. In either case the effect of load currents is likely to increase in significance because at high frequencies the inductance of wires causes larger voltage drops, and with heavy currents the resistance causes similar large voltage drops.

In this circuit load current flows through $T_3$ emitter and collector, the return path then being along HJ, through the battery to A and back through B, D, and E to F. From the point of view of $T_1$, the signal is that which appears between its base and its emitter, i.e. the potential difference from C to E, if the losses in $C_1$ and $C_2$ are ignored. The input is, however, intended to be $V_{CB}$, so that $T_1$ has a spurious input of $V_{BE}$. Since the load current flows along BE, this spurious input can be large and may add to or subtract from the true input according to the frequency concerned. (At medium frequencies the feedback would be positive, giving increased gain and probable oscillation.)

There are several remedies. Perhaps the simplest is to remove the link AB and connect A direct to F so that no load current flows along BE. Another is to reconnect capacitor $C_3$ to B instead of E (and preferably take $R_4$ to B also). It may happen that because of some existing printed board the above changes cannot be made. A possible solution then is to add $R_5C_4$, as shown in Fig. 16.9, in $T_2$ collector circuit with $C_4$ returned direct to F. This last modification has the further advantage of avoiding load currents in the positive line and

![Fig. 16.8 Simple amplifier circuit](image)

![Fig. 16.9 Improved layout for simple amplifier circuit (Fig. 16.8)](image)

is therefore useful in protecting any other circuits, which may have to share the line, from stray effects.

The best arrangement, to be used where possible, would be a combination of all three methods, as shown in Fig. 16.9.

In more complicated amplifiers employing several stages the difficulties considered above become multiplied in number and magnitude: in multistage high-gain circuits the voltage drop caused by output load current flowing in a few inches of printed circuit track can be as large as the intended input signal to the amplifier. Since it is not always possible to avoid such a voltage drop, it is essential to use methods similar to those just described in order to isolate the critical stages.

It would clearly be impossible to illustrate remedies to suit all amplifier circuits, but the following hints should prove helpful.

The problem usually has to be approached from two directions. First, the output end of the circuit must be so arranged that the heavy currents flowing into the load and supply lines are localized and do not cause unwanted voltage drops in the more sensitive stages. Secondly, the sensitive stages should be sensitive only to the intended signal input and not to spurious voltage drops in connecting leads or track. This is easier said than done, but much can be achieved by simply ensuring that the emitter, base, and signal return paths of any one stage are each separately returned to one point in the wiring; if they have to be routed to different points along the same wire, then no other currents, especially large-signal currents, should be allowed to flow along the same path.
This latter point brings up a question which is really a circuit design rather than a layout problem, but since its understanding requires a similar approach it seems appropriate for discussion here. Referring to Fig. 16.9, the question which could have arisen in initial design is whether C2 should be returned to earth as shown, or to the negative line, or the positive line. After considering the above rules, it is obvious that 'earth' is here the correct choice, since only in this way can the emitter, base, and signal returns be made to the same point. Connection to the negative line would mean that any signal content on that line (caused for instance by the change of current which occurs in R4 when the output appears) would directly add to or subtract from the input, i.e. the true base-emitter signal voltage, which is the signal to which the transistor responds, would not be the intended input signal. The same is true of connection to the negative line.

Another, and in practice more important, point is that at switch-on with C2 connected to the negative line, and with a low-impedance source attached to the input, the current which could flow through C2 and T1 would be virtually unlimited, because initially C2 would remain with zero charge and would probably destroy T1. Conversely, with C2 connected to the positive line, switch-on would drag T1 emitter down to the negative rail, damaging T1 if its reverse base-emitter rating is less than the supply voltage.

Appendix 1

HALF-WAVE DIODE RECTIFICATION

![Half-wave rectifier circuit and input waveform](image)

Charge lost by C in one cycle \[= \frac{TV_{out}}{R} \] \(\text{(A.1)}\)

Charge gained by C in one cycle \[= \int_{-\pi/2}^{\pi/2} \frac{V_{out}}{R_s} \sin \omega t \, dt \]
\[= \left[ \frac{V_{out}}{R_s} \frac{\sin \omega t - \tau V_{out}}{\omega} \right]_{-\pi/2}^{\pi/2} \]
\[= \frac{1}{R_s} \left[ \frac{V_{out}}{\pi} \sin \frac{\pi T}{2} - \frac{T}{R_s} V_{out} \right] \] \(\text{(A.2)}\)

(Note: \(T = \frac{2\pi}{\omega}\))

Equating expressions (A.1) and (A.2)

\[V_{out} = \frac{V_{out}}{\pi} \frac{\sin (\pi T/F)}{R_s + \tau/T} \] \(\text{(A.3)}\)
Also

\[ V_{out} = V_s \cos (\pi \tau / T) \]  

directly, \hspace{1cm} (A.4)

therefore

\[ \frac{R_s}{R} = \frac{1}{\pi} \left[ \tan \frac{\pi \tau}{T} - \frac{\pi \tau}{T} \right] \]  \hspace{1cm} (A.5)

From equations (A.3) and (A.5), \( (V_{out}/V_s) \) may be plotted against \( R_s/R \) and against \( \tau/T \) (see Fig. 1.9).

Power in \( R_s \) in 1 cycle, \( P_{R_s} = \frac{1}{R_s T} \int_{-\pi/2}^{\pi/2} (V_s - V_{out})^2 \, dt \)

\[ = \frac{V_s^2}{R_s T} \left[ \frac{\pi \tau}{T} + \frac{\pi \tau}{T} \sin \frac{2 \pi \tau}{T} + \frac{\pi \tau}{T} \cos \frac{2 \pi \tau}{T} \right] \]

\[ = \frac{V_s^2}{R_s T} \left[ \frac{\pi \tau}{T} \cos \frac{2 \pi \tau}{T} - \frac{\pi \tau}{T} \sin \frac{2 \pi \tau}{T} \right] \]

Therefore

\[ P_{R_s} = \frac{V_s^2}{R_s T} \left[ \frac{\pi \tau}{T} \cos \frac{2 \pi \tau}{T} - \frac{\pi \tau}{T} \sin \frac{2 \pi \tau}{T} \right] \]

Now, power in load = \( P_L = \frac{V_{out}^2}{R} \), and \( R_s = \frac{1}{\pi} \left[ \tan \frac{\pi \tau}{T} - \frac{\pi \tau}{T} \right] \).

Therefore

\[ \frac{P_{R_s}}{P_L} = \frac{P_{R_s} R}{V_s^2} = \left( \frac{\cos (\pi \tau / T)}{\cos^2 (\pi \tau / T)} \right) \]

Full-wave (Fig. 1.7) and Bridge (Fig. 1.8) Rectifier Circuits

Assuming input isolation is needed, each circuit requires an input transformer. The transformer \( T_{R1} \) for Fig. 1.7 must deliver twice the voltage of the transformer \( T_{R2} \) for Fig. 1.8. Since each half-secondary of \( T_{R1} \) passes current only on alternate half-cycles, identical cores and bobbins may be used for \( T_{R1} \) and \( T_{R2} \), the latter using secondary wire of twice the area but half the number of turns.

For economy, Fig. 1.7 is thus preferred, since diode cost is halved.

However, in Fig. 1.8 \( R_s \) is effectively half that of Fig. 1.7 if transformer secondary resistance predominates over other contributors to source resistance.

Technically, Fig. 1.8 is then superior, having in the limit a regulation performance twice as good as that of Fig. 1.7.

Appendix 2

ANALYSIS OF BIAS CIRCUIT (FIG. 2.5)

Assuming an emitter current \( I_e \), collector current is \( (\alpha I_e + I_{cb0}) \), giving a base current (Kirchhoff's first law) of \((1 - \alpha)I_e - I_{cb0}\) in the direction shown.

Therefore

\[ V_b = R_b[(1 - \alpha)I_e - I_{cb0}] \]

Now, \( I_e = \frac{V_e - V_{be} - V_b}{R_e} \)

Therefore

\[ I_e = \frac{V_e - V_{be} - R_b[(1 - \alpha)I_e - I_{cb0}]}{R_e} \]

\[ i.e. \quad I_e = \frac{V_e - V_{be} + R_b I_{cb0}}{R_e + R_b(1 - \alpha)} \] \hspace{1cm} (A.6)

Now, \( I_e = \alpha I_e + I_{cb0} \)

Therefore

\[ I_e = \frac{\alpha(V_e - V_{be}) + I_{cb0}(R_e + R_b)}{R_e + R_b(1 - \alpha)} \] \hspace{1cm} (A.7)

and

\[ V_L = I_e R_L \]

\[ i.e. \quad V_L = \frac{R_L}{R_e + R_b(1 - \alpha)}[\alpha(V_e - V_{be}) + I_{cb0}(R_e + R_b)] \] \hspace{1cm} (A.8)

Special Cases

(a) When

\[ R_b = 0 \]

\[ I_e = \frac{V_e - V_{be}}{R_e} \] \hspace{1cm} (A.9)

\[ I_e = \alpha \frac{V_e - V_{be}}{R_e} + I_{cb0} \] \hspace{1cm} (A.10)
and \[ V_L = R_L \left[ \frac{V_e - V_{be}}{R_e} + I_{ebo} \right] \] ...(A.11)

(b) When \[ R_b \to \infty \]

\[ (1 - a)I_e - I_{ebo} = 0 \]

Therefore

\[ I_e = \frac{I_{ebo}}{1 - a} \] ...(A.12)

\[ I_e = I_e = aI_e + I_{ebo} \]

i.e.

\[ I_e = \frac{I_{ebo}}{1 - a} \] ...(A.13)

and

\[ V_L = R_L \frac{I_{ebo}}{1 - a} \] ...(A.14)

(c) When \[ R_e = 0 \]

\[ R_b(1 - a) - I_{ebo} + V_{be} = V_e \]

Therefore

\[ I_e = \frac{V_e - V_{be} + R_bI_{ebo}}{R_b(1 - a)} \] ...(A.15)

Also,

\[ I_e = aI_e + I_{ebo} \]

Therefore

\[ I_e = \frac{a(V_e - V_{be}) + R_bI_{ebo}}{R_b(1 - a)} \] ...(A.16)

and

\[ V_L = R_L \left[ \frac{a(V_e - V_{be}) + R_bI_{ebo}}{R_b(1 - a)} \right] \] ...(A.17)

Notes

Special case (a), \( R_b = 0 \). \( I_e \) is independent of \( a \), \( I_{ebo} \) and \( V_{be} \) provided that \( V_e \gg V_{be} \).

\( I_e \) somewhat dependent on \( I_{ebo} \) and \( a \): only slight dependence if \( I_e \gg I_{ebo} \) and \( a \approx 1 \).

Special case (b) \( R_b \to \infty \). \( I_e \) and \( I_{ebo} \) critically dependent on \( a \) and directly proportional to \( I_{ebo} \).

Special case (c) \( R_e = 0 \). \( I_e \) and \( I_{ebo} \) critically dependent on \( a \). Dependence on \( I_{ebo} \) is great if \( R_bI_{ebo} \gg V_e \), negligible if \( R_bI_{ebo} \ll V_e \).

\[ v_j \] may be expressed by three different equations, from which the required values of \( Z_{in} = \frac{v_j}{I_s} - Z_a \), \( Z_{out} e = \frac{\partial v_e}{\partial I_e}I_{s'}, Z_{out} e = \frac{\partial v_e}{\partial (I_e + I_c)}I_{s'} \) and \((v_c/v_s)\) may be obtained.

**Appendix 3**

**ANALYSIS OF EMITTER FOLLOWER AND GROUNDED EMITTER AMPLIFIER**

\[ v_j = -i_d(r_e + Z_a) \] ...(A.18)

\[ v_j = v_j - i_d(Z_a + r_b) \] ...(A.19)

\[ v_j = r_d[i_s + (1 - a)i_d] + (i_s + i_d)Z_L \] ...(A.20)

**Input Impedance**

From equations (A.18) and (A.19),

\[ v_j - i_d(Z_a + r_b) + i_d(r_e + Z_a) = 0 \] ...(A.21)

From equations (A.18) and (A.20)

\[ i_d(r_e + Z_a) + r_d[i_s + (1 - a)i_d] + (i_s + i_d)Z_L = 0 \]
Therefore \( i_d[r_e + Z_e + (1 - a)r_c + Z_L] + i_d[Z_e + Z_L] = 0 \) \( \ldots \) (A.22)

From equations (A.21) and (A.22)

\[
Z_{in} = \frac{\partial v_e}{\partial i_d} - Z_e = r_b + \frac{(r_e + Z_e)(r_e + Z_L)}{r_e + Z_e + Z_L + (1 - a)r_c} \ldots \text{(A.23)}
\]

If \( (Z_e + Z_L) \ll (1 - a)r_c \), \( \ldots \) (A.24)

this reduces to

\[
Z_{in} \approx r_b + \frac{r_e + Z_e}{1 - a}
\]

e.i.

\[
Z_{in} = \beta(Z_e + 1/g_m) \ldots \text{(A.25)}
\]

This of course applies for both emitter follower and earthed emitter amplifier.

**Emitter Follower Voltage Gain** \( v_e/v_s \)

From equations (A.21) and (A.22)

\[
v_e = -i_e \left[ r_e + Z_e + \frac{(Z_e + r_b)(r_e + Z_e + Z_L + (1 - a)r_c)}{r_e + Z_L} \right]
\]

Therefore \( \frac{v_e}{v_s} \) is:

\[
\frac{v_e}{v_s} = -\frac{Z_e}{r_e + Z_e + (Z_e + r_b)(r_e + Z_e + Z_L + (1 - a)r_c)} \ldots \text{(A.26)}
\]

If \( (Z_e + Z_L) \ll (1 - a)r_c \),

this reduces to

\[
\frac{v_e}{v_s} = \frac{Z_e}{r_e + Z_e + (Z_e + r_b)(1 - a) \approx Z_e + 1/g_m + Z_e/\beta} \ldots \text{(A.27)}
\]

**Emitter Follower Output Impedance**

This is given by \( v_e/i_d \) for constant \( v_s \), \( Z_e \) being varied; hence, \( \frac{\partial v_e}{\partial i_d} \) is required in a form which does not include \( Z_e \).

Now,

\[
v_e = i_d r_e + v_j \ldots \text{(A.28)}
\]

From equations (A.19) and (A.20)

\[
v_e - i_d(Z_e + r_b) = r_e i_d + (1 - a)i_d + (r_e + i_d)Z_L
\]

i.e.

\[
v_e = i_d[Z_e + r_b + r_e + Z_L] + i_d[Z_L + (1 - a)r_c] \ldots \text{(A.29)}
\]

**Earthed Emitter Amplifier Voltage Gain** \( v_e/v_s \)

From equations (A.22), (A.23) and (A.34)

\[
v_e = Z_s + r_b + [(r_e + Z_e)/(1 - a) + (r_e + Z_e + Z_L)/(r_e + Z_L)] \ldots \text{(A.30)}
\]

Therefore

\[
\frac{v_e}{v_s} = \frac{Z_L}{Z_s + r_b + (r_e + Z_e)/(r_e + Z_e + Z_L) + (1 - a)r_c + (2 - a)Z_L + r_e + Z_a}
\]

\[
\times \frac{r_e + Z_a - ar_c}{(1 - a)[r_e + (r_e + Z_e + Z_L)/(1 - a)]}
\]

If \( (Z_e + Z_L) \ll (1 - a)r_c \), this reduces to

\[
\frac{v_e}{v_s} = \frac{Z_L}{r_e + Z_e + (Z_e + r_b)(1 - a) = 1/g_m + Z_e + Z_e/\beta} \ldots \text{(A.31)}
\]

If \( Z_e = 0 \), \( Z_L > r_e \), \( \frac{v_e}{v_s} \approx \frac{Z_L}{r_b} \) (maximum voltage gain/stage).

**Earthed Emitter Amplifier Output Impedance**

This is given by

\[
-\frac{\partial v_e}{\partial (i_s + i_d)} \times v_s
\]

so that \( v_e \) is required in terms of \( (i_s + i_d) \) but without \( Z_L \), which is variable. The negative sign comes about because of the assumed direction of
Therefore
\[ Z_{\text{out} e} = \frac{I_e + Z_b + (1 - a)r_c + r_c r_e + Z_e}{Z_e + r_b} \]

\text{and if } r_e \approx (1 - a) r_b, \quad Z_{\text{out} e} \to 2 r_c \beta

The practical interpretation of these results is that \( Z_{\text{out}} \) from the collector circuit is very high \( (r_c) \) if \( R_e \) is large (which leads to low voltage gain) and moderately high \( (r_c/\beta) \) if \( R_e \) is zero. For intermediate values of \( R_e, Z_{\text{out} e} \) lies between these extremes.

\text{ANALYSIS OF EARTHED BASE AMPLIFIER}

\text{Fig. A3.2} Earthed base amplifier and equivalent circuit

For this circuit \( Z_{\text{in} e} \) and \( v_{\text{out}}/v_I \) are required. \( Z_{\text{out} e} \) is identical with the earthed emitter case, since the circuits are identical when \( v_k \) is constant.

By Kirchhoff's second law
\[ v_k = (Z_b + r_b)(I_e - i_e) \quad \text{(A.40)} \]
\[ v_J = r_c (i_k - a l_0) + i_e Z_L \quad \text{(A.41)} \]
\[ v_g = (r_e + Z_b) i_e + v_J \quad \text{(A.42)} \]

\text{Input Impedance } v_k / l_e - Z_b

From equations (A.40) and (A.41)
\[ i_e (r_e + Z_b + a r_c) = i_e (r_e + Z_b + r_c + Z_L) \quad \text{(A.43)} \]

From equations (A.40) and (A.42)
\[ v_g = i_e (Z_b + r_b + Z_b + r_b) - (r_b + Z_b) i_e \quad \text{(A.44)} \]

Therefore, from equations (A.43) and (A.44)
\[ Z_{\text{in} e} = \frac{v_g}{I_e} = Z_e = r_e + (r_b + Z_b) \frac{Z_L + (1 - a) r_c}{Z_b + r_b + r_c + Z_L} \quad \text{(A.45)} \]

Note that this result is identical with \( Z_{\text{out} e} \) for the emitter follower, provided \( Z_b \) is replaced by \( Z_e \) (see equation (A.32)). If \( Z_L \ll (1 - a) r_c \) and \( r_b + Z_b \ll r_c \), this reduces to
\[ Z_{\text{in} e} = r_e + (r_b + Z_b)(1 - a) = 1/g_m + R_b/\beta \]

\text{Voltage Gain } v_{\text{out}}/v_I

From equations (A.41) and (A.42)
\[ v_I = i_e (Z_b + r_c - a r_c) + i_e (r_c + Z_L) \quad \text{(A.46)} \]
From equations (A.43) and (A.46)

\[(r_e + Z_L)I_e = v_b - \alpha[Z_e + r_e - ar_c] - r_b + Z_b + r_e + Z_L\]

Therefore

\[v_b = I_e \frac{r_e + Z_b + r_e + Z_L}{r_e + Z_b + r_c + Z_L} \]

therefore

\[\frac{v_{out}}{v_e} = \frac{I_e Z_L}{v_e} \]

\[\frac{(r_b + Z_b + r_e + Z_L)}{(r_e + Z_b + r_c + Z_L) + (r_b + Z_b + Z_L + (1 - a)r_c)} \] (A.47)

If \(Z_L \approx (1 - a)r_c\) and \(Z_b + r_b \ll r_a\), this reduces to

\[\frac{v_{out}}{v_e} = \frac{a Z_L}{r_e + Z_e + (Z_b + r_b)(1 - a) \approx 1/|g_m + Z_e + Z_b|} \] (A.48)

Note the identity with equation (A.35) for the earthed emitter amplifier if \(Z_e\) is replaced by \(Z_b\) and \(Z_b\) by \(Z_e\).

**HIGHER-FREQUENCY T-EQUIVALENT CIRCUITS**

Although the simple T-equivalent circuit is adequate for low-frequency analysis, there are several additional parameters which become important at higher frequencies. The most significant of these are the collector depletion capacitance \(C_c\) and the change of \(a\) with frequency. The effects of these can be represented approximately by the modified equivalent circuit of Fig. A3.3.

![Fig. A3.3 Simple high-frequency T-equivalent circuit](image)

Here the collector capacitance \(C_c\) is shown in parallel with \(r_e\) and \(a\) is replaced by \(a_0(1 + j \omega_0 f_0)\) where \(a_0\) is the low-frequency value of \(a\) and \(f_0\) is the 'cut-off' frequency at which \(a\) has fallen to \(a_o/\sqrt{2}\) with an accompanying phase lag of 45 degrees.

**APPENDIX 3**

This circuit is only approximate, since \(C_c\) is more correctly connected to a point along \(r_b\), i.e. between the base and the \(r_d/r_h/r_e\) junction. Also, the frequency law for \(a\) has been assumed to behave like a single \(CR\) network in having 45 degrees phase angle when its impedance changes by \(\sqrt{2}: 1\); this is near the truth provided \(f \geq f_o\).

In general, more correct equivalent circuits are very restricted in practical application. Any increased basic accuracy they possess is offset by the difficulty in analysis, and at frequencies where the deficiences of Fig. A3.3 are noticeable the designer should be using a higher-\(f_o\) or a lower-\(C_c\) transistor.

As so often in circuit analysis, there is no point in highly accurate calculation, because \(f_o\) and \(C_c\) have large variations from one unit to another of the same type. Usually the designer will need to know the effects merely to ensure that they will be insignificant.

**HIGH-FREQUENCY ANALYSIS**

Calculations of gains and impedances using the simpler equivalent circuit are correct for Fig. A3.3, provided \(r_e\) is replaced by \(r_e(1 + j \omega_c r_c)\) and \(a\) by \(a_0(1 + j \omega_0 f_0)\). The practical difficulty of interpreting the results is now very great, and to obtain useful answers still more assumptions must be made. One is that \(Z_L \approx r_e[1 + j \omega_c r_c]\). Although this was used in the low-frequency analysis, its significance is now different, because \(r_e(1 + j \omega_c r_c)\) can well be only \(r_e/10\) at some frequency in the band of operation; however, \(\beta\) is much less than its low-frequency value \(\beta_0\) as soon as \(j \omega f_0\) rises to even a very small fraction of unity. In practice \(Z_L \ll r_e[1 + j \omega_c r_c]\) is therefore still valid for most circuits.

**Effect of Frequency on \(\beta\)**

\[\beta = \frac{\alpha}{1 - \alpha} \approx \frac{\alpha}{1 - \alpha} \]

Now,

\[a = \frac{a_0}{1 + j \omega f_0}\]

Therefore

\[\beta = \frac{a_0}{1 - a_0 + j \omega f_0} = \frac{a_0(1 - a_0)}{1 + j \omega f_0} \approx 1 + j \omega f_0\]

Hence,

\[\beta \text{ becomes } \beta_0\sqrt{2} \text{ when } f = f_0/\beta_0\]

**Effect of Frequency on \(g_m\)**

\[g_m = \frac{1}{r_e + r_b/\beta} = \frac{1}{r_e + r_b(1 + j \omega_0 f_0)/\beta_0} = \frac{1}{r_e + (r_b/\beta_0) + r_b(j \omega_0 f_0)}\]
Therefore $|g_m|$ becomes $|g_m|/\sqrt{2}$ when
\[ f = f_0 \left( \frac{1}{\beta_0} + \frac{r_e}{r_b} \right) \]

therefore $g_m$ may be written
\[ g_m = \frac{1}{\beta_0} + \frac{r_e}{r_b} \]

The following results are derived from the simplified low-frequency analysis.

**Input Impedance to Base Circuit (Z\text{in b})**
\[ Z_{\text{in b}} = \beta g_m + \beta Z_e \]
\[ = \frac{\beta_0}{1 + jf/\omega_0} \left[ \frac{1 + jf(\beta_0/\beta + r_e/r_b)}{g_m} \right] + Z_e \]
\[ = \beta_0 \frac{1 + g_m Z_e + jf(\beta_0/\beta + r_e/r_b)}{g_m(1 + jf/\omega_0)} \]

Therefore $Z_{\text{in b}} = \frac{\beta_0 Z_e}{1 + jf/\omega_0}$

which falls by $\sqrt{2}$ in magnitude, and has a phase lag of 45 degrees when
\[ f = f_0/\beta. \]

If $Z_e = 0$
\[ Z_{\text{in b}} = \beta_0 g_m \left( \frac{1 + jf(\beta_0/\beta)}{1 + jf/\omega_0} \right) \]

which falls as $f$ rises.

**Emitter Follower Gain ($v_e/v_b$)**
\[ \frac{v_e}{v_b} = \frac{Z_e}{Z_e + \beta_0 + Z_e/\beta} \]

This falls by $\sqrt{2}$ (i.e. is 3 dB down) when
\[ Z_\epsilon + \left( 1 + jf(\beta_0/\beta + r_e/r_b) \right) = \frac{Z_e}{Z_\epsilon + \left( 1 + jf(\beta_0/\beta + r_e/r_b) \right)} \]

i.e.
\[ f_3 \text{ab} = \left( f_0/\beta_0 \right) \frac{Z_e + \left( 1/g_m + Z_e/\beta_0 \right) \beta_0 + r_e/r_b}{Z_e + \left( 1/g_m + Z_e/\beta_0 \right) \beta_0 + r_e/r_b} \]

When $Z_e \gg \beta_0 + 1/g_m$, this gives $f_3 \text{ab} = f_0 Z_e g_m (1/\beta_0 + r_e/r_b)$.

However, many assumptions made hold only for $f \ll f_0$, so the only safe conclusion here is that in this case
\[ f_3 \text{ab} \approx f_0 Z_e g_m (1/\beta_0 + r_e/r_b) \] but is never $> f_0$.

If $Z_e < (1/g_m + [Z_e/\beta])$, then $f_3 \text{ab} \approx f_0/\beta_0$.

**Emitter Output Impedance ($Z_{\text{out e}}$) for emitter follower or earthed emitter amplifier**

**Emitter Input Impedance ($Z_{\text{in e}}$) for earthed base amplifier**

These two quantities are identical provided $Z_e$ in the $Z_{\text{out e}}$ formula is replaced by $Z_0$ in the $Z_{\text{in e}}$ formula.

\[ Z_{\text{in e}} = Z_{\text{out e}} = \frac{Z_0 + jf(1/\beta_0 + r_e/r_b)}{Z_0 + jf(1/\beta_0 + r_e/r_b)} \]

Therefore
\[ Z_{\text{in e}} = \frac{Z_0 + jf(1/\beta_0 + r_e/r_b)}{Z_0 + jf(1/\beta_0 + r_e/r_b)} \]

This rises by a factor $\sqrt{2}$ and has a phase lead of 45 degrees when
\[ f \approx f_0/\beta. \]

**Earthed Emitter Amplifier/Earthed Base Amplifier Voltage Gain ($v_e/v_b$)**

**Earthed emitter**

From (A.35)
\[ \frac{v_e}{v_b} = \frac{Z_L}{Z_e + 1/g_m + Z_0/\beta} \]

Therefore
\[ f_3 \text{ab} \approx \frac{Z_L + f_0 Z_e g_m (1/\beta_0 + r_e/r_b)}{Z_e + (1/g_m + [Z_e/\beta]) (1 + jf(1/\beta_0 + r_e/r_b))} \]

When $Z_e = Z_0 = 0$
\[ f_3 \text{ab} = \frac{f_0 Z_e g_m (1/\beta_0 + r_e/r_b)}{Z_e + (1/g_m + [Z_e/\beta]) (1 + jf(1/\beta_0 + r_e/r_b))} \]

When $Z_e \gg 1/g_m$ and $Z_0 = 0$
\[ f_3 \text{ab} = \frac{f_0 Z_e g_m (1/\beta_0 + r_e/r_b)}{Z_e + (1/g_m + [Z_e/\beta]) (1 + jf(1/\beta_0 + r_e/r_b))} \]

Since previously we have assumed $f \ll f_0$ for the equivalent circuit to be valid, this equation is correct only if $f_3 \text{ab} < f_0$. 

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Earthed base

As above, but $Z_e$ is now called $Z_s$.

Therefore when $Z_s = 0$ and $Z_e = 0$,

$$f_3 \approx f_0$$

When $Z_s \approx \frac{1}{\mu_m}$

$$f_3 \approx f_0 \frac{\mu_m Z_s}{\beta}, \text{ provided } f_3 \approx f_0$$

Assessing the effects of $C_c$

The above equations are derived under the assumption that $r_c/(1 + j\omega C r c)\beta Z_L$. Many cases arise where this is untrue, and the analysis becomes too unwieldy to have practical value. It is simpler to analyse as above and then assume an external emitter–collector capacitance from collector to base and an additional external base resistor. This is in many ways more correct than the circuit of Fig. A3.3, as was indicated. The value of the external resistor $r_{be x}$ can be deduced from the $I_c$ or $I_e$ against $V_{be}$ curves which give $g_m$. At high currents (i.e. near the limit for the particular device) the $r_c$ component is small and the internal $r_b$ is also small. Hence $r_{be x}$ is given by $\beta/g_m$ at high current. Alternatively it is given by the slope of the $I_b$ against $V_{be}$ curve directly, again at high current. The circuit now becomes as shown in Fig. A3.4; $C_e$ and $r_{be x}$ can now be treated as external components, $C_e$ acting as a negative feedback or ‘Miller’ capacitance and $r_{be x}$ as part of the source resistance.

Fig. A3.4 Earthed emitter amplifier with $r_{be x}$ and $C_e$

Appendix 4

Analysis of feedback amplifier

Voltage gain $v_{out}/v_{in}$

By superposition

$$v_1 = v_{out} = \frac{R R_1/(R + R_1)}{R_1 + [R R_1/(R + R_1)]} + v_{in} \frac{R R_2/(R + R_2)}{R_1 + [R R_2/(R + R_2)]} \quad \ldots \ (A.49)$$

and

$$v_1 = - \frac{v_{out}}{A} \quad \ldots \ (A.50)$$

From equations (A.49) and (A.50)

$$v_{out} = \frac{R R_1/(R + R_2)}{A + R_2 + R R_1/(R + R_1)} = \frac{R R_2/(R + R_2)}{R_1 + [R R_1/(R + R_1)]} \left[ \frac{R_2 + R R_1/(R + R_1)}{A R R_1/(R + R_1)} \right] + 1$$

$$= \frac{R_2/R_1}{1 + R_2 + R R_1/(R + R_1) \left[ A R R_1/(R + R_1) \right]} \quad \ldots \ (A.51)$$
If $R > R_1$, this reduces to

$$\frac{v_{out}}{v_{in}} = -\frac{R_2 R_1}{1 + [(R_1 + R_2)/AR_1]} \quad \ldots \text{(A.52)}$$

When $AR_1/(R_1 + R_2) \gg 1$,

$$\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1} \quad \ldots \text{(A.53)}$$

**Frequency Response**

If $A = A_0/(1 + if/f_0)$ (i.e. the 3 dB high-frequency cut-off is $f_0$), then from equation (A.51)

$$\frac{v_{out}}{v_{in}} = -\frac{R_2 R_1}{1 + [(1 + if/f_0) R_2 + RR_1/(R + R_1)]}$$

This is 3 dB down when the real and imaginary parts of the denominator are equal. (If $z = 1/(x + iy)$ then $|z| = 1/\sqrt{x^2 + y^2} = 1/\sqrt{2}(x)$

when $x = y$.) This occurs at $f = f'_0$, if $v_{out}/v_{in}$ is expressed in the form

$$\frac{v_{out}}{v_{in}} = \frac{1 + if/fo}{1 + if/fo}$$

Now $f'_0 = f_0 = \frac{A_0 R_1 R_2/(R_1 + R_2) + R_2 + RR_1/(R + R_1)}{R_2 + RR_1/(R + R_1)}$

i.e. $f'_0 = \left\{1 + A_0 \frac{RR_1/(R + R_1)}{R_2 + RR_1/(R + R_1)}\right\} f_0$

If $R \to \infty$ and $A_0 R_1/(R_1 + R_2) \gg 1$, this reduces to

$$f'_0 = f_0 \left\{A_0 R_1/(R_1 + R_2)\right\}$$

A similar result is obtained for low-frequency response, i.e. improved by a factor $A_0 R_1/(R_1 + R_2)$.

**Input Impedance**

$$Z_{in} = \frac{\partial v_{in}}{\partial v_{out}}$$

Now

$$i_{in} = \frac{v_{in} - v_1}{R_1} = \frac{v_{in} + v_{out}/A}{R_1}$$

(from equation (A.50))

Therefore, from equation (A.51)

$$i_{in} = v_{in} \left\{1 + \frac{1}{AR_1} \left\{\frac{-R_2/R_1}{1 + R_2 + RR_1/(R + R_1)}\right\}\right\}$$

$$= v_{in} \left\{\frac{A + 1 + R_2 R_1}{R_1} \left[\frac{1 + R_2 + RR_1/(R + R_1)}{AR_1/(R_1 + R_2)}\right]\right\}$$

Therefore

$$Z_{in} = \frac{\partial v_{in}}{\partial v_{out}} = R_1 + A + 1 + (R_2/R) \quad \ldots \text{(A.54)}$$

If $A + 1 + R_2/R \gg R_2/R_1 \text{ (i.e. } AR_1/R_2 \gg 1)$,

$$Z_{in} = R_1 \quad \ldots \text{(A.55)}$$

**Output Impedance when Amplifier Output Impedance is $Z_{out}$**

By superposition

$$v_1 = v_2 \frac{R_3}{R_1 + R_2} + v_{out} \frac{R_1}{R_1 + R_2} \quad \ldots \text{(A.56)}$$

and

$$v_{out} - (i_{in} + i_{out})Z_{out} = -Av_1 \quad \ldots \text{(A.57)}$$

Also

$$i_{in} = \frac{v_{in} - v_{out}}{R_1 + R_2} \quad \ldots \text{(A.58)}$$

Therefore, from equations (A.56), (A.57) and (A.58)

$$v_{out} \left[\frac{AR_1}{R_1 + R_2} + 1 + \frac{Z_{out}}{R_1 + R_2}\right] = i_{out} Z_{out} + \frac{v_{in}}{R_1 + R_2} (Z_{out} - AR_2/Z_{out})$$

Therefore output impedance =

$$\frac{\partial v_{out}}{\partial i_{out}} = \frac{Z_{out}}{1 + A[(R_1 + Z_{out})/(R_1 + R_2)]} \quad \ldots \text{(A.59)}$$

If $Z_{out} \ll R_1$ and $A \frac{R_1 + R_2}{R_1} \gg 1$, this reduces to

$$Z_{out} \approx \frac{Z_{out}}{A(R_1/(R_1 + R_2))} \to 0$$

**Positive Feedback Within a Negative Feedback Loop**

Here $A_1$ and $A_2$ are the ‘normal’ stages of the loop amplifier. $A_2$ has positive feedback by $R_4$ and the overall gain is negative in sign.
Consider the positive feedback amplifier alone. By superposition

\[
v_3 = v_2 \frac{R_4}{R_3 + R_4} + v_4 \frac{R_3}{R_3 + R_4}
\]

...(A.60)

and

\[
\frac{v_4}{A_2} = v_3
\]

...(A.61)

Therefore

\[
\frac{v_4}{v_2} = -1 + \frac{R_4}{R_3} + \frac{1}{(R_3 + R_4)/A_2 R_3}
\]

...(A.62)

Returning to the whole system,

\[
\frac{v_5}{v_1} = -A_1 A_3 \frac{v_4}{v_2} = -1 + \frac{A_1 A_3 R_4}{R_3} + \frac{1}{(R_3 + R_4)/A_2 R_3}
\]

...(A.63)

The expression for \(v_5/v_1\) may be written for \(-A\) in equations (A.49)-(A.59) for a 'normal' feedback system. In each case the value of \(A_2\) equal to \((R_3 + R_4)/R_3\) is equivalent to an infinite \(A\).

Therefore

\[
\frac{v_{out}}{v_{in}} = -\frac{R_8}{R_1}
\]

\[Z_{out} = 0\]

\[Z_{in} = R_1\]

\[f_0' \to \infty\]

However, slight departures from \(A_2 = (R_3 + R_4)/R_3\) have great effect on \(v_5/v_1\), so that \(Z_{out}\) can be negative; \(Z_{in}\) can be less or greater than \(R_1\); and \(f_0'\) in practice is not infinite but depends critically on the cut-off frequency for \(A\).

---

**Appendix 5**

**DIODE PUMP STAIRCASE GENERATOR**

After the first input pulse,

\[
\epsilon_{1(out)} = \frac{C_1}{C_1 + C_2} V_{in}
\]

...(A.64)

If, after \((n - 1)\) pulses, \(\epsilon_{(out)}\) is denoted \(\epsilon_{n-1(out)}\) then the \(n(\text{th})\) pulse increases \(\epsilon_{(out)}\) by \([C_1/(C_1 + C_2)](V_{in} - \epsilon_{n-1(out)})\).

Therefore

\[
\epsilon_{n(out)} - \epsilon_{n-1(out)} = \frac{C_1}{C_1 + C_2} (V_{in} - \epsilon_{n-1(out)})
\]

...(A.65)

Similarly

\[
\epsilon_{n-1(out)} - \epsilon_{n-2(out)} = \frac{C_1}{C_1 + C_2} (V_{in} - \epsilon_{n-2(out)})
\]

...(A.66)

Equation (A.65) may be written

\[
\epsilon_{n(out)} - \epsilon_{n-1(out)} = \frac{C_1}{C_1 + C_2} V_{in}
\]

...(A.67)

Equation (A.66) may be written

\[
\epsilon_{n-1(out)} - \epsilon_{n-2(out)} = \frac{C_1}{C_1 + C_2} V_{in}
\]

...(A.68)

This sequence may be continued until finally, when \(n = 2\),

\[
\epsilon_{1(out)} - \epsilon_{0(out)} = \frac{C_1}{C_1 + C_2} V_{in}
\]

...(A.69)
which confirms equation (A.64) provided $e_{\text{out}} = 0$, i.e. $C_2$ carries no initial charge.

By multiplying equation (A.68) by $C_2/(C_1 + C_2)$, the next equation in the sequence by $[C_2/(C_1 + C_2)]^2$ etc., and finally equation (A.69) by $[C_2/(C_1 + C_2)]^{n-1}$, the following equation for $e_{\text{in(out)}}$ is obtained by addition:

$$\frac{e_{\text{in(out)}} - e_{\text{out}}}{C_1 + C_2} = \frac{V_{\text{in}}}{C_1 + C_2} \left[ 1 + \frac{C_2}{C_1 + C_2} + \ldots + \left( \frac{C_2}{C_1 + C_2} \right)^{n-1} \right]$$

Therefore

$$e_n = e_{\text{in(out)}} \left( \frac{C_2}{C_1 + C_2} \right)^{n-1} + \frac{V_{\text{in}}}{C_1 + C_2} \left[ 1 - \left( \frac{C_2}{C_1 + C_2} \right)^n \right]$$

i.e. $e_n = V_{\text{in}}(1 - [C_2/(C_1 + C_2)]^n)$, if $e_{\text{out}} = 0$

Appendix 6

LOW FREQUENCY RESPONSE OF HIGH IMPEDANCE BOOTSTRAP CIRCUIT

In the following analysis of a nominally unity-gain amplifier, using bootstrap feedback to obtain high input impedance, three commonly encountered non-ideal conditions are considered. These are the presence of source resistance $R_s$, parallel resistance $R$ from amplifier input to earth (i.e. any un-bootstrapped component), and non-unity gain in the amplifier (i.e. imperfect bootstrapping).

For normal conditions it is shown that these have little effect on either the frequency or the gain at the response peak.

![Fig. A6.1](image)

Fig. A6.1 Amplifier with bootstrap feedback

Note that it is inadmissible to state that $C_s$ and $C_b$ will be made so large as to be unimportant over the signal frequency band, since however large they are there is always a frequency at which a peak occurs. Even though this frequency may be outside the intended signal band, it can still cause trouble by overloading following stages after an input transient, caused by switch-on or changing the input source. In an extreme case where $C_s$ and $C_b$ are much larger than normal design would dictate, and where $C_b/C_s$ is also large, an amplifier intended for audio amplification can block for several seconds after an input transient.
Analysis

The current in \( R_b2 \), namely \( v_1/R_b2 \), is the sum of two currents, one from \( R_b1 \), the other from \( C_b \).

Hence,
\[
\frac{v_1}{R_b2} = \left[ \frac{v_b - v_1}{R_b1} + \left( 1 - \delta \right) v_b - v_1 \right] j\omega C_b
\]

Therefore
\[
v_b R_b2 \left[ 1 + j\omega C_b \left( 1 - \delta \right) R_b1 \right] = v_1 \left( R_b1 + R_b2 \left[ 1 + j\omega C_b R_b1 \right] \right)
\]

Now, \( v_b \) is given by
\[
v_b = R_b R
\]

and also by
\[
v_b - v_1 = R_b1 \left( is - i_0 \right)
\]
\[
= R_b1 \left( is - \frac{v_b}{R} \right)
\]

Therefore
\[
v_b \left[ 1 + \frac{R_b1}{R} \right] = v_1 + R_b1 i_0 \quad \text{...(A.70)}
\]

Therefore, from equations (A.70) and (A.71)
\[
v_b \left[ 1 + \frac{R_b1}{R} - \frac{R_b01 \left[ 1 + j\omega C_b R_b1 \left( 1 - \delta \right) \right]}{R_b01 + R_b2 \left[ 1 + j\omega C_b R_b1 \right]} \right] = i_0 R_b1 \quad \text{...(A.72)}
\]

Now,
\[
v_b = i_b \left( is + \frac{1}{j\omega C_b} \right) + v_b \quad \text{...(A.73)}
\]

Therefore, from equations (A.72) and (A.73)
\[
v_b = \frac{v_{out}}{v_s} \left[ 1 + \left( \frac{R_b + \frac{1}{j\omega C_s}}{R} \right) - \frac{R_b \left[ 1 + j\omega C_b R_b1 \left( 1 - \delta \right) \right]}{R_b1 + R_b2 \left[ 1 + j\omega C_b R_b1 \right]} \right]
\]

Therefore \( G = \frac{v_{out}}{v_s} = \frac{(1 - \delta) v_b}{v_s} \quad \text{...(A.74)}\)

\[
= \frac{1 - \delta}{1 + \left( \frac{R_b + \frac{1}{j\omega C_s}}{R} \right) - \frac{R_b \left[ 1 + j\omega C_b R_b1 \left( 1 - \delta \right) \right]}{R_b1 + R_b2 \left[ 1 + j\omega C_b R_b1 \right]}}
\]

or
\[
\omega = \frac{1 + \frac{R_b1 + R_b2}{R}}{\sqrt{C_s C_b R_b2 \left[ R_b1 \left( 1 + \frac{R_b}{R} \right) + R_b \delta \right]}} \quad \text{...(A.76)}
\]

For this value of \( \omega \), \( |G|_{\text{max}} \) is given by, from equation (A.75),
\[
|G|_{\text{max}} = (1 - \delta) \left( \frac{(R_b1 + R_b2)^2 + \left[ 1 + \left( \frac{R_b1 + R_b2}{R} \right) C_s R_b2 R_b1 \delta \right]}{R_b1 + R_b2} \right) \frac{R_b \left[ 1 + j\omega C_b R_b1 \left( 1 - \delta \right) \right]}{R_b1 + R_b2 \left[ 1 + j\omega C_b R_b1 \right]}
\]

In the typical case where
\[
R \gg R_b1 + R_b2 \quad \text{(a)}
\]
\[
R \gg R_s \quad \text{(b)}
\]
\[
R_\delta \ll R_b1 \quad \text{(c)}
\]
and
\[
\delta \ll 1 \quad \text{(d)}
\]

equation (A.76) reduces to \( \omega = \frac{1}{\sqrt{C_s C_b R_b2 R_b2}} \quad \text{...(A.77)} \)
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\[ |G|_{\text{max}} = \sqrt{(R_{b1} + R_{b2})^2 + \left|\frac{C_b}{C_a}\right| R_{b1} R_{b2}} \]

Therefore

\[ |G|_{\text{max}} = \sqrt{\left|\frac{C_b}{C_a}\right| R_{b1} R_{b2} \left(\frac{R_{b1} + R_{b2}}{R_{b1}}\right)^2} \]

...(A.78)

These simplified results are combined in Fig. A6.2.

Fig. A6.2 Frequency response of bootstrap circuit

All the assumptions are valid in normal use.

(a) Implies that spurious parallel components of input impedance, which are not bootstrapped, are to be much greater than the total value of the base resistors \( R_{b1} \) and \( R_{b2} \); this would usually be true since there would otherwise be little point in bootstrapping \( R_{b1} \).

(b) Implies that the direct attenuation caused by source resistance and spurious parallel \( R \) is negligible.

(c) Further implies that the attenuation caused by the source resistance coupled to the bootstrapped value of \( R_{b3} \) is negligible.

(b) and (c) are always true when the bootstrap circuit is intended to give approximately unity-gain and is therefore designed so as not to load \( R_a \) appreciably.

(d) Means that the bootstrap feedback is nearly unity, a necessary condition to make the circuit effective.

The assumption made that \(|G|\) is maximum when the second denominator term in equation (A.75) vanishes is valid unless \( C_b/C_a \) is very much greater than unity. In such cases the second numerator term shifts the peak frequency; this effect is usually negligible.

**Inductance Analogy**

The reason for the peculiar response of this circuit (Fig. A6.1) can be seen by considering the result of a step input. This step appears at the amplifier input \((v_b)\), at the output \((v_{out})\), and also at \(v_1\). Thus initially there is no change of current in \( R_{b1} \). While the step remains at \(v_1\) and (assuming only slow discharge of \( C_1 \)) at \(v_b\), \( C_b \) begins to discharge, so that \(v_1\) returns towards its original potential. The current in \( R_{b1} \) therefore increases exponentially and finally becomes \(1/e\) times its original value. (This is not strictly true, since by this time \( C_b \) has discharged thus modifying the exponential.)

The above sequence shows that the current in \( R_{b1} \) begins at a low value and steadily increases, which is similar to the behaviour of an inductance. At the frequency where \( C_a \) resonates with this inductance there will therefore be a peak in the response and the peak frequency will be given by an equation of the form

\[ f = \frac{1}{2\pi\sqrt{L/C_a}} \]

Examination of the results of the analysis shows that the equivalent \( L \) is approximately \( R_{b1} R_{b2} C_a \).

If the equivalent inductance is assumed to have internal series resistance \( r \), then simple sine wave analysis shows that the gain at the resonant frequency is

\[ \sqrt{1 + \frac{C_b R_{b1} R_{b2}}{C_a r^2}} \]

This is identical with the simplified result of the main analysis if \( r = (R_{b1} + R_{b2}) \).

**Analysis of \( LC \) series circuit**

\[
\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{j\omega L + r}{r + j(\omega L - 1/\omega C_a)}
\]

Therefore

\[
\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = \frac{\sqrt{\left(\omega L + r \right)^2}}{\sqrt{\left(\omega L^2 + (\omega L - 1/\omega C_a)^2\right)}}
\]

At resonance \( \omega L = 1/(\omega C_a) \)

\[
\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = \frac{1 + \omega L^2}{\sqrt{\omega L^2}} = \sqrt{\frac{1}{1 + \frac{L}{C_a^2}}}
\]

If

\[
L = R_{b1} R_{b2} C_a
\]

and

\[
\left|\frac{v_{\text{out}}}{v_{\text{in}}}\right| = \sqrt{1 + \frac{C_b R_{b1} R_{b2}}{C_a R_{b1} R_{b2} C_a (R_{b1} + R_{b2})^2}}
\]
For frequency calculations near the response peak the circuit to the right of \( C_8 \) in Fig. A6.1 can therefore be represented by an inductance of \( R_{b1}R_{w}C_8 \) and series resistance of \( (R_{b1} + R_{w}) \) to earth, as shown in Fig. A6.3.

![Fig. A6.3 Equivalent circuit of amplifier with bootstrap feedback (Fig. A6.1) for approximate analysis near response peak](image)

Note that for a given low-frequency limit at which normal bootstrapping is to be effective, \( C_8R_{w} \) will be constant, and that \( (R_{b1} + R_{w}) \) will be determined by bias conditions and will also be constant. The effective \( L \) is therefore proportional to \( R_{b1} \) and the ratio \( R_{b1}/R_{w} \) should be as small as possible consistent with \( R_{b1}/8 \) representing sufficiently high input impedance.

Transistor data

For details of specific transistor types it is advisable to consult the appropriate manufacturer who is always pleased to supply up-to-date information.

Most of the symbols used for transistor parameters are defined by the manufacturer but the terms \( f_0, f_a, f_{b1}, f_r, \) and \( f_{b1}, f_h, f_{h1e} \) often cause confusion. All are concerned with the behaviour of current gain at high frequencies.

\( f_0 = f_a = f_{h1b} \), often called the '\( a \)' cut-off frequency', is the frequency at which \( a \), known as \( h_{f1b} \) in \( h \) parameters, has fallen by 3 dB from its low frequency value.

\( f_b = f_{h1e} \), often called the '\( b \)' cut-off frequency', is the frequency at which \( b \), or \( h_{f1e} \), has fallen by 3 dB from its low frequency value.

\( f_r \), known as the 'gain-bandwidth product' is the product of \( b \) and the frequency at which it is measured, provided this frequency is much higher than \( f_b \). A simpler but less flexible definition of \( f_r \) is the frequency at which \( b \) has fallen to unity.

Relative Magnitude of \( f_a, f_b, \) and \( f_r \)

It is easy to calculate \( f_0 \) in terms of \( f_a \) from the equations \( \beta = \frac{\alpha}{1 - \alpha} \)

and \( \alpha = \frac{\alpha_0}{1 + J(f/f_0)} \). For normal cases where \( \beta \gg 1 \) this gives \( f_0 = \frac{1}{\beta} f_a \).

Similarly \( f_r \approx f_a \).

Practical Uses of \( f_a, f_b, \) and \( f_r \)

As shown in Appendix 3, the gain at high frequencies of an earthed emitter amplifier is 3 dB down compared with its low frequency gain.
according to the following criteria, assuming that collector capacitance is negligible:

(a) Emitter fully decoupled, 3 dB down when \( f \approx f_b \approx \frac{1}{\beta} f_a \approx \frac{1}{\beta} f_T. \)

(b) Emitter undecoupled with \( R_s \gg r_e, \) 3 dB down when \( f \approx f_a \approx \beta f_a \approx f_T. \)

For example, an amplifier using a transistor with \( f_T = 0.5 \) MHz and typical \( \beta \) of 50 would be 3 dB down at 10 kHz if fully decoupled, but 3 dB down at 0.5 MHz if an emitter load of a few kilohms were present. This is approximate and assumes that the collector load does not exceed a few kilohms. Note that there is no implication that the gain at, say, 100 kHz falls as a result of decoupling the emitter; it does in fact rise but the gain at, say, 5 kHz, rises very much more.

Use of \( f_T \)

In circuit equations \( f_a \) and \( f_b \) appear but \( f_T \) rarely occurs. The reason for its use in manufacturers’ data is the relative ease with which it may be measured especially where frequencies in the GHz region are involved. Realistic comparisons between transistors are readily made and highly stable test frequencies are not required. Since \( \beta \) is measured at a high frequency where its law is well defined \( (\beta = f_T f) \) rather than at a 3 dB point repeatability is good.

TRANSISTOR RATINGS

Most transistor ratings are easily understood provided the manufacturers’ definitions are clear. It is important which collector voltage rating is stated since its permissible value depends on whether the base is open circuit, the emitter open circuit or the base emitter junction reverse biased. The symbols used for these cases are usually \( BV_{ces}, BV_{cbo}, \) and \( BV_{crr}. \)

Power ratings are often quoted in a confusing manner. The statement that a transistor can withstand 90 W at 25°C case temperature is, by itself, useless information since the user would have to possess an infinite heat sink in an ambient temperature of 25°C to hold the case at this temperature. A statement of permissible dissipation in free air (i.e. without heat sink) is more practical but is still insufficient unless the actual air temperature coincides with that quoted on the data sheet.

To arrive at practical figures the designer needs to know firstly that the number of watts flowing through a body of thermal conductivity \( \theta \) degC/W causes a temperature difference across the body of \( W/\theta \) degrees, and secondly that the temperature of a transistor junction must not exceed a certain figure \( T_{j(max)} \) typically 90 to 100°C for germanium and 150 to 200°C for silicon.

Taking a practical example, assume that a power transistor has a thermal conductivity from junction to case \( (\theta_{ja}) \) of 1.5 degC/W and a \( T_{j(max)} \) of 90°C. This is to be bolted to a heat sink using a mica insulating washer of thermal conductivity \( \theta_{ha} \) (case-to-heat sink) of 0.5 degC/W. The heat sink has a thermal conductivity \( \theta_{ha} \) (heat sink to air) of 2 degC/W.

In this example, three values of \( \theta \) are involved and these are simply added to give the combined \( \theta \) from junction to air \( \theta_{ja} \) of \( 1.5 + 0.5 + 2 \), namely 4 degC/W.

If the maximum air temperature in which the equipment has to operate is, for example, 50°C, then a drop between junction and air of \( T_{j(max)} - T_a = 90 - 50 = 40 \) degC can be allowed. Since \( \theta_{ja} \) is 4 degC/W, 10 W is the safe limit for transistor power dissipation. Conversely if 15 W has to be dissipated, a temperature drop of \( 15 \times 4 = 60 \) degC will occur and the maximum safe ambient temperature is 30°C.

By using this thermodynamic equivalent of Ohm’s law the required figures can be derived even if not given explicitly in the data.
Although many of the following do not deal directly with transistor circuits, they help the designer by their approach to related design problems. Littauer's work is particularly well written.


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